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Quantum Faraday Effect in Aharonov-Bohm Loops

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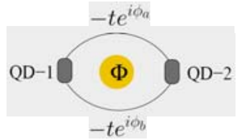

Reference: *KK, arXiv:1102.5261*

Gwangju, Korea

A Paradox?

The following two (well accepted) statements look contradictory:

1. The wave function of an Aharonov-Bohm (AB) ring is arbitrary

   (Its local phase factor depends on the choice of gauge).

2. Wave function (density matrix, in general) of a system can be reconstructed by the quantum state tomography (QST).

What happens if we try to reconstruct the wave function of an AB ring by the QST?

Outline

- Backgrounds; a paradox
- "Faraday" Phase Shift in Double-Dot Aharonov-Bohm Loop
  - for a Fast Switching of the Flux
  - for an Adiabatic Switching of the Flux
- Conclusion

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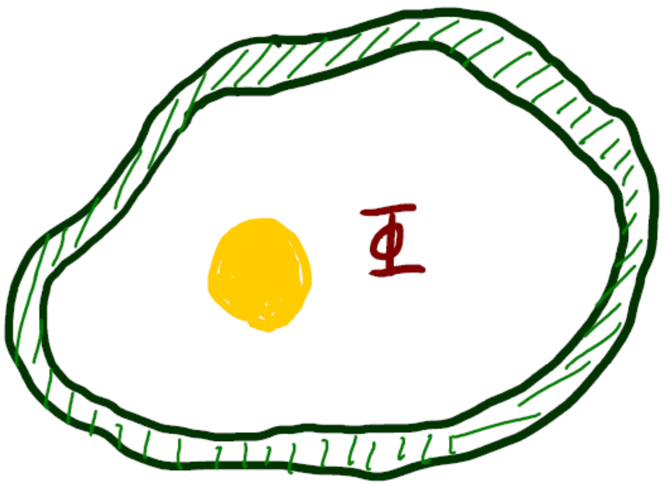
Aharonov-Bohm loop (at equilibrium)



$$\oint \mathbf{A} \cdot d\mathbf{l} = \Phi$$

- The problem is invariant under the gauge transformation:

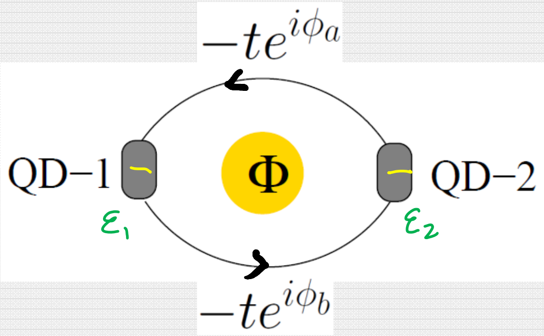
$\vec{B} = \nabla \times \vec{A}$

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\chi,$$
$$\psi \to \psi' = \psi e^{ie\chi/\hbar c}$$

$\chi = \chi(\vec{r})$: arbitrary, single-valued

- Any physical quantity is periodic in Φ with period Φ_0 ($= hc/e$) *(Byers-Yang's theorem)*
- Local phase factor of $\psi(\mathbf{r})$ is arbitrary

Double-dot Aharonov-Bohm loop



$$\phi_a + \phi_b = \phi \; (= 2\pi\Phi/\Phi_0)$$

$$H = \begin{pmatrix} \varepsilon_1 & t_\phi \\ t_\phi^* & \varepsilon_2 \end{pmatrix}$$

$$t_\phi = -2t\cos(\phi/2)e^{i(\phi_a-\phi_b)/2}$$

: gauge-dependent, no Φ_0 -periodicity

Pseudo-spin representation

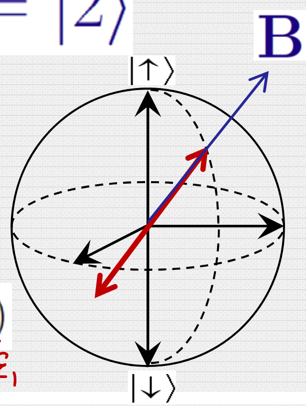
$$|\uparrow\rangle = |1\rangle, \quad |\downarrow\rangle = |2\rangle$$

$$H = -\frac{1}{2}\vec{\sigma}\cdot\mathbf{B}$$

gauge-dep.

$$\mathbf{B} = (-2\mathrm{Re}(t_\phi), 2\mathrm{Im}(t_\phi), \Delta\varepsilon)$$

