ELECTRON AND ATOM MOTION IN MOLECULES AND NANOSTRUCTURES



Peter Saalfrank

University of Potsdam, Germany

TIME- AND LENGTH-SCALES IN CHEMISTRY



ELECTRONS AND ATOMS IN MOTION

• Atom motion



Baskin, Zewail, J. Chem. Ed. 78, 737 (2001)

femtosecond chemistry

ELECTRONS AND ATOMS IN MOTION





Baskin, Zewail, J. Chem. Ed. 78, 737 (2001)

femtosecond chemistry

• Electron motion



Mauritsson *et al.*, PRL **100**, 073003 (2008) http://www.youtube.com/

attosecond physics

Krausz, Corkum, ...

• Born-Oppenheimer separation

OUTLINE

• Laser-driven electron dynamics

Methods

- \bullet A simple example: H₂
- **2** Charge transfer and transport in MIM contacts
- **3** Charge transfer in molecular systems
- **4** Dissipative electron dynamics

OUTLINE

• Laser-driven electron dynamics

Methods

- \bullet A simple example: H₂
- **2** Charge transfer and transport in MIM contacts
- Charge transfer in molecular systems
- **4** Dissipative electron dynamics
- Nuclear dynamics with and without lasers Methods
 - An STM-driven switch: COD/Si(100)
 - 2 Vibrational relaxation and excitation: H/Si(100), CO/Cu(100)

LASER-DRIVEN ELECTRON DYNAMICS



• The N-electron time-dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi(\underline{x}_1, \dots, \underline{x}_N, t)}{\partial t} = \left[\hat{H}_{el}(\underline{x}_1, \dots, \underline{x}_N) - \underline{\hat{\mu}}\underline{E}(t)\right] \Psi(\underline{x}_1, \dots, \underline{x}_N, t)$$

$$\boxed{\underline{\hat{\mu}} = -\sum_{i}^{N} \underline{r}_{i} + \sum_{A}^{N_{A}} Z_{A} \underline{R}_{A}}$$

• The N-electron time-dependent Schrödinger equation

$$i\hbar\frac{\partial\Psi(\underline{x}_{1},\ldots,\underline{x}_{N},t)}{\partial t} = \left[\hat{H}_{el}(\underline{x}_{1},\ldots,\underline{x}_{N}) - \underline{\hat{\mu}}\underline{E}(t)\right]\Psi(\underline{x}_{1},\ldots,\underline{x}_{N},t)$$

• Solution techniques

- One-electron approaches
- Single-determinant methods
 - TD-HF: $\Psi(t) = \psi_0(t)$
 - TD-DFT: $\Psi(t) = \psi_0^{KS}(t)$



- good for valence excitations $(n \to \pi^*, \pi \to \pi^*)$
- Rydberg states?
- charge transfer states?
- static correlation: conical intersections etc.?

• The N-electron time-dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi(\underline{x}_1, \dots, \underline{x}_N, t)}{\partial t} = \left[\hat{H}_{el}(\underline{x}_1, \dots, \underline{x}_N) - \underline{\hat{\mu}}\underline{E}(t)\right] \Psi(\underline{x}_1, \dots, \underline{x}_N, t)$$

• Solution techniques

- One-electron approaches
- Single-determinant methods
 - TD-HF: $\Psi(t) = \psi_0(t)$
 - TD-DFT: $\Psi(t) = \psi_0^{KS}(t)$
- Multi-determinant methods



- TD-CASSCF²: $\Psi(t) = C_0(t)\psi_0(t) + \sum_{ar} C_a^r(t)\psi_a^r(t) + \sum_{ab,rs} C_{ab}^{rs}(t)\psi_{ab}^{rs}(t) + \cdots$

¹ Klamroth *et al.*, Appl. Phys. A **78**, 189 (2004); PRB **68**, 245421 (2003); Schlegel *et al.* ² Nest *et al.*, JCP **122**, 124102 (2005); PRA **73**, 023613 (2006); Scrinzi *et al.*; Kono *et al.*



• The N-electron time-dependent Schrödinger equation

$$i\hbar\frac{\partial\Psi(\underline{x}_{1},\ldots,\underline{x}_{\mathrm{N}},t)}{\partial t} = \left[\hat{H}_{el}(\underline{x}_{1},\ldots,\underline{x}_{\mathrm{N}}) - \underline{\hat{\mu}}\underline{E}(t)\right]\Psi(\underline{x}_{1},\ldots,\underline{x}_{\mathrm{N}},t)$$

