Surface Acoustic Solitary Waves

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Outline

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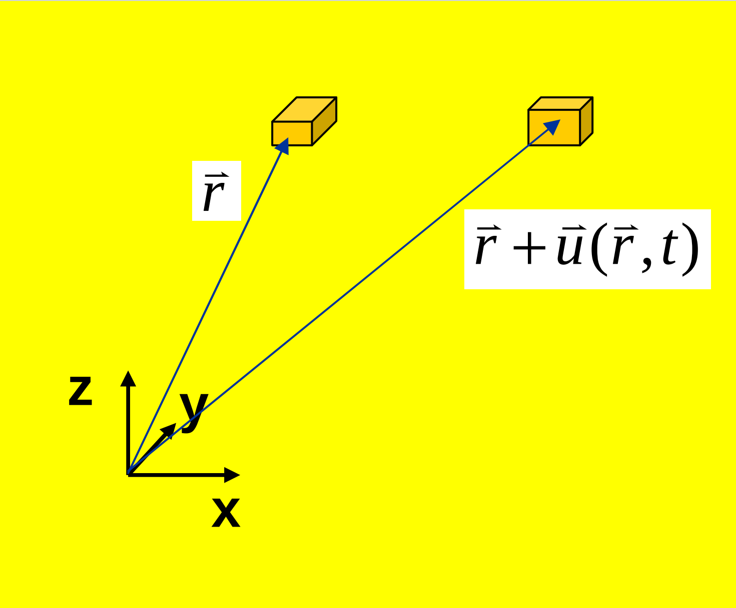
- Linear surface acoustic waves (SAWs)
- Pulse evolution on homogeneous elastic media
- Solitary wave pulses on coated substrates
- Edge acoustic waves

Elasticity Theory

Displacement field: $\vec{u}(\vec{r},t)$



$\vec{r}$: Position of mass element in undeformed medium

$\vec{r}+\vec{u}(\vec{r},t)$:

Position of same mass element in deformed medium

Elasticity Theory

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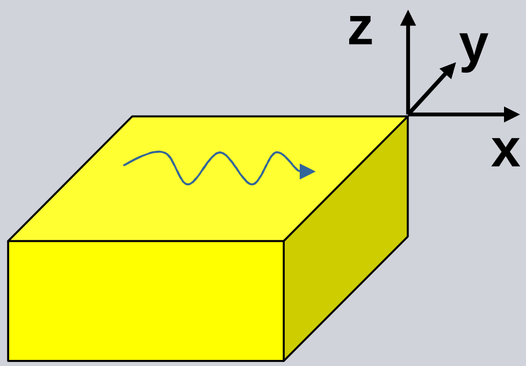
Displacement field: $\vec{u}(x,z,t)$

Displacement gradients: $u_{\alpha,\beta} = \frac{\partial u_\alpha}{\partial x_\beta}$

Piola-Kirchhoff stress tensor:

$$T_{\alpha\beta} = C_{\alpha\beta\,\gamma\delta} u_{\gamma,\delta} + \tfrac{1}{2} S_{\alpha\beta\,\gamma\delta\,\varepsilon\zeta} u_{\gamma,\delta} u_{\varepsilon,\zeta} + O(u^3_{..,..})$$



$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Equation of motion: $\rho \ddot{u}_\alpha = T_{\alpha\beta,\beta}$

Boundary conditions:

$$T_{\alpha 3}\big|_{z=0} = 0$$

$$u \xrightarrow[z \to -\infty]{} 0$$

Linear SAW

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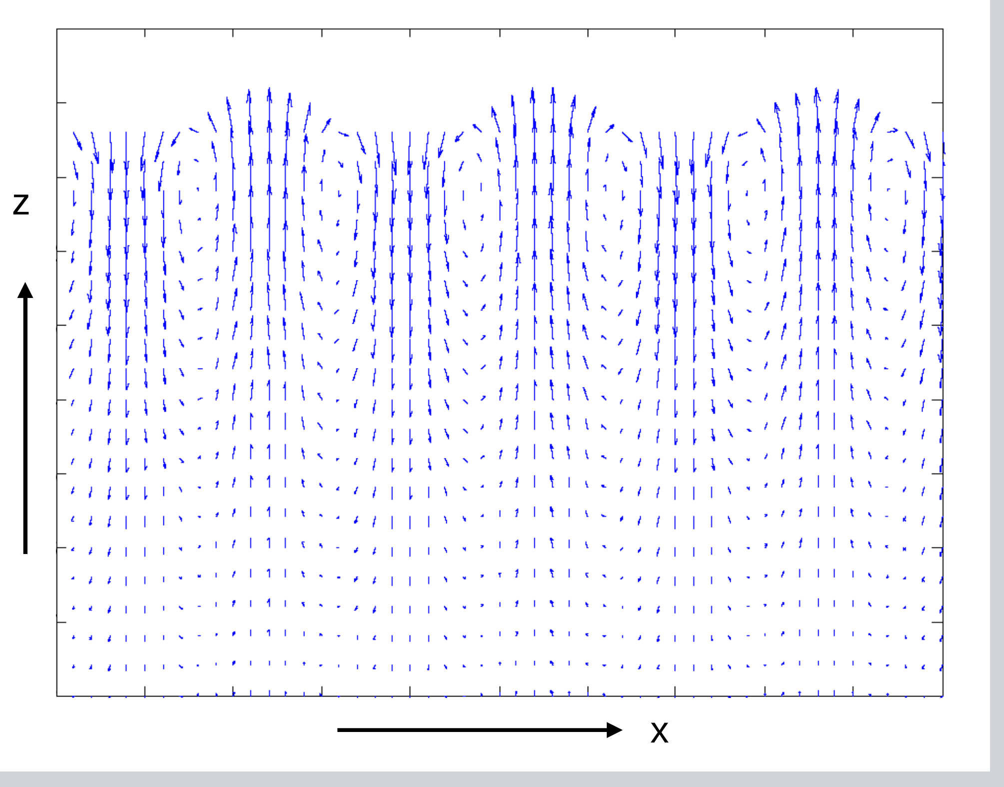
Displacement field:

$$u_\alpha(x,z,t) = \exp[iq(x - v_R t)]\; w_\alpha(z\,|\,q) + c.c.$$

depth profile



$$u_1(x,z,t) = \cos[q(x - v_R t)] W_1(z\,|\,q)$$
$$u_3(x,z,t) = \sin[q(x - v_R t)] W_3(z\,|\,q)$$

Applications

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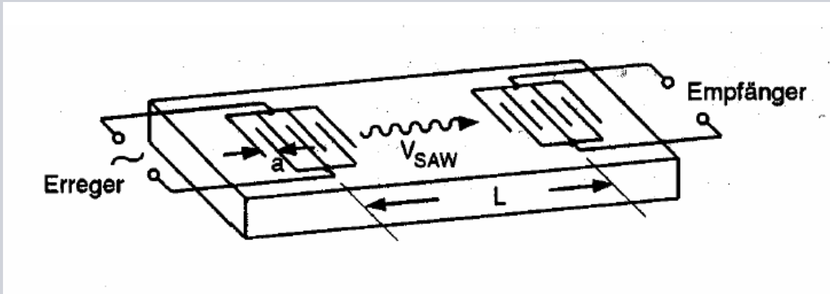
Wavelength

km — Geophysics (seismics, earthquakes)

µm — Non-destructive testing
Sensors
Electronic devices
(frequency filters, delay lines,…)