$\varepsilon_2 - \varepsilon_1$



Eigenstates are gauge-dependent

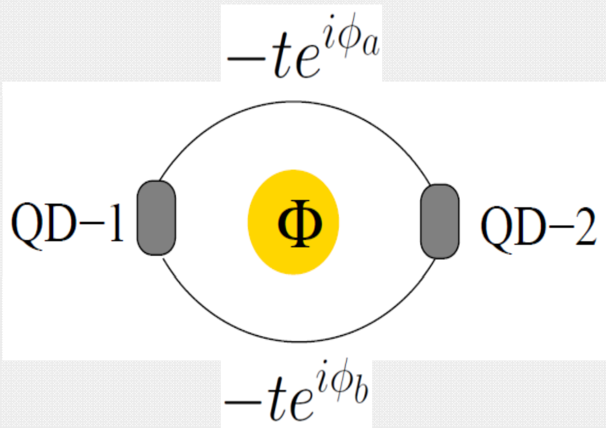
$$|\pm\rangle = \alpha_\pm|\uparrow\rangle + \beta_\pm|\downarrow\rangle$$

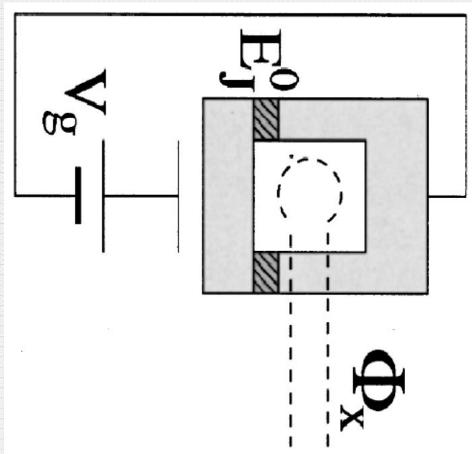
$$\alpha_\pm = \frac{t_\phi}{(\varepsilon_1 - E_\pm)^2 + |t_\phi|^2}, \quad \beta_\pm = -\frac{\varepsilon_1 - E_\pm}{(\varepsilon_1 - E_\pm)^2 + |t_\phi|^2}$$

$\alpha_\pm/\beta_\pm$: gauge - dependent

Josephson charge qubit with a flux

(*equivalent to a double-dot loop*)





Makhlin, Schon, & Shnirman (1999)

2 QD levels	$\Leftrightarrow$	2 charge (Cooper pair) states
Tunnel coupling	$\Leftrightarrow$	Josephson coupling
$\Phi_0 = hc/e$	$\Leftrightarrow$	$\Phi_s = hc/2e$

Quantum state tomography (QST)?

"*is the process of reconstructing the quantum state (density matrix) for a source of quantum systems by (ensemble) measurements on the systems coming from the source*."

Tomography [Greek]

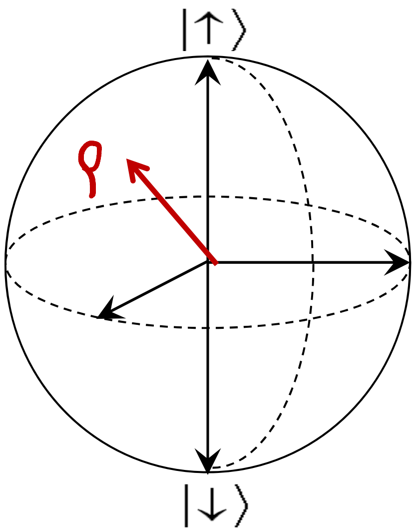
= tomos (slice) + grapherin (to write)

: imaging by sectioning

Cf. X-ray computed tomography (CT)

Image taken from "Wikipedia"

Quantum state tomography (of a qubit)



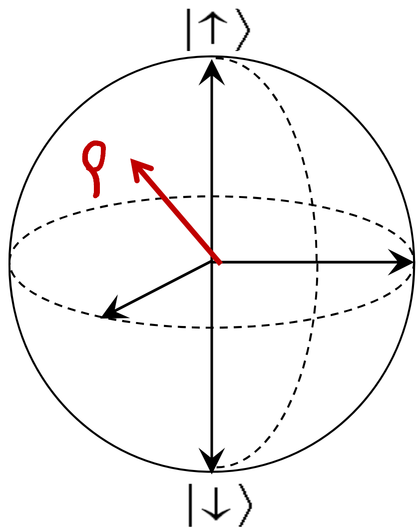
$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \frac{1}{2}\sum_{k=0}^{3} a_k \sigma_k$$