• Solution techniques

- One-electron approaches
- Single-determinant methods
 - TD-HF: $\Psi(t) = \psi_0(t)$
 - TD-DFT: $\Psi(t) = \psi_0^{KS}(t)$
- Multi-determinant methods



- TD-CI: $\Psi(t) = C_0(t)\psi_0 + \sum_{ar} C_a^r(t)\psi_a^r + \sum_{ab,rs} C_{ab}^{rs}(t)\psi_{ab}^{rs} + \cdots$
- TD-CASSCF: $\Psi(t) = C_0(t)\psi_0(t) + \sum_{ar} C_a^r(t)\psi_a^r(t) + \sum_{ab,rs} C_{ab}^{rs}(t)\psi_{ab}^{rs}(t) + \cdots$

TD-CI:TD-CIS, TD-CIS(D), TD-CISD, ... TD-CISD··N=Full-CI (FCI)TD-CASSCF(N,M):TD-CASSCF (N,N/2) = TD-HF, ..., TD-CASSCF(N,K)=FCI

A SIMPLE EXAMPLE: THE H_2 MOLECULE

• **TD-CISD** (=FCI) treatment: aug-cc-pV5Z; $|0\rangle \rightarrow |1\rangle$ laser excitation $\sin^2 \pi$ pulses $E_z(t) = E_0 \sin^2(\pi t/2\sigma) \cos(\omega_{10}t)$ with FWHM σ



A SIMPLE EXAMPLE: THE H_2 MOLECULE

• **TD-CISD** (=FCI) treatment: aug-cc-pV5Z; $|0\rangle \rightarrow |1\rangle$ laser excitation $\sin^2 \pi$ pulses $E_z(t) = E_0 \sin^2(\pi t/2\sigma) \cos(\omega_{10}t)$ with FWHM σ

"long pulse": $\sigma = 1000 \ \hbar/E_h$





A SIMPLE EXAMPLE: THE H_2 MOLECULE



• Possible processes • Metal-Insulator-Metal junctions $(\hbar\omega = 1.5 \text{ eV})$ 30 nm Al 15 nm Ag ZnSe Internal Photoemission 2 nm AlOxid Tunneling of Excited 70 MM Electrons TOWN Photon-Assisted 3.9 eV Tunneling 2.4 eV hv = 1.5 eV AI_2O_3 Al Ag

Klamroth et al., Appl. Phys. A (2003), PRB 61, 245421 (2003)

• Metal-Insulator-Metal junctions • Possible processes $(\hbar\omega = 1.5 \text{ eV})$ 30 nm Al 15 nm Ag ZnSe Internal Photoemission 2 nm AlOxid Tunneling of Excited to MA Electrons TOWN 3.9 eV Photon-Assisted Tunneling 2.4 eV hy = 1.5 eV"Control parameters" Al_2O_3 Al Ag 1. bias V 2. laser pulses $E_z(t)$ 3. film thickness

Klamroth et al., Appl. Phys. A (2003), PRB 61, 245421 (2003)

• Experiment: 2-pulse-correlation Pfeiffer et al., Würzburg





• Experiment: 2-pulse-correlation



Pfeiffer et al., Würzburg

• Theory TD-CIS, 1D-Jellium





Klamroth et al., Appl. Phys. A (2003); PRB 61, 245421 (2003)

V-TRANSPORT: MOLECULAR JUNCTIONS

• Break junctions





• Benzene-dithiolates



V-TRANSPORT: BENZENE-DITHIOLATES

• Transmission N(E)

• I(V) curves



EHT Greens' functions with absorbing boundaries

Kopf, P.S., CPL **386**, 17 (2004)

MOLECULAR JUNCTIONS: PHOTOSWITCHING

• E.g., molecular dipole switch



LiCN: A MOLECULAR DIPOLE SWITCH?



LiCN: A MOLECULAR DIPOLE SWITCH?



 Li^+

• Goal: Laser-pulse controlled dipole switch

LiCN: DIPOLE SWITCH



• What about environmental effects (spontaneous emission)?