Source: IMTEK, Uni Freiburg

nm — Surface physics

Nonlinearity / Dispersion

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Nonlinearity

Piola-Kirchhoff stress tensor: $(T_{\alpha\beta})$

$$T_{11} = C_{1111}u_{1,1} + \ldots + \tfrac{1}{2} S_{111111}u_{1,1}u_{1,1} + \ldots + O(u^3)$$

$$u_{\alpha,\beta} = \frac{\partial u_\alpha}{\partial x_\beta}$$ displacement gradients

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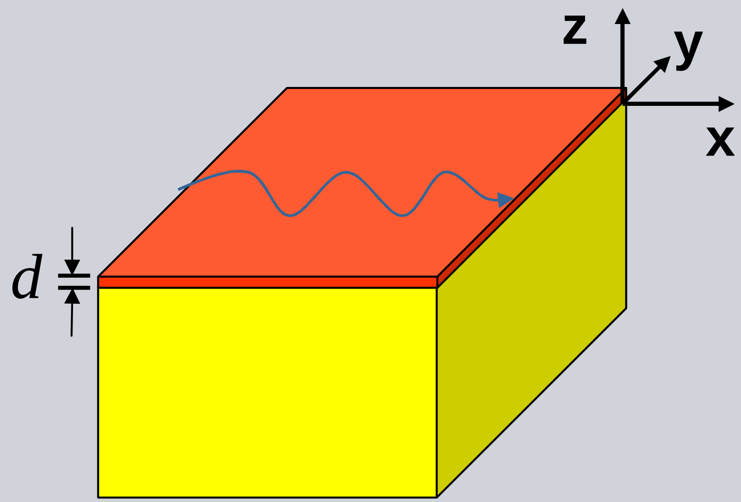
Dispersion

due to coating of substrate (S) by film (F) of different material



⇨ Phase velocity depends on wavelength

SAW frequency: $\omega(q) = v_R q + \Delta_0 q^2 + \Delta_1 q^3 + O(q^4)$

Nonlinear Pulse Evolution

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Nonlinearity is small: $\left| u_{\alpha,\beta} \right| < 0.01$

Asymptotic expansion for displacement gradient: $u_{\alpha,1} = \varepsilon\, u_{\alpha,1}^{(1)} + \varepsilon^2\, u_{\alpha,1}^{(2)} + O\left(\varepsilon^3\right)$

with "stretched" time coordinate: τ

Fourier transform of displacement gradients at the surface:

$$u_{\alpha,1}^{(1)}(x,0,t) = \int_0^\infty \exp\left[iq(x - v_R t)\right] B(q;\tau)\, \frac{dq}{2\pi} + c.c.$$

strain amplitude

Evolution equation
for strain amplitudes

(Reutov, Kalyanasundaram, Lardner, Parker)

Evolution Equation

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for homogeneous medium (no dispersion):

$$i\frac{\partial}{\partial\tau}B(q) =$$

$$qv_R\left[\int_0^q F\left(k/q\right)B(k)B(q-k)\,dk + 2\int_q^\infty \left(q/k\right) F\left(q/k\right)^* B(k)B(k-q)^*\,dk\right]$$

Depends on ratios of 2nd-order and 3rd-order elastic moduli:

Influence of anisotropy !

Non-local nonlinearity !

Evolution Equation

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Evolution equation for **weakly dispersive SAW**:

$$i\frac{\partial}{\partial\tau}B(q)=q^2\Delta_R(q)\;B(q)$$
$$+qv_R\left[\int_0^q F\left(k/q\right)\,B(k)B(q-k)\,dk+2\int_q^\infty \left(q/k\right)\,F\left(q/k\right)^*\;B(k)B(k-q)^*\,dk\right]$$

Evolution equation for **weakly dispersive BAW**: **KdV equation**

$$i\frac{\partial}{\partial\tau}B(q)=q^3\Delta_B\,B(q)+qv_B\left[\int_0^q B(k)B(q-k)\,dk+2\int_q^\infty B(k)B(k-q)^*\,dk\right]$$

$$\Leftrightarrow\quad \frac{\partial}{\partial\tau}V-\Delta_B\frac{\partial^3}{\partial\xi^3}V+2v_B\;V\frac{\partial}{\partial\xi}V=0$$

Evolution Equation

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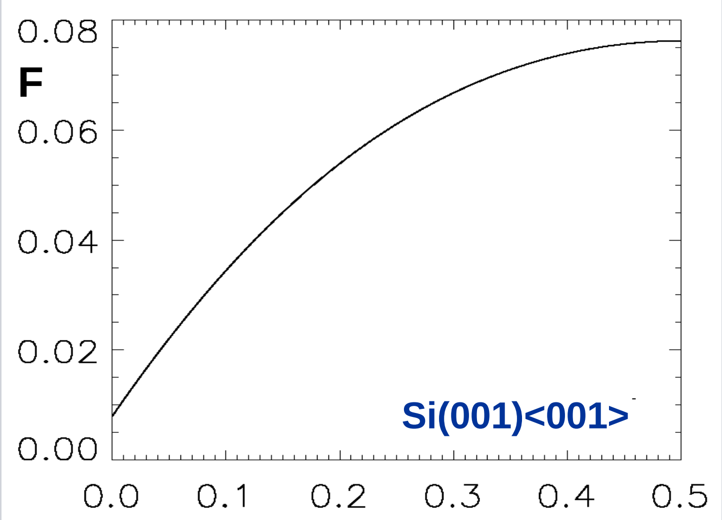
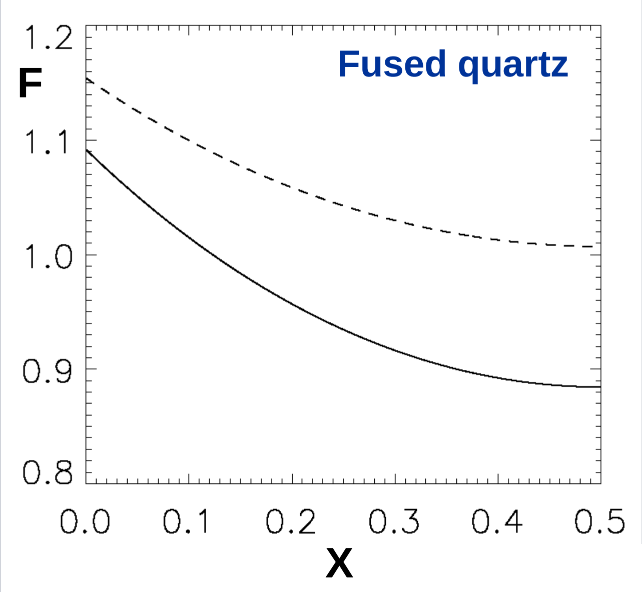
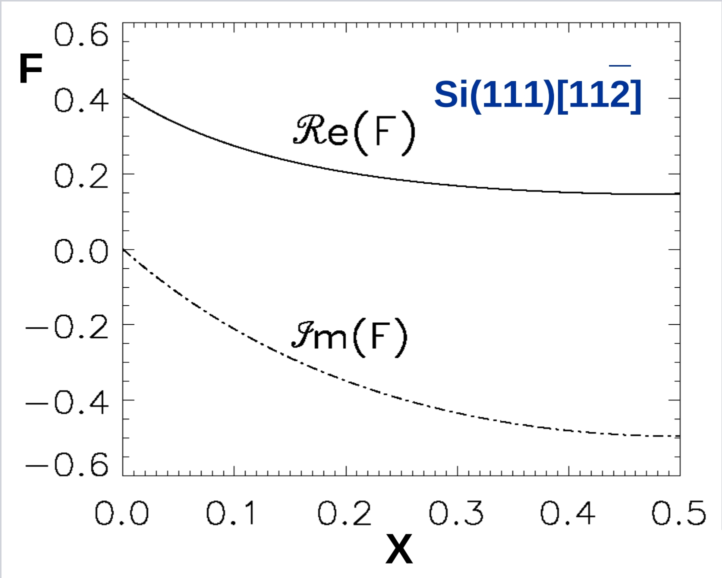
Function **F** in evolution equation for $u_{1,1}$:

$$F(X) = F(1 - X)$$







Pulse Evolution without Dispersion

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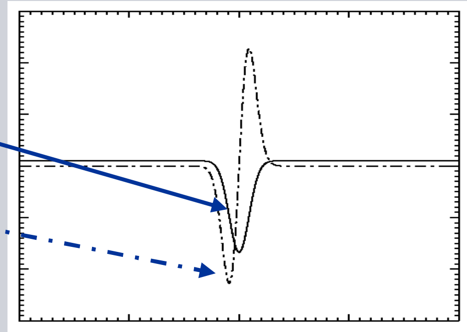
Initial surface elevation profile $u_3(x,0)$