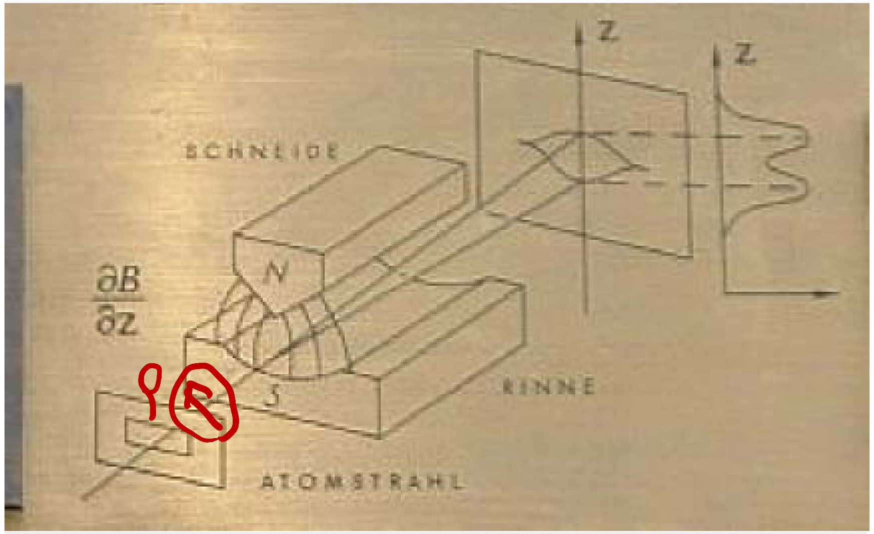
$$\longrightarrow \quad a_k = \mathrm{Tr}(\rho\sigma_k) = \langle \sigma_k \rangle$$

$$(\mathrm{Tr}(\sigma_i\sigma_j) = 2\delta_{ij})$$

- Density matrix of a quantum system can be reconstructed by measurement of $\langle\sigma_1\rangle, \langle\sigma_2\rangle, \langle\sigma_3\rangle$

Quantum state tomography (of a qubit)





Stern-Gelach-like experiment:

- $\langle\sigma_1\rangle, \langle\sigma_2\rangle, \langle\sigma_3\rangle$ can be measured by three different choices of measurement axis

- Individual measurements collapse the quantum state
- Many identical copies are needed to reconstruct the state

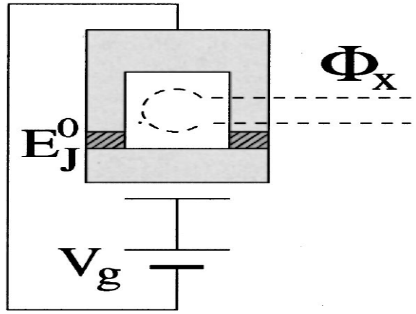
Quantum state tomography (of a qubit)

QST for a Joshepson charge qubit with AB flux

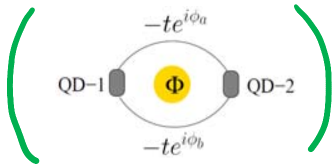
(Liu et al., PRB (2005))

- Charge detection → $\langle\sigma_3\rangle$

- Pseudospin rotation + charge detection → $\langle\sigma_1\rangle, \langle\sigma_2\rangle$
  (involves voltage and flux switching)



or



However, what is measured when one tries to measure something that is arbitrary (density matrix of an AB loop) ?

Outline

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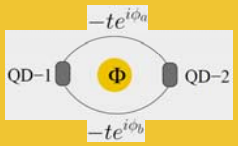


A flux-switching & charge oscillation

- *to investigate the flux dependence of the state*

<u>The procedure of the experiment:</u>

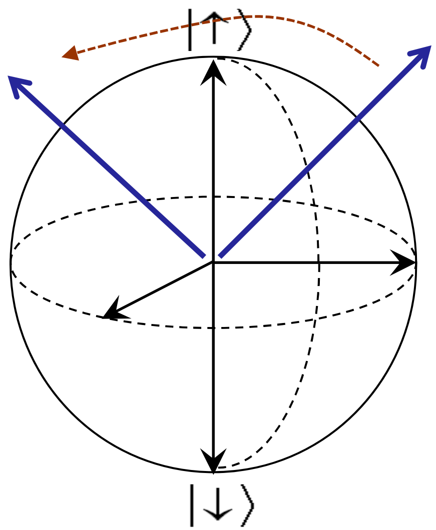
1. Prepare an initial ground state at

   $\Phi = \Phi_i$

2. Sudden switching of the flux

   $\Phi = \Phi_i \rightarrow \Phi_f \; (=0)$

3. Measure the time-dependent charge at one of the QDs.



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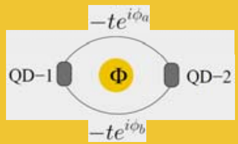