P. Krause, T. Klamroth, PS, JCP 123, 074105 (2005)

ELECTRON DYNAMICS IN AN ENVIRONMENT

• Liouville-von Neumann equation for laser-driven electrons



ELECTRON DYNAMICS IN AN ENVIRONMENT

• Liouville-von Neumann equation for laser-driven electrons



• Lindblad dissipation, eigenstate basis

Populations: Diagonal elements of $\hat{\rho}_s$

dipole coupling $V_{mn}(t) = -\underline{\mu}_{mn}\underline{E}(t)$ energy relaxation rates $\Gamma_{n \to m}$

dephasing enters $\dot{\rho}_{mn}$ via dephasing rates γ_{mn}

• Switching of LiCN: $\Gamma_{2\rightarrow 0}^{-1} = 430$ fs; 5 and 75 fs π pulses



J.C. Tremblay, T. Klamroth, PS, JCP **129**, 084320 (2008)

NUCLEAR (ATOM) DYNAMICS



SWITCHING AGAIN: COD ON Si(100)

• STM-induced conformational switching at low T

1,5-cyclooctadiene



SWITCHING AGAIN: COD ON Si(100)

• STM-induced conformational switching at low T





COD@Si₁₅H₁₆ cluster, B3LYP-6/31G*

SWITCHING AGAIN: COD ON Si(100)

• STM-induced conformational switching at low T

1,5-cyclooctadiene



COD@Si₁₅H₁₆ cluster, B3LYP-6/31G*

Experiment¹:

- switching at T = 5 K
- rate $R_{sw}(+1.5\mathrm{V}, 0.7\mathrm{nA}) \approx 3.7 \mathrm{Hz}$
- $R_{sw} \propto I$
- inelastic electron tunneling (IET)?

1 Nacci, Fölsch, PRB 77, 121405(R) (2008)

Theory:

- Activation energy $E_a = 0.179 \text{ eV}$
- Eyring: $R_{sw}^{therm}(5\text{K}) < 10^{-100} \text{ s}^{-1}$

• Ground state: B3LYP/6-31G* cluster model









Left Right



• Use *localized* basis: $|n_{L,R}\rangle = 1/\sqrt{2}(|n_+\rangle \pm |n_-\rangle)$







• LvN equation in *localized* state basis







0.5

• LvN equation in *localized* state basis



• Interlevel transition rates

 $\Gamma_{i \to f} = \Gamma_{i \to f}^{relax} + \Gamma_{i \to f}^{IET}(I, V)$







• LvN equation in *localized* state basis



- Interlevel transition rates Vibrational excitation and relaxation

$$\Gamma_{i \to f} = \Gamma_{i \to f}^{relax} + \Gamma_{i \to f}^{IET}(I, V)$$

$$\Gamma_{i \to f}^{IET} = I \ \mu_{if}^2 = \Gamma_{f \to i}^{IET}$$



Persson, Demuth, Solid State Commun. 57, 769 (1986)







• LvN equation in *localized* state basis



- Interlevel transition rates Vibrational excitation and relaxation

$$\Gamma_{i \to f} = \Gamma_{i \to f}^{relax} + \Gamma_{i \to f}^{IET}(I, V)$$

$$\Gamma_{i \to f}^{IET} = I \ \mu_{if}^2 = \Gamma_{f \to i}^{IET}$$

$$\Gamma_{i \to f}^{relax} = \tau_{vib}^{-1} \ i \ \delta_{f,i-1}$$









IET

 $\tau_{\rm vib}$

• LvN equation in *localized* state basis



- Interlevel transition rates
- Vibrational excitation and relaxation





COD@Si(100): RESULTS

• Switch rate: T = 0



- $R_{sw}(1nA) \approx Hz$
- $R_{sw} \propto I$
- IET: $|0_L\rangle \rightarrow |4_L\rangle \rightarrow \text{switch}$

COD@Si(100): RESULTS



- $R_{sw}(1nA) \approx Hz$
- $R_{sw} \propto I$
- IET: $|0_L\rangle \rightarrow |4_L\rangle \rightarrow \text{switch}$

- low T: IET tunneling
- high T: Thermal Arrhenius over-barrier
- crossover temperature T_c

COD@Si(100): RESULTS



- $R_{sw}(1nA) \approx \text{Hz}$
- $R_{sw} \propto I$
- IET: $|0_L\rangle \rightarrow |4_L\rangle \rightarrow \text{switch}$

- low T: IET tunneling
- high T: Thermal Arrhenius over-barrier
- crossover temperature T_c

I-controlled switching from *classical* to *quantum* regime

Nacci, Fölsch, Zenichowski, Klamroth, Dokic, Saalfrank, Nano Letters 9, 2996 (2009)

CAN WE CALCULATE τ_{vib} ?

CAN WE CALCULATE τ_{vib} ?