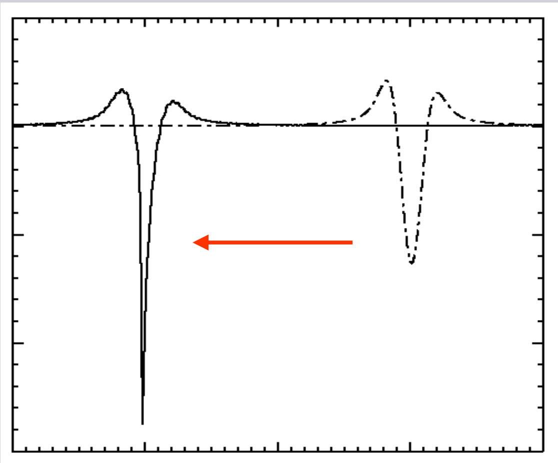
surface slope $u_{3,1}(x,0)$



$x - v_R t$

Si(111)$[11\bar{2}]$

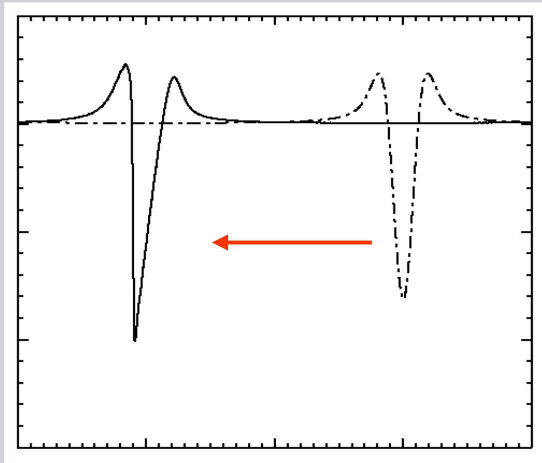
$u_{1,1}(x,0)$



$x - v_R t$

Si(001)[100]

$u_{1,1}(x,0)$

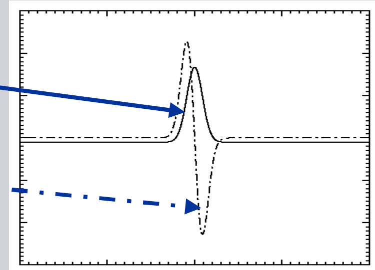


$x - v_R t$

Pulse Evolution without Dispersion

Initial surface elevation profile $u_3(x,0)$

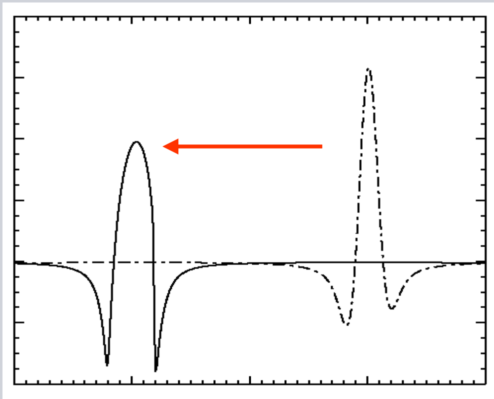
surface slope $u_{3,1}(x,0)$



$x - v_R t$

Si(111)[11$\bar{2}$]

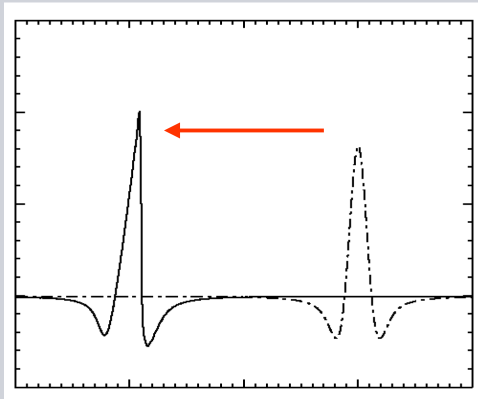
$u_{1,1}(x,0)$



$x - v_R t$

Si(001)[100]