A flux-switching & charge oscillation

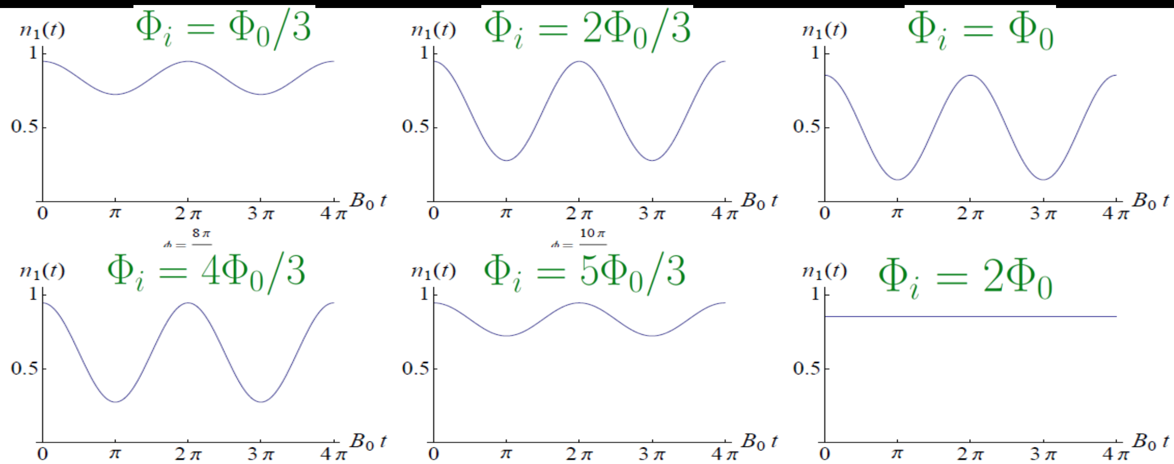
$\Phi = \Phi_i \rightarrow 0$

Ave. el. number of QD-1

(for the symmetric gauge)

($\phi_a = \phi_b = \phi/2$)

: 4π periodicity

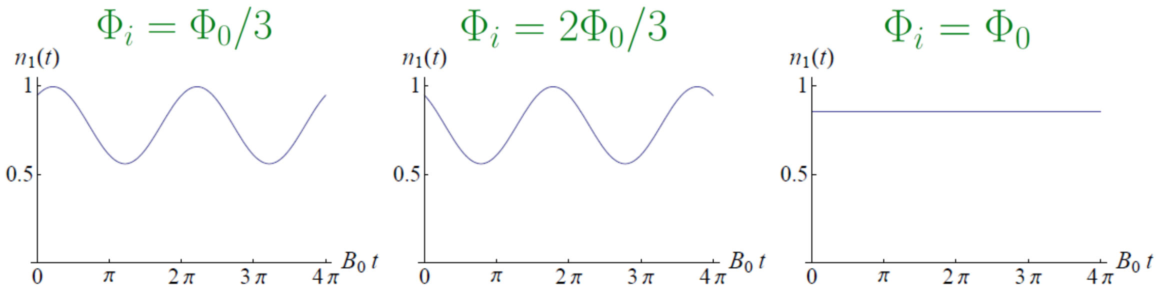


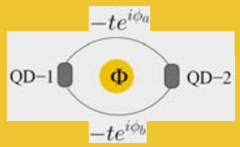
$$t_\phi = -2t\cos(\phi/2)e^{i(\phi_a-\phi_b)/2}$$

Ave. el. number of QD-1

(for the "$\phi_b = 0$" gauge)