H / Si(100): VIBRATIONAL RELAXATION

• A "system-bath" model for H on Si(100)





H / Si(100): VIBRATIONAL RELAXATION

• A "system-bath" model for H on Si(100)





¹ force field: D. Brenner, PRB **42**, 9458 (1990); NMA: I. Andrianov, PS, JCP **124**, 034710 (2006)

H/Si(100): PERTURBATION THEORY

• The Golden Rule of quantum mechanics

$$\Gamma_{n \to m} = \frac{2\pi}{\hbar} \sum_{i} \sum_{f} w_i(T) \left(1 - w_f(T)\right) \left| \left\langle m, f | \hat{H}_{sb} | n, i \right\rangle \right|^2 \delta(\varepsilon_f^{ph} - \varepsilon_i^{ph} - \hbar \omega_{nm})$$

H/Si(100): PERTURBATION THEORY

• The Golden Rule of quantum mechanics

$$\Gamma_{n \to m} = \frac{2\pi}{\hbar} \sum_{i} \sum_{f} w_i(T) \left(1 - w_f(T)\right) \left| \left\langle m, f | \hat{H}_{sb} | n, i \right\rangle \right|^2 \delta(\varepsilon_f^{ph} - \varepsilon_i^{ph} - \hbar \omega_{nm})$$



- stretch mode: $\tau_{vib} = \Gamma_{1 \to 0}^{-1} = ns$
- bending mode: ps
- $\Gamma_{n \to m} \approx \tau_{vib}^{-1} n \delta_{m,n-1}$: $\Delta n = -1$
- exponential population decay

Non-perturbative RELAXATION H:Si(100), MCTDH

- Solve $i\hbar \frac{\partial \Psi}{\partial t} = (\hat{H}_s + \hat{H}_b + \hat{H}_{sb})\Psi$ with Multi Configuration TD Hartree method¹
- Wavefunction:

$$\Psi(q_1 \dots q_F, t) = \sum_{j_1}^{n_1} \dots \sum_{j_F}^{n_F} A_{j_1 \dots j_F}(t) \Phi_{j_1 \dots j_F}(t)$$

Non-perturbative RELAXATION H:Si(100), MCTDH

• Solve $i\hbar \frac{\partial \Psi}{\partial t} = (\hat{H}_s + \hat{H}_b + \hat{H}_{sb})\Psi$ with Multi Configuration TD Hartree method¹

• Wavefunction:

$$\Psi(q_1 \dots q_F, t) = \sum_{j_1}^{n_1} \dots \sum_{j_F}^{n_F} A_{j_1 \dots j_F}(t) \Phi_{j_1 \dots j_F}(t)$$

- "Relaxation" dynamics² (50+2 DOF)
 - non-exponential decay
 - recurrences



• Alternatives: $TD-SCF^3$ and $LCSA^4$

¹ Meyer *et al.*, JCP **97**, 3199 (1992)
 ² I. Andrianov, PS, CPL **433**, 91 (2006)

³ Paramonov, Andrianov, PS, J. Phys. Chem. C 111, 5432 (2007)
 ⁴ Martinazzo, Nest, PS, Tantardini, JCP 125, 194102 (2006)

H/Si: IR MODE-SELECTIVE CHEMISTRY

Science **312**, 1024 (2006): Desorption of H from Si(111) by Resonant Excitation of the Si-H Vibrational Stretch Mode

Zhiheng Liu,^{1,2} L. C. Feldman,^{2,3} N. H. Tolk,² Zhenyu Zhang,^{3,4} P. I. Cohen^{1*}

Past efforts to achieve selective bond scission by vibrational excitation have been thwarted by energy thermalization. Here we report resonant photodesorption of hydrogen from a Si(111) surface using tunable infrared radiation. The wavelength dependence of the desorption yield peaks at 0.26 electron volt: the energy of the Si-H vibrational stretch mode. The desorption yield is quadratic in the infrared intensity. A strong H/D isotope effect rules out thermal desorption mechanisms, and electronic effects are not applicable in this low-energy regime. A molecular mechanism accounting for the desorption event remains elusive.