$u_{1,1}(x,0)$



$x - v_R t$

Laser Excitation

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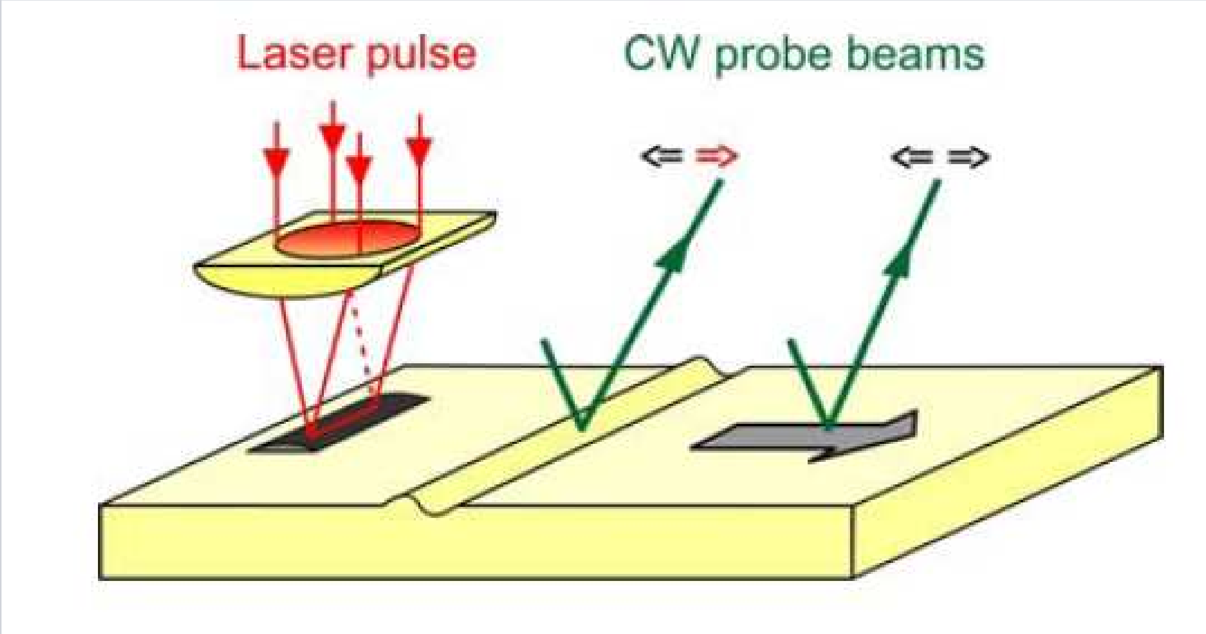
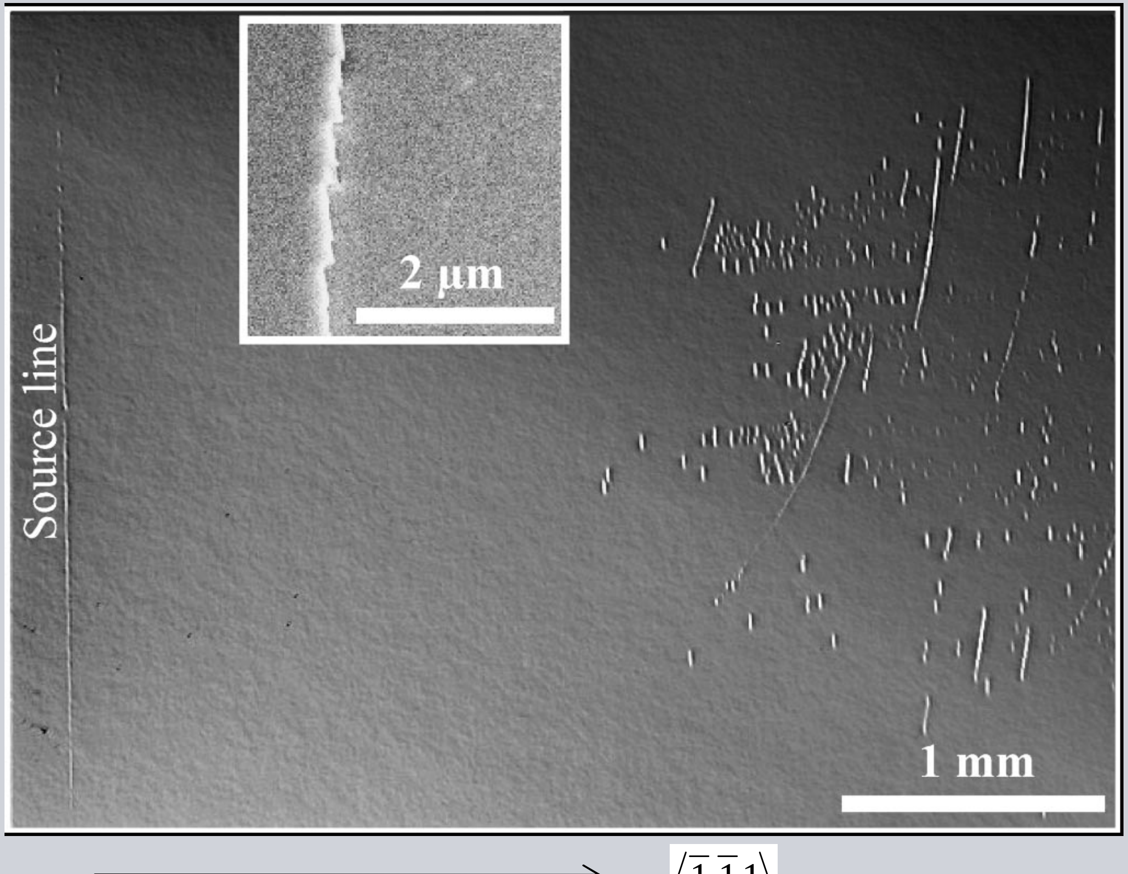


Experimental setup

A.M. Lomonosov, V.V. Kozhushko and P. Hess

Crack formation

Optical microscope image of Si(112) surface after propagation of a nonlinear SAW pulse in the <-1 -1 1> direction



$\langle \bar{1}\,\bar{1}\,1 \rangle$

V.V. Kozhushko, A.M. Lomonosov, and P. Hess

PRL **98**, 195505 (2007)

Crack formation

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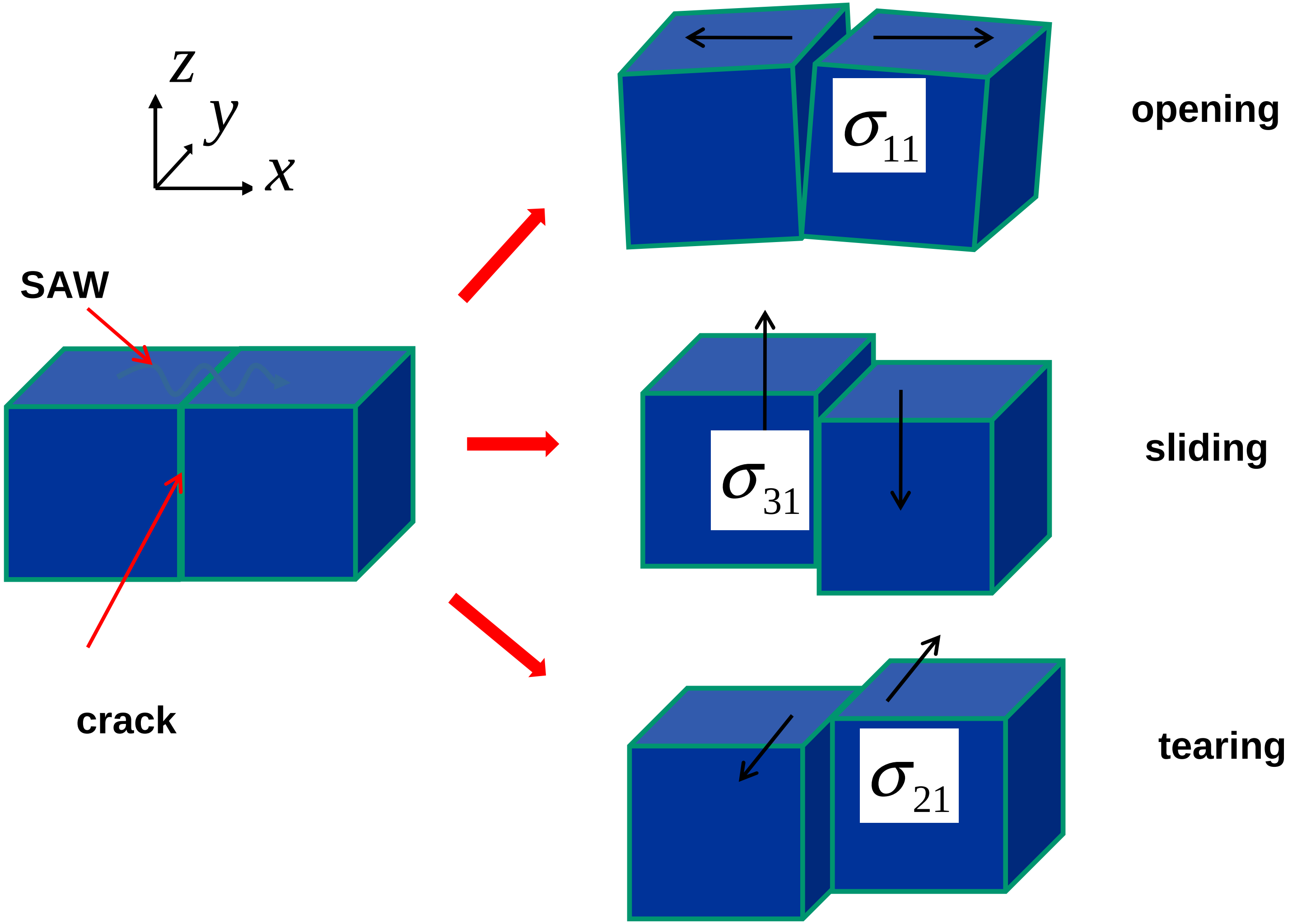
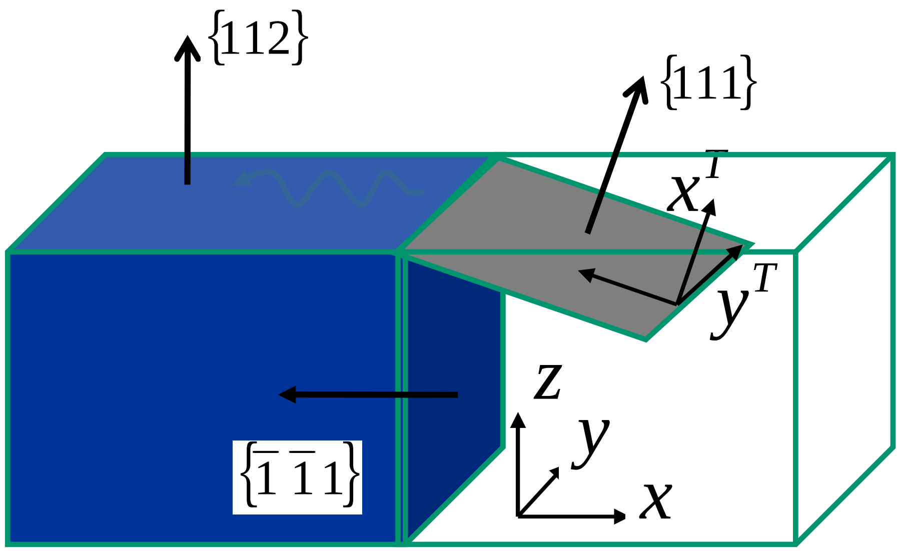
Fracture modes of Si {111} surfaces