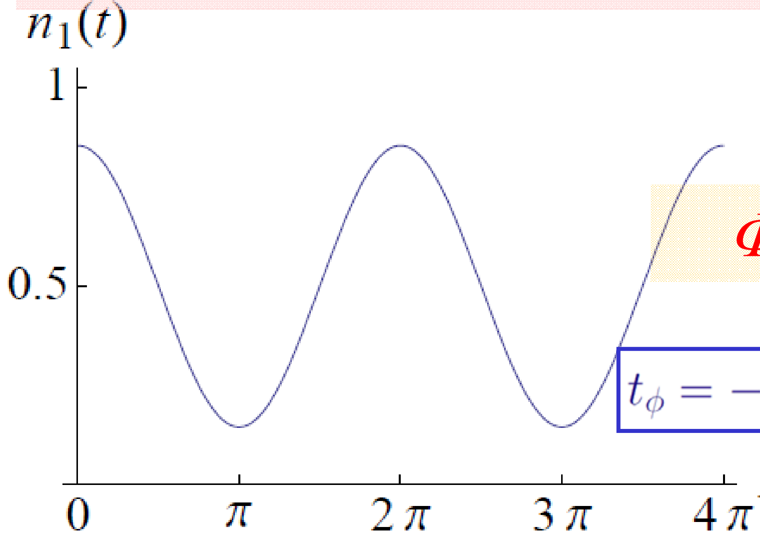
: 2π periodicity



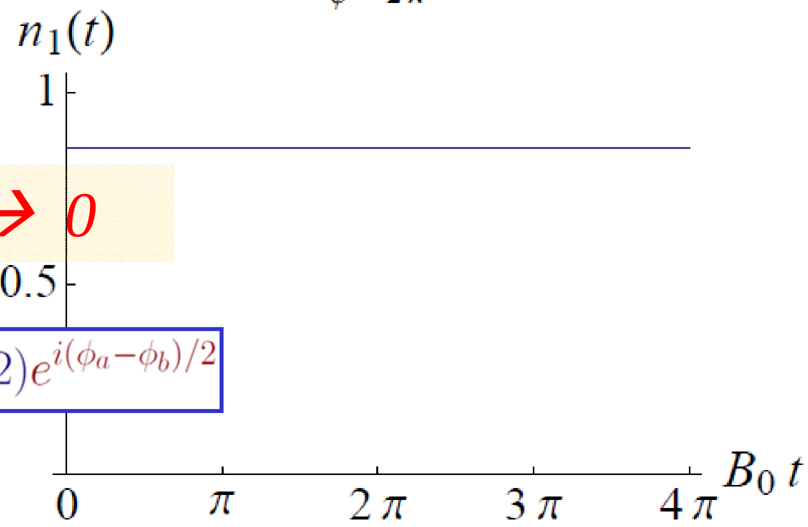


A flux-switching & charge oscillation

For the symmetric gauge ($\phi_a = \phi_b$)



For the "$\phi_b = 0$" gauge



$\Phi = \Phi_0 \rightarrow 0$

$$t_\phi = -2t\cos(\phi/2)e^{i(\phi_a-\phi_b)/2}$$

In general, it gives an arbitrary result depending on the gauge :<

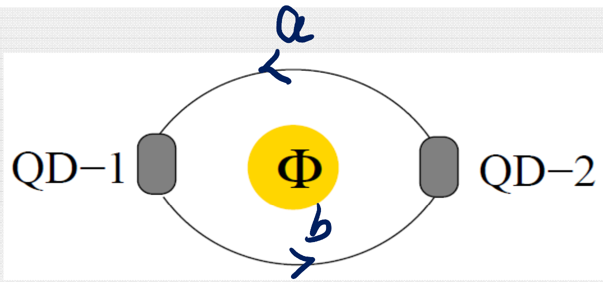
What about the gauge invariance?

- *Φ(t) is not enough* to decide the physics of this problem

Faraday effect:
additional constraint on the gauge

A gauge should be chosen to give the correct inductive field:

$$\mathbf{E} = -\nabla V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$



Time-dependent part of E-field

$$\int_a \mathbf{E}\cdot d\mathbf{r} = -\frac{1}{c}\frac{\partial}{\partial t}\int_a \mathbf{A}\cdot d\mathbf{r} = -\frac{1}{c}\frac{\partial \Phi_a}{\partial t}$$
$$\int_b \mathbf{E}\cdot d\mathbf{r} = -\frac{1}{c}\frac{\partial}{\partial t}\int_b \mathbf{A}\cdot d\mathbf{r} = -\frac{1}{c}\frac{\partial \Phi_b}{\partial t}$$

For e.g., a symmetric ring with circular symmetric flux satisfies:

$$\delta\Phi_a = \delta\Phi_b = \delta\Phi/2$$

$$(\Phi_a + \Phi_b = \Phi)$$

A flux-switching & charge oscillation

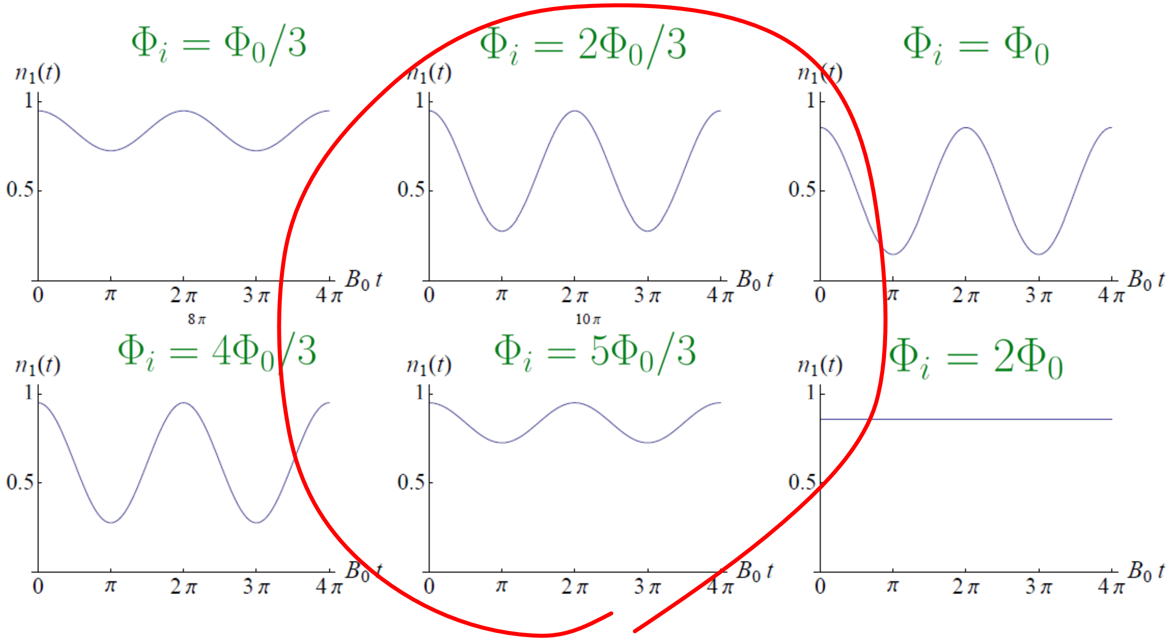
$\Phi = \Phi_i \rightarrow 0$ (for a symmetric ring)