H/Si: IR MODE-SELECTIVE CHEMISTRY

Science **312**, 1024 (2006): Desorption of H from Si(111) by Resonant Excitation of the Si-H Vibrational Stretch Mode

Zhiheng Liu,^{1,2} L. C. Feldman,^{2,3} N. H. Tolk,² Zhenyu Zhang,^{3,4} P. I. Cohen^{1*}

Past efforts to achieve selective bond scission by vibrational excitation have been thwarted by energy thermalization. Here we report resonant photodesorption of hydrogen from a Si(111) surface using tunable infrared radiation. The wavelength dependence of the desorption yield peaks at 0.26 electron volt: the energy of the Si-H vibrational stretch mode. The desorption yield is quadratic in the infrared intensity. A strong H/D isotope effect rules out thermal desorption mechanisms, and electronic effects are not applicable in this low-energy regime. A molecular mechanism accounting for the desorption event remains elusive.





(534 + 2 DOF)

desorption by mode-selective excitation

G.K. Paramonov, I. Andrianov, PS, J. Phys. Chem. C 111, 5432 (2007)

SUMMARY

• Electron dynamics

1 correlated wavefunction-based many-electron methods

2 ultrashort pulses:

multi-photon processes and electronic wavepackets

- **3** transport
- **4** dissipation

SUMMARY

• Electron dynamics

• correlated wavefunction-based many-electron methods

2 ultrashort pulses:

multi-photon processes and electronic wavepackets

3 transport

4 dissipation

• Nuclear (atom) dynamics

• open-system density matrices and system-bath Schrödinger equations

- **2** applications to surface and nano science
- **❸** single molecules: switching conformations, and quanticity
- **4** mode-selectivity despite of decoherence

THANKS TO ...

• ... the group:



• ... the sponsors:

Deutsche Forschungsgemeinschaft DFG

- Deutsche Forschungsgemeinschaft
 - SFB 450, SFB 658, SPP 1145, UniCat, Sa 547/7





 BMBF
 Bundesministerium für Bildung und Forschung



t-DEPENDENT CONFIGURATION INTERACTION

- TD-CIS (Singles)
- 1. Field-free Hartree-Fock

$$\hat{f}(1)\phi_a(1) = arepsilon_a\phi_a(1)$$
 $\phi_a = \sum_{\mu}^K d_{\mu a} arphi_{\mu}$

2. Field-free CIS



3. TD Schrödinger equation with field

$\Psi(t) = \sum C_i(t)\psi_i$
i

$$i\hbar \underline{C}(t) = \underline{\underline{H}}(t) \ \underline{C}(t)$$
$$\hat{H} = \hat{H}_{el} - \underline{\hat{\mu}}\underline{\underline{E}}(t) \quad ; \quad \psi(0) = \psi_0$$

• Extensions

- TD-CISD, TD-CIS $(D)^1$, ..., FCI
- TD-CASSCF (=MCTDHF)

- TD-CASSCF (N,M): N electrons in M spatial orbitals



 1 M. Head-Gordon $et~al.,~{\rm CPL}~{\bf 219},~{\bf 21}~(1994)$

CIS(D) METHOD

• Perturbative treatment of double-excitations

$$E_i^{(D)} = -\frac{1}{4} \sum_{\text{abrs}} \frac{\left(u_{\text{ab},i}^{\text{rs}}\right)^2}{\left(\Delta_{\text{ab}}^{\text{rs}} - E_i^{CIS}\right)} + \sum_{\text{ar}} D_{\text{a},i}^{\text{r}} v_{\text{a},i}^{\text{r}}$$

• Example: H_2

$$\frac{\text{aug-cc-pV5Z}}{E_0 (E_h)} \quad \begin{array}{c} \text{CIS} & \text{CIS}(D) \\ \hline \text{CISD} & \text{``exact''} \\ \hline -1.1336 \\ \hline -1.1673 \\ \hline -1.1742 \\ \hline -1.178 \\ \hline \end{array}$$

Head-Gordon *et al.*, CPL **219**, 21 (1994)

• Dipole moments

\sin^2 laser pulses: $E_x(t), E_z(t)$



P. Krause, T. Klamroth, PS, JCP **123**, 074105 (2005)

MOLECULAR JUNCTIONS: THEORY

• Landauer transport theory $\mathbf{N}(\mathbf{E})$: Miller-Seideman

$$I(V) = \frac{2e}{h} \int_{E_F - \frac{eV}{2}}^{E_F + \frac{eV}{2}} N(E, V) dE$$

$$N(E) = \frac{1}{\hbar^2} \text{Tr}[\underline{\underline{\Gamma}}^L \underline{\underline{G}}(E) \underline{\underline{\Gamma}}^R \underline{\underline{G}}^{\dagger}(E)]$$