Crack formation

Critical planes: $\{111\}$ $\{\bar{1}\bar{1}1\}$ $\{1\bar{1}\bar{1}\}$ $\{\bar{1}1\bar{1}\}$

Example: Si(112) surface



Crack formation

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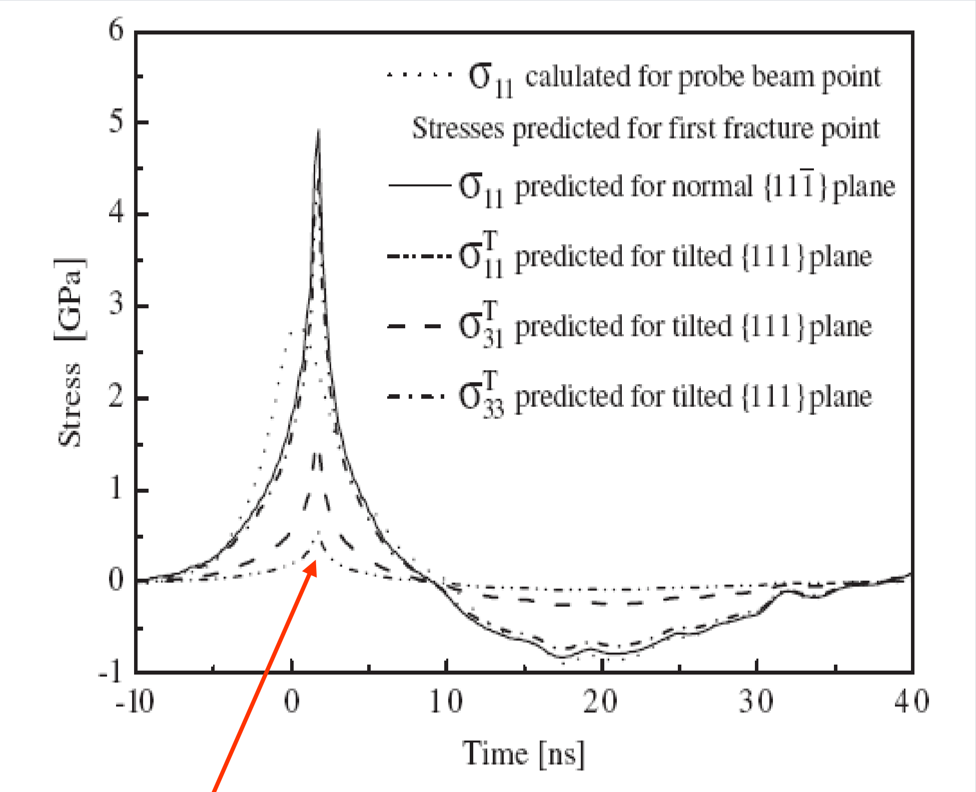
Calculated stress at the Si(112) surface:



Pulse shape depends on excitation mechanism and elastic nonlinearity

V.V. Kozhushko, A.M. Lomonosov, and P. Hess

PRL **98**, 195505 (2007)

Critical strength (Ab-initio calculations): **22 GPa**

Experimental strength: **5-7 GPa**

Solitary Pulses

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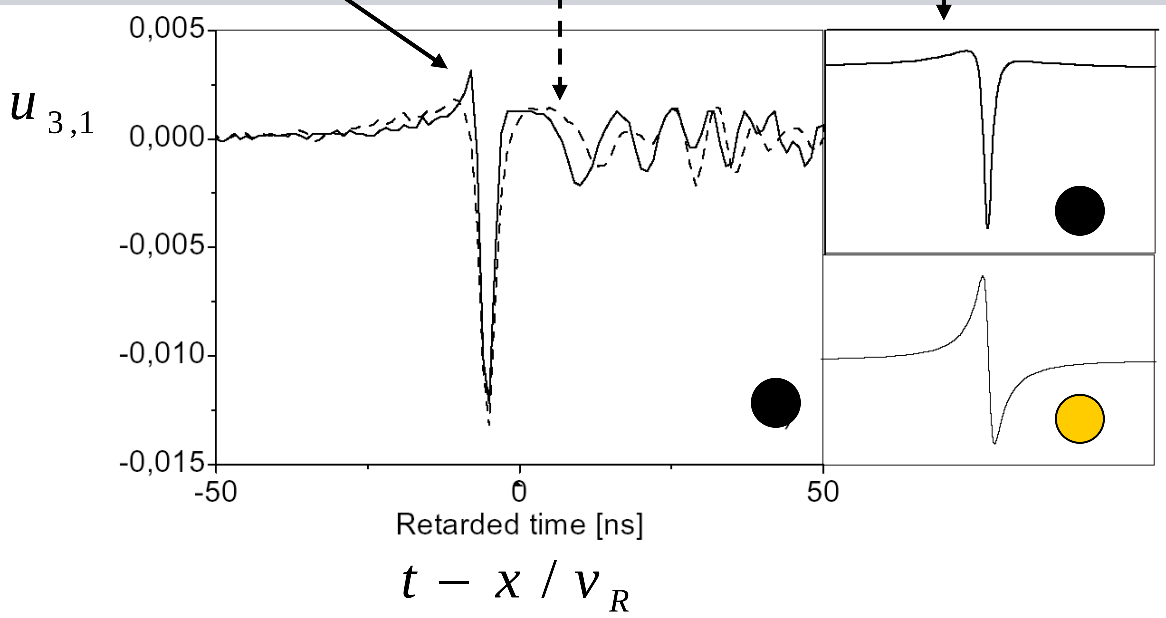
Pulse evolution after laser excitation
(**Lomonosov & Hess**)

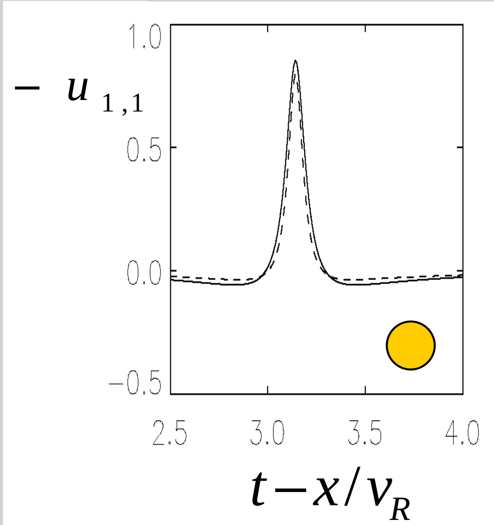
experimental simulated pulse shape stationary solitary solutions