For the symmetric gauge

($\phi_a = \phi_b = \phi/2$)

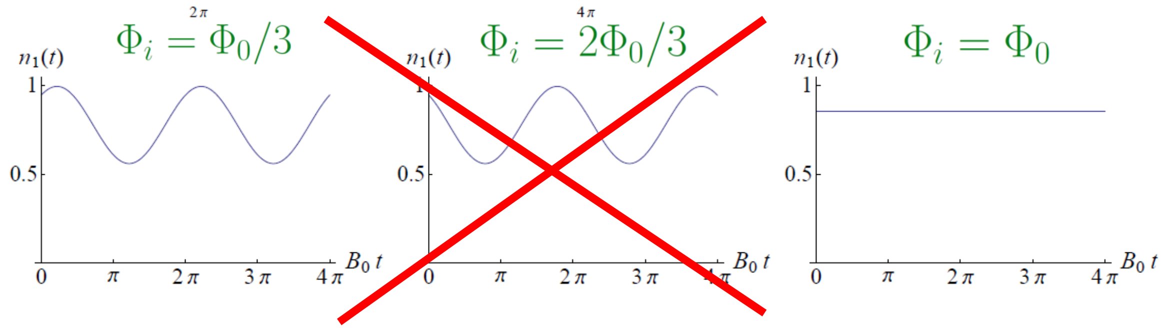
: *4π periodicity*

$$t_\phi = -2t\cos(\phi/2)$$

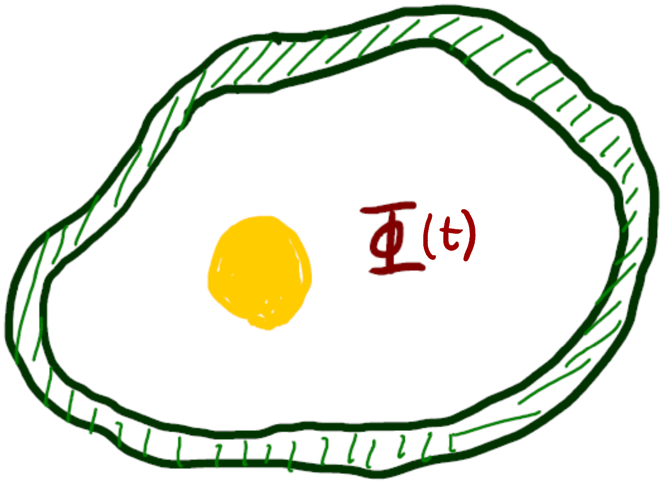


For the "$\phi_b = 0$" gauge

: *2π periodicity*



Faraday-induced phase



$$\mathbf{A} = \mathbf{A}_i \rightarrow \mathbf{A}_i + \Delta\mathbf{A}$$

Faraday-induced momentum kick

$$\Delta\mathbf{p} = e\int \mathbf{E}\,dt = -\frac{e}{c}\Delta\mathbf{A}$$

Faraday-induced phase shift (local)

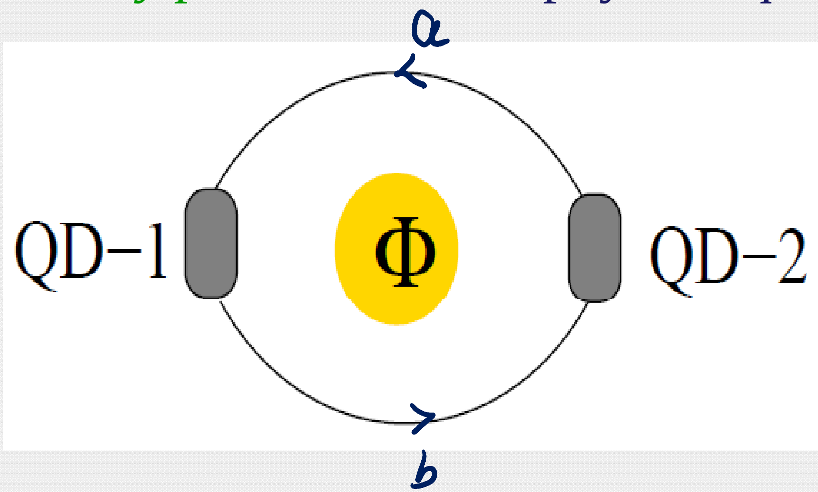
$$\phi_{Fa}(\mathbf{r}) = \frac{1}{\hbar}\Delta\mathbf{p}\cdot\mathbf{r} = -\frac{e}{\hbar c}\Delta\mathbf{A}(\mathbf{r})\cdot\mathbf{r}$$