• LCAO-MO approach

$$\underline{\underline{G}}(E) = \left[E\underline{\underline{S}} - (\underline{\underline{H}} - \frac{i\hbar}{2}(\underline{\underline{\Gamma}}^{L} + \underline{\underline{\Gamma}}^{R}))\right]^{-1}$$

$$\underline{\underline{H}} = \text{Hamiltonian matrix} \\ \underline{\underline{\underline{S}}} = \text{overlap matrix} \\ \underline{\underline{\Gamma}}^{\overline{L}/R} = \text{left / right absorbers}$$

MOLECULAR JUNCTIONS: TESTING

• Quasi-1D Hückel (tight binding) model





REDUCED DENSITY MATRIX (RDM) THEORY

$$\begin{split} \langle \hat{A} \rangle(t) &= \operatorname{tr} \{ \hat{\rho}(t) \ \hat{A} \} \\ \uparrow \\ \\ \text{Liouville-von Neumann eq.} \quad \boxed{\frac{\partial \hat{\rho}(t)}{\partial t} = -\frac{i}{\hbar} \left[\hat{H}_s + \hat{H}_b + \hat{H}_{sb}, \hat{\rho} \right]}_{\begin{array}{c} \hat{\rho} = \sum_n w_n(T) |\psi_n\rangle \langle \psi_n | \\ \text{density operator} \\ \end{array} \\ \hline \begin{array}{c} \text{energy relaxation} \\ \text{system, } \hat{H}_s \\ \text{dephasing} \\ \end{array} \\ \begin{array}{c} \text{bath, } \hat{H}_b \\ \end{array} \end{split}$$

REDUCED DENSITY MATRIX (RDM) THEORY

L

N

H / Si(100): VIBRATIONAL RELAXATION

• A "system-bath" model for H on Si(100)



¹I. Andrianov, PS, JCP **124**, 034710 (2006)

²Stokbro et al., Surf. Sci. **415**, L1037 (2000)

RELAXATION H:Si(100), TD-SCF

- Solve $i\hbar \frac{\partial \Psi}{\partial t} = (\hat{H}_s + \hat{H}_b + \hat{H}_{sb})\Psi$ with TD Self Consistent Field method
- Wavefunction: $\Psi(r, \phi, q_1, \dots, q_M, t) = \Phi_s(r, \phi, t) \Pi_{i=1}^M \chi_i(q_i, t)$
- "Relaxation" dynamics



• Half-life times $T_{1/2}$

state	TD-SCF	PT	MCTDH
	M = 534	M = 534	M = 50
	(ps)	(ps)	(ps)
(0,1)	0.92	0.94	3.54
(0,2)	0.48	0.48	1.13
(0,3)	0.34	0.33	0.87

Paramonov, Andrianov, PS, J. Phys. Chem. C 111, 5432 (2007)

OPTIMAL CONTROL IN AN OPEN SYSTEM^{(1),(2)}

• Liouville-von Neumann equation:

 $i\hbar \frac{\partial}{\partial t} |\hat{\rho}(t)\rangle\rangle = (\mathcal{L}_H + \mathcal{L}_D) |\hat{\rho}(t)\rangle\rangle$ forward from $t = 0, |\hat{\rho}(0)\rangle\rangle = |\hat{\rho}_0\rangle\rangle$

• Maximize constrained target functional:

$$J = \langle \langle \hat{O} | \hat{\rho}(t_f) \rangle \rangle - \int_0^{t_f} \alpha(t) |E(t)|^2 dt - \int_0^{t_f} dt \langle \langle \hat{\sigma}(t) | \frac{\partial}{\partial t} + \frac{i}{\hbar} [\mathcal{L}_H + \mathcal{L}_D] | \hat{\rho}(t) \rangle \rangle$$

• Solve in addition to LvN equation:

 $i\hbar \frac{\partial}{\partial t} |\hat{\sigma}(t)\rangle\rangle = (\mathcal{L}_H + \mathcal{L}_D)^{\dagger} |\hat{\sigma}(t)\rangle\rangle$ backward from $t = t_f, |\hat{\sigma}(t_f)\rangle\rangle = |\hat{O}\rangle\rangle$

• Field:

$$E(t) = -\frac{1}{\hbar\alpha(t)} \operatorname{Im} \langle \langle \hat{\sigma}(t) | \hat{\mu} | \hat{\rho}(t) \rangle \rangle$$

(1) Y. Ohtsuki et al., JCP **109**, 9318 (1998); (2) Y. Ohtsuki et al. JCP **110**, 9825 (1999)