Stationary pulse shape calculated with

full F(X) ——

F(X) replaced by F(1/2) - - -



● **Si(111)[$\bar{1}\bar{1}2$]**

● **Si(001)[100]**

Solitary Pulses

2-parameter family:

$$u_{\alpha,1}^{(1)}(x,0,t)=\kappa S_\alpha\left(\kappa(x-v_R t\pm\kappa v_R\tau-x_0)\right)$$

for linear dispersion law $\omega(q)-v_R q=\Delta_0 q^2$

$$u_{\alpha,1}^{(1)}(x,0,t)=\kappa S_\alpha\left(\sqrt{\kappa}(x-v_R t\pm\kappa v_R\tau-x_0)\right)$$

for linear dispersion law $\omega(q)-v_R q=\Delta_1 q^3$

Can be computed as limiting cases
of periodic pulse-train solutions.

Depth Profile

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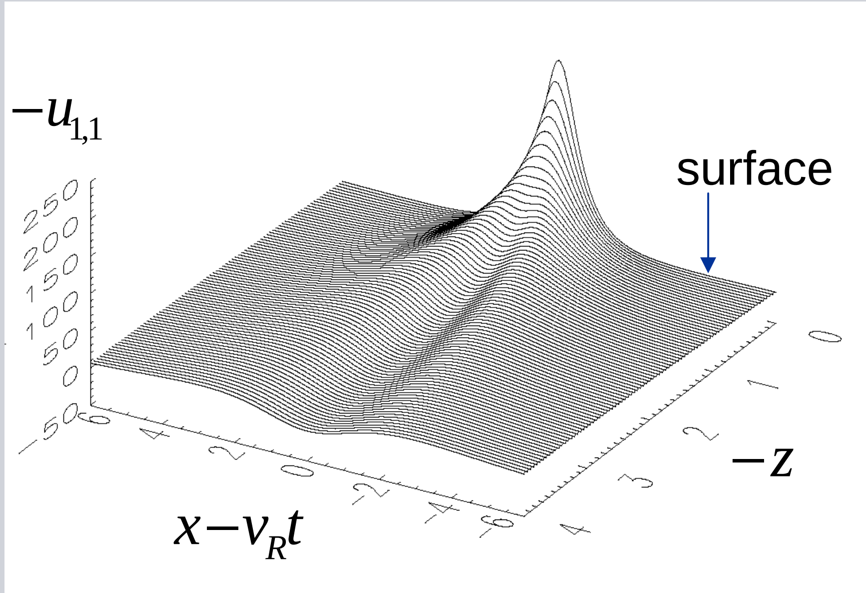
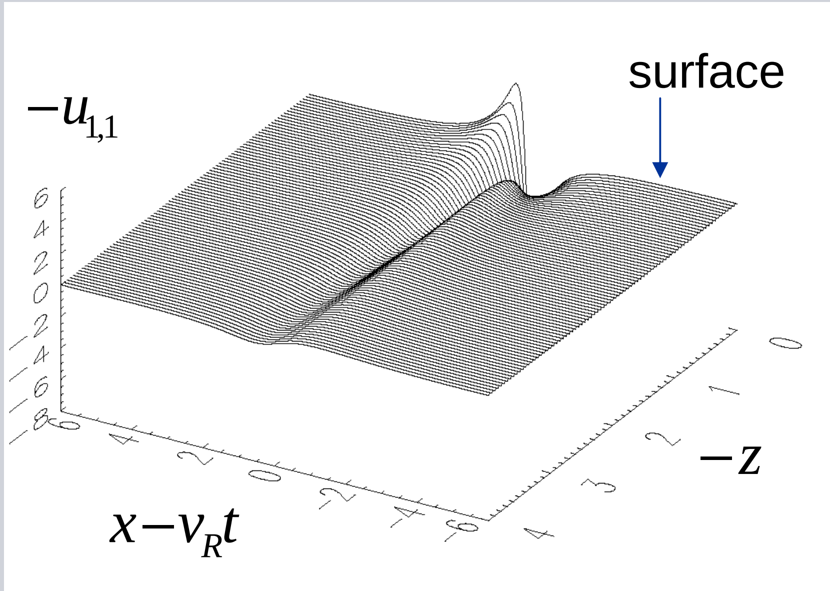
Strain / Particle Velocity





Fused silica

Si(111)[$\bar{1}\bar{1}$2]

Depth Profile

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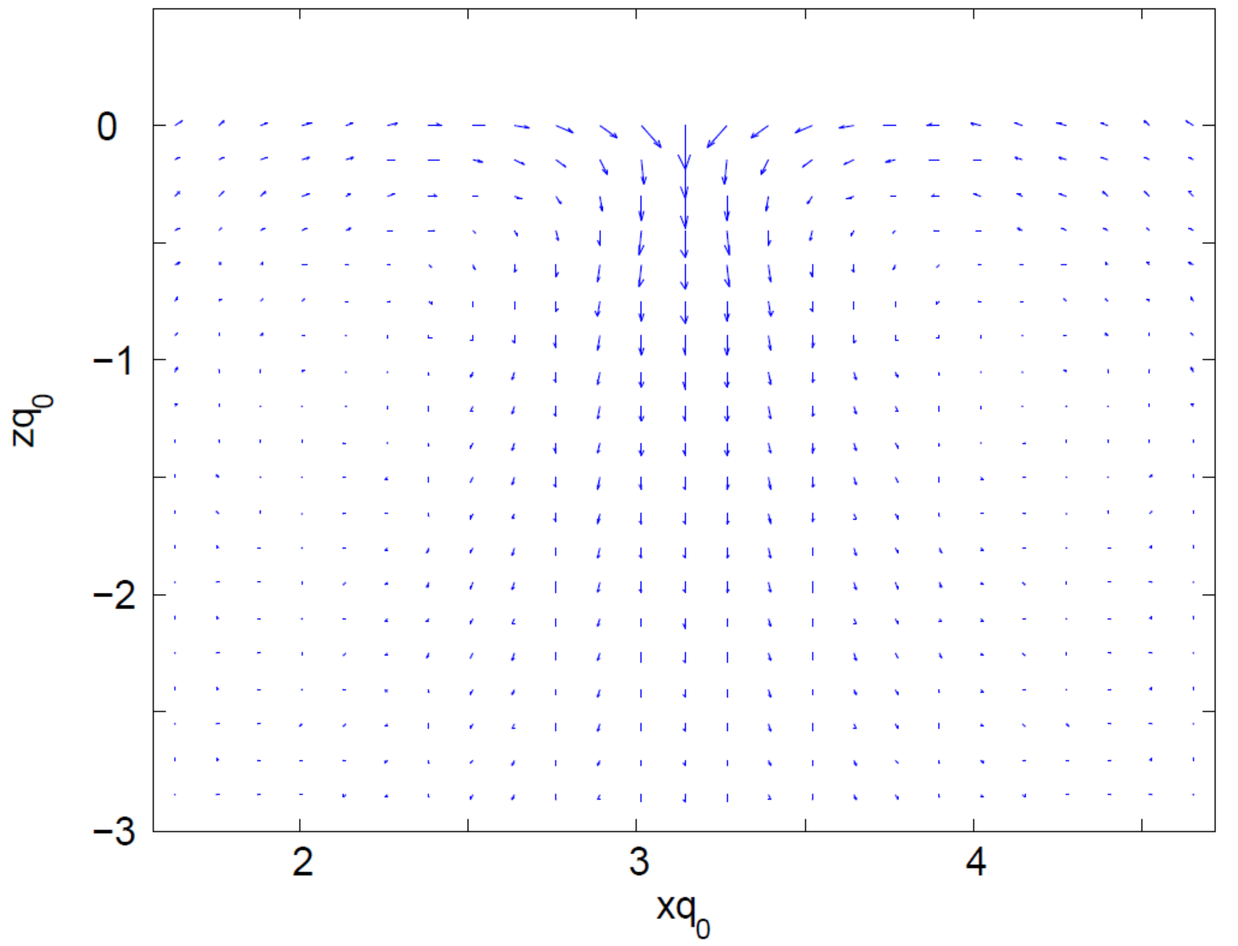
Displacement Pattern