* For one loop:

$$\delta\phi_{Fa} = -2\pi\Delta\Phi/\Phi_0$$ (= - (*change of the AB phase*))

Faraday-induced phase

In general, local "*Faraday phase*" is also a physical quantity (gauge-invariant):



$$\delta\phi_{Fa}(\text{path a}) = \delta\phi_{Fa}(\text{path b}) = -\pi\Delta\Phi/\Phi_0$$

(for a symmetric double-dot ring)

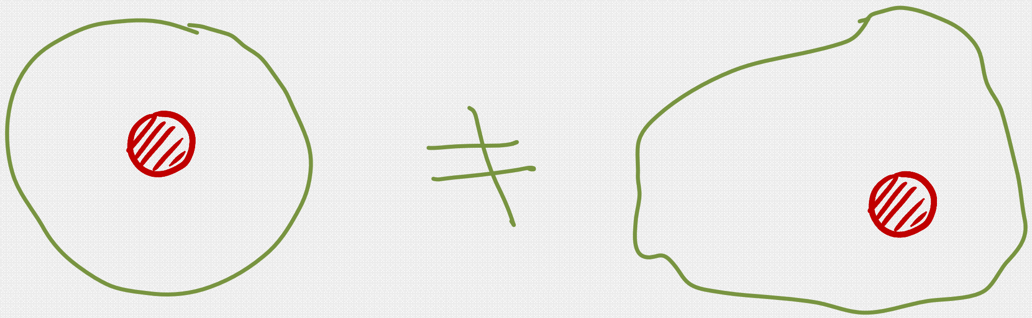
- *$2\Phi_0$ periodicity*

Faraday-induced phase

- Geometric phase shift
  - depends only on *ΔA(r) (initial & final configurations)*

$$\phi_{Fa}(\mathbf{r}) = \frac{1}{\hbar}\Delta\mathbf{p}\cdot\mathbf{r} = -\frac{e}{\hbar c}\Delta\mathbf{A}(\mathbf{r})\cdot\mathbf{r}$$

- Not a topological one



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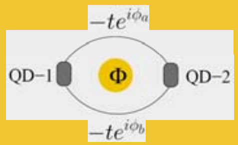
Adiabatic switching of the flux



$$\hbar/\Delta E \ll \Delta t_{sw} \lesssim \text{Dephasing time}$$

<u>Procedure:</u>

1. Start from the ground state at

   $\Delta\varepsilon=0,\ \Phi = \Phi_i$

2. Initialize a nonstationary state by

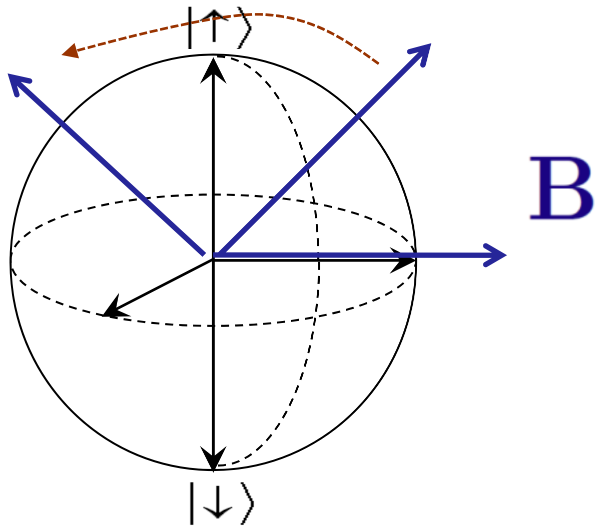
   sudden switching of $\Delta\varepsilon$

3. Adiabatic switching of the flux

   $\Phi = \Phi_i \rightarrow \Phi_i + \Delta\Phi$

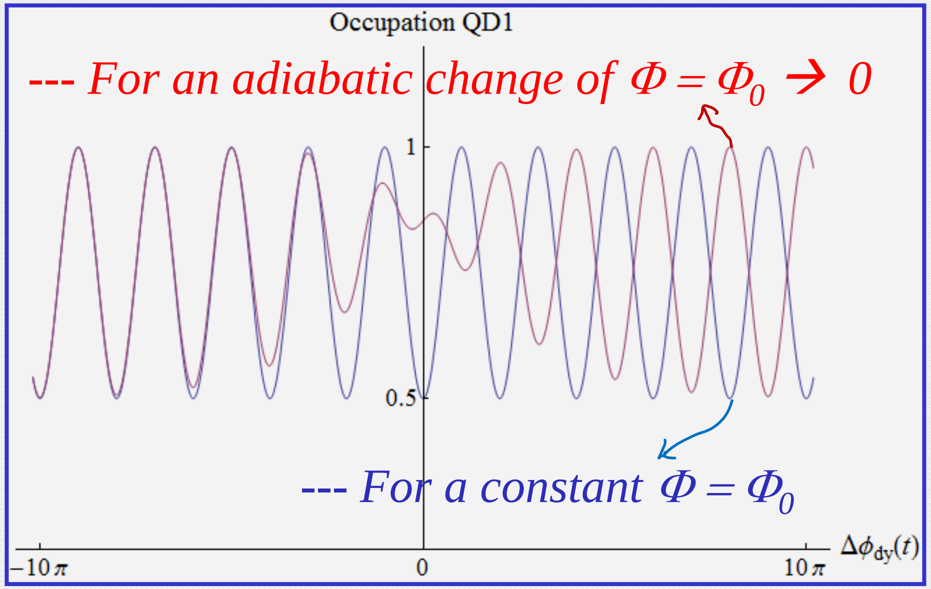
4. Measure the time-dependent

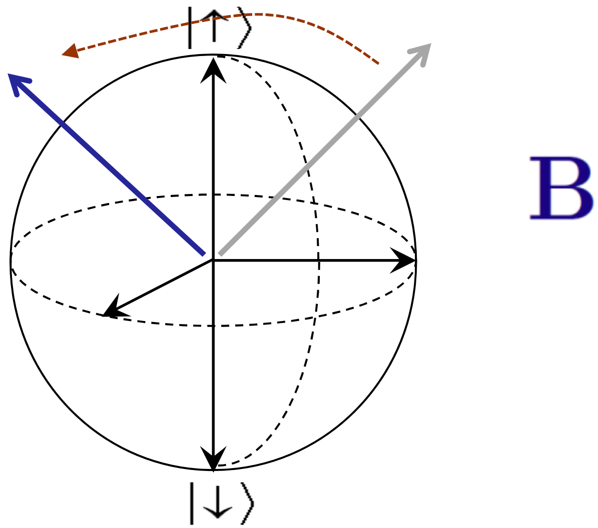
   charge at one of the QDs.



Adiabatic switching of the flux

$\hbar/\Delta E \ll \Delta t_{sw} \lesssim \text{Dephasing time}$

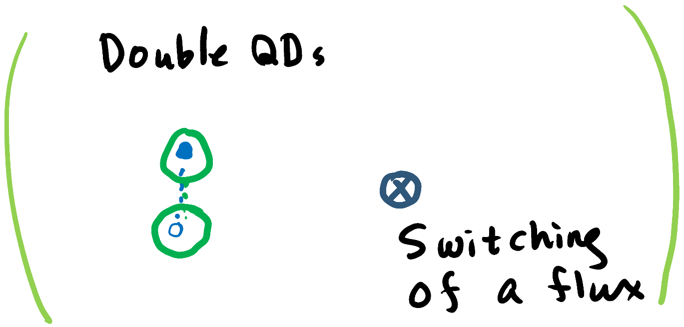




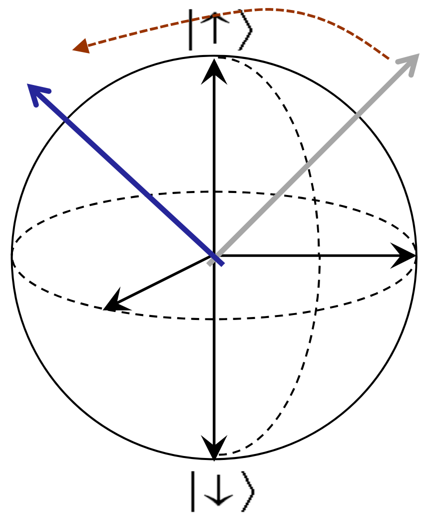
Out of phase oscillation

← *Faraday-induced phase shift*

Faraday phase without a loop



$$\Delta\phi_{Fa} = -\frac{e}{\hbar c}\int \Delta\vec{A}\cdot d\vec{r}$$



Conclusion

When one tries to reconstruct (by a QST) the wave function (of an AB loop) which is arbitrary:

- Its local phase is determined by *the law of Faraday induction*, not by the arbitrary choice of gauge.
- The induced phase is geometric, but non-topological
- Double-dot loop is only one example

Reference: *KK, arXiv:1102.5261*

Conclusion

“No progress without a paradox.”

- J. A. Wheeler