Fused silica

Solitary Pulses

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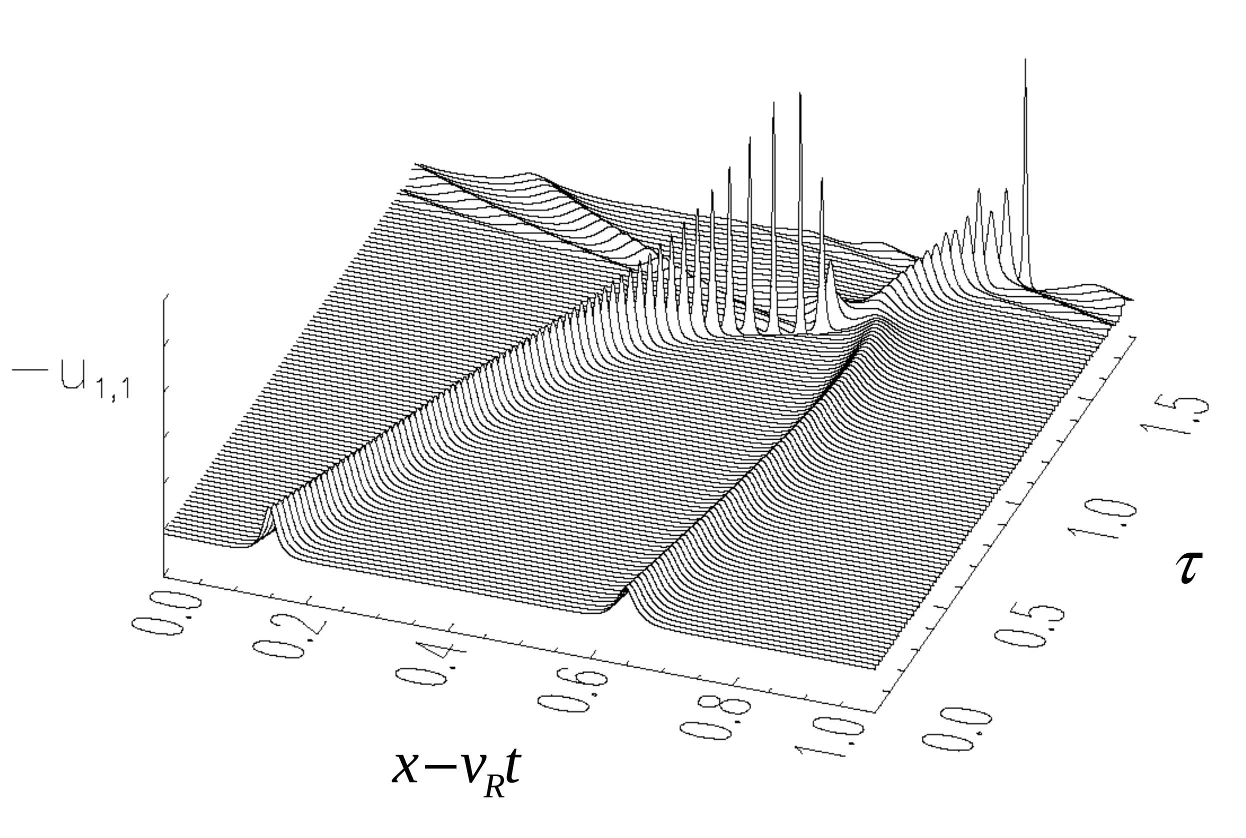
Definitions:

Solitary pulses propagate without change of their shape.

Solitons are solitary pulses which survive collisions with each other as intact pulses.

Pulse Collisions



Nonlinearity:

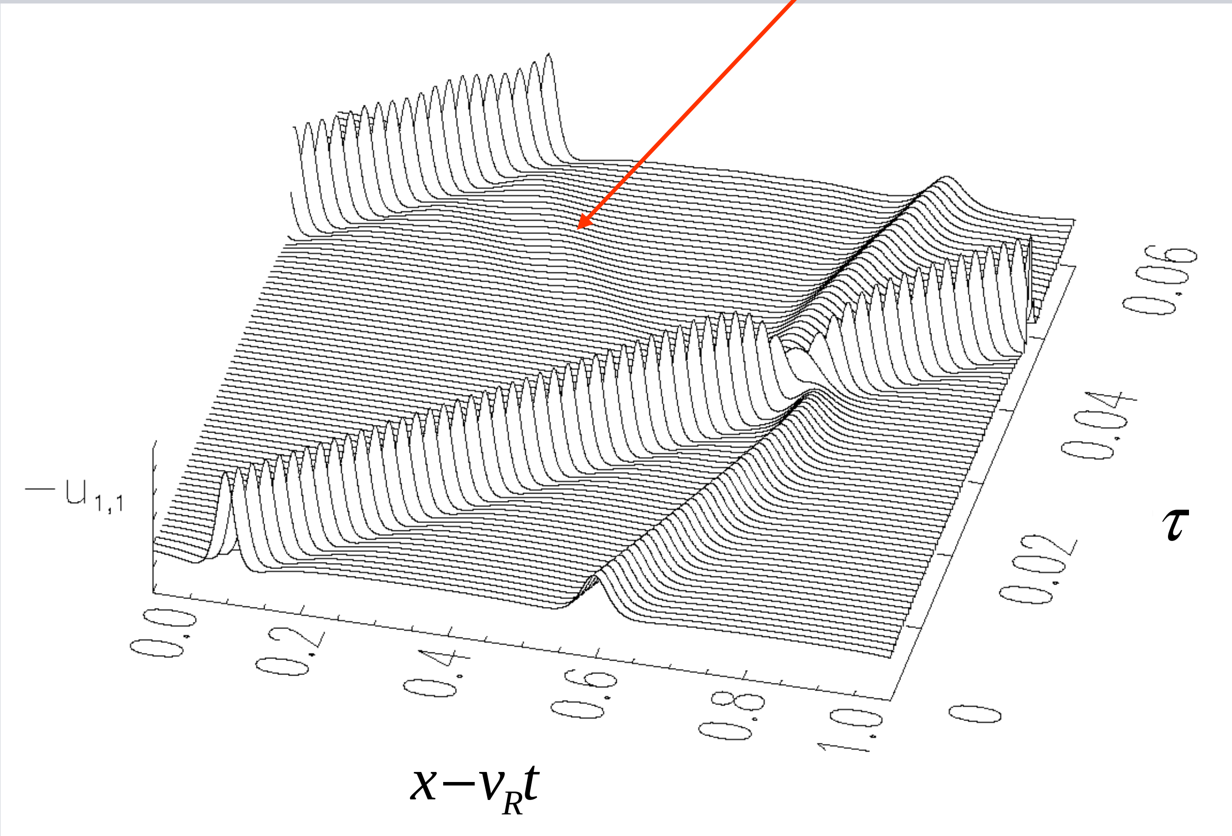
F = const.

Dispersion:

$$\omega(q) - v_R q = \Delta_0 q^2$$

Pulse Collisions

weak “radiation”



Nonlinearity:

F = const.

Dispersion:

$$\omega(q) - v_R q = \Delta_1 q^3$$

Mathematical Analysis

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Family of evolution equations:

$$\frac{\partial}{\partial\tau}U = \frac{1}{4}(5\lambda-1)\frac{\partial^3}{\partial\xi^3}U + (\lambda-1)\hat{H}\left[\frac{\partial}{\partial\xi}U\right]\frac{\partial}{\partial\xi}U + \frac{\partial}{\partial\xi}\left\{\hat{H}\left[U\frac{\partial}{\partial\xi}U\right] + (2\lambda-1)U\hat{H}\left[\frac{\partial}{\partial\xi}U\right] - \frac{4}{9}\lambda U^3\right\}$$

has solitary wave solution **independent of parameter λ !**

$$U(\xi,\tau) = \frac{-3\sqrt{12\kappa}}{4\kappa(\xi-\kappa\tau-\xi_0)^2+3}$$

$\hat{H}$: Hilbert transform w.r.t. ξ

$\lambda = 0$: **Evolution equation for nonlinear dispersive SAW**
$U \propto u_3$ **, approximation: F=const.**

$\lambda = 1$: **BO_3-equation**
has multi-soliton solutions.

1D Non-Dispersive Acoustic Waveguides

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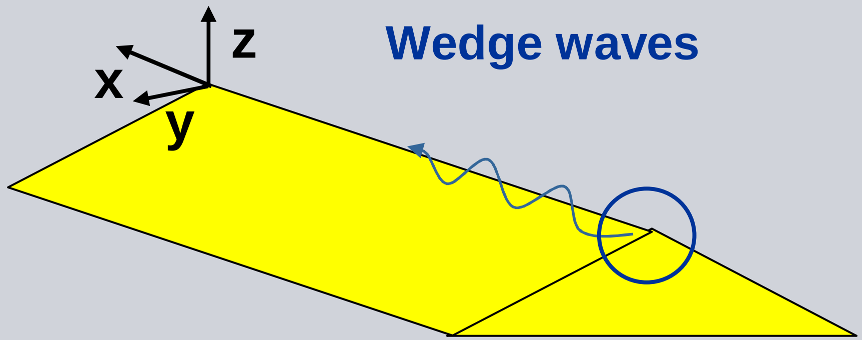
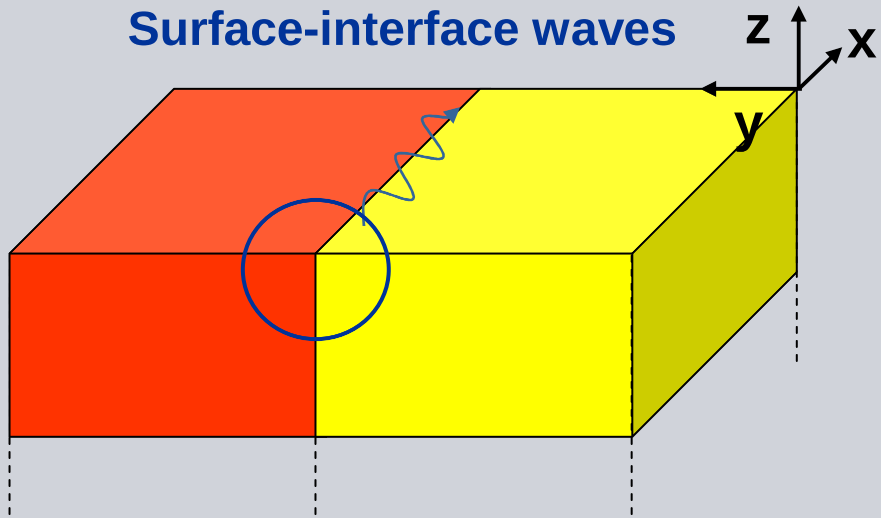


Displacement gradient associated with linear guided waves:

$$u_{\alpha,1}(x,y,z,t) =$$
$$\exp[iq(x - vt)]\, w_\alpha(y,z \mid q)\, B(q) + c.c.$$



Wedge Acoustic Waves

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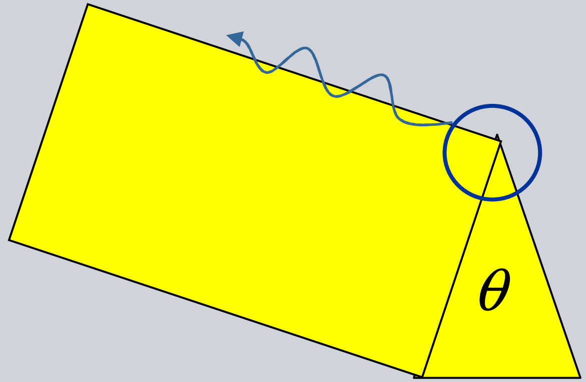
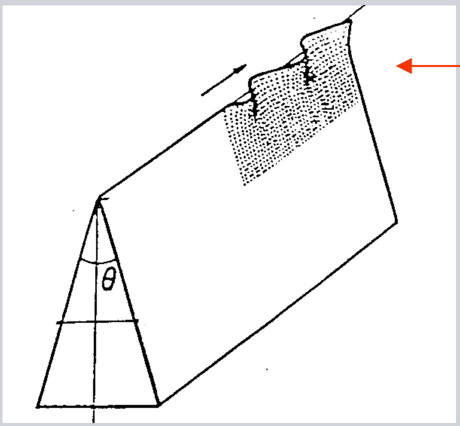


Predicted 1972 by

Lagasse
(FEM)

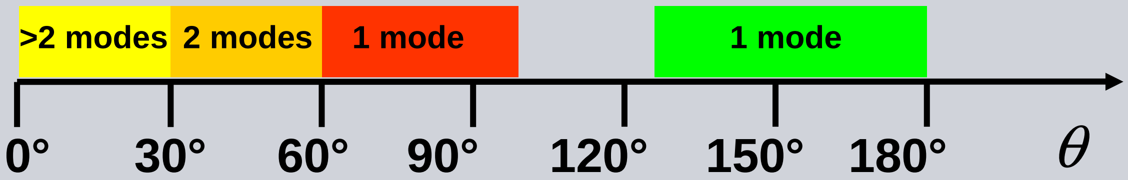
Maradudin et al.
(Laguerre function expansion)

Findings for isotropic medium in Poisson case $(\lambda = \mu)$:



Odd (flexural) modes

Even modes
(weakly localized)



Evolution Equation

for even wedge modes:

$$i\frac{\partial}{\partial\tau}B(q)=q^2\Delta_W(q)\;B(q)$$

$$+qv_W\left[\int_0^q F\left(\frac{k}{q}\right)B(k)B(q-k)\,dk+2\int_q^\infty\left(\frac{q}{k}\right)^2 F\left(\frac{q}{k}\right)^* B(k)B(k-q)^*\,dk\right]$$

(Krylov, Parker)

Summary

- Theoretical description of nonlinear acoustic pulse evolution on solid surfaces
- Strong influence of anisotropy on nonlinear pulse evolution
- Frequency up-conversion relative to down-conversion more efficient than in case of BAWs,
  (would be even more efficient in 1D acoustic waveguides)
- Solitary SAW and solitary edge pulses exist.
  They are not solitons.

Cooperations

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