Understanding Kondo-Peak Splitting
via nonperturbative dynamical theory

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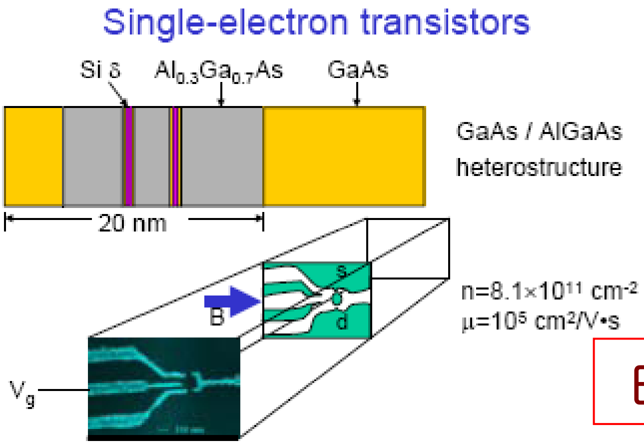
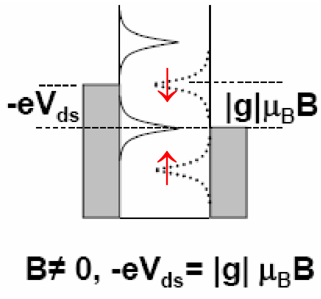
1. Experiment on Single Electron Transistor under Bias and Field (Nonequilibrium Kondo Phenomenon)
2. Theoretical Difficulties & New Approach
3. Nonperturbative Dynamical Theory: SIAM & SET
4. Understanding Nonequilibrium Kondo Phenomenon & Explaining Kondo–Peak Splitting

T. U. Dresden, Jan. 15, 2008

Kondo-peak Splitting by B-field

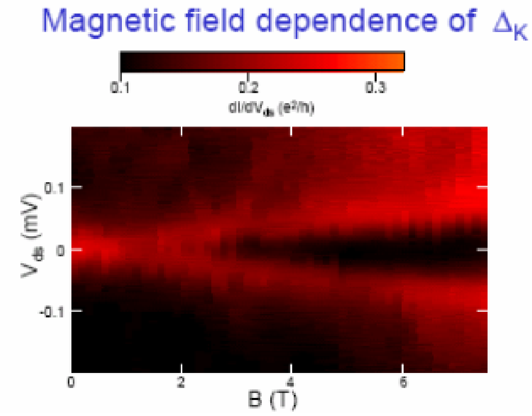


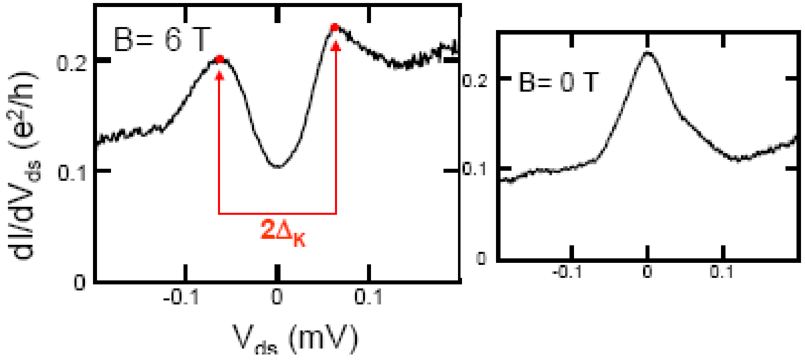


$B\neq0$

Change V_g

$V_{ds}\neq0$



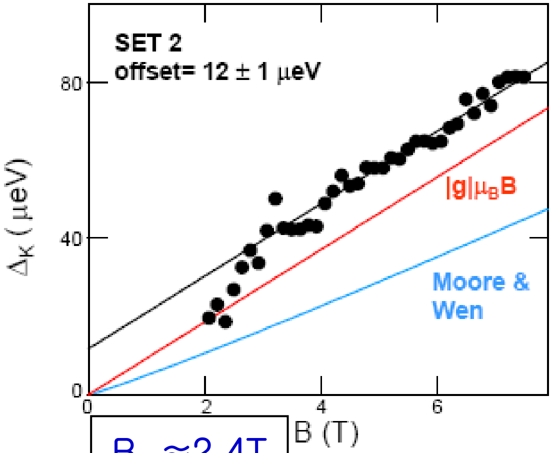


Kogan et al., **PRL 2004**
Amasha et al., **PRB 2005**

There are something more than Zeeman Splitting with $V_{ds}\neq0$

Features of Kondo-peak Splitting

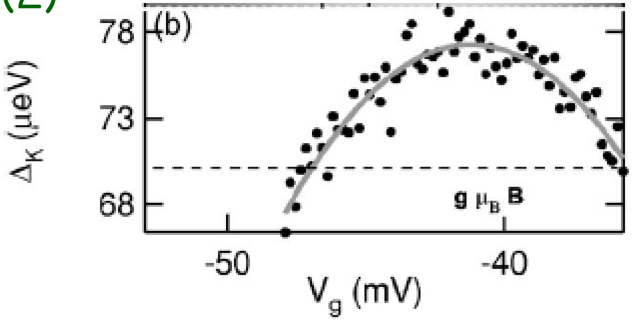
(1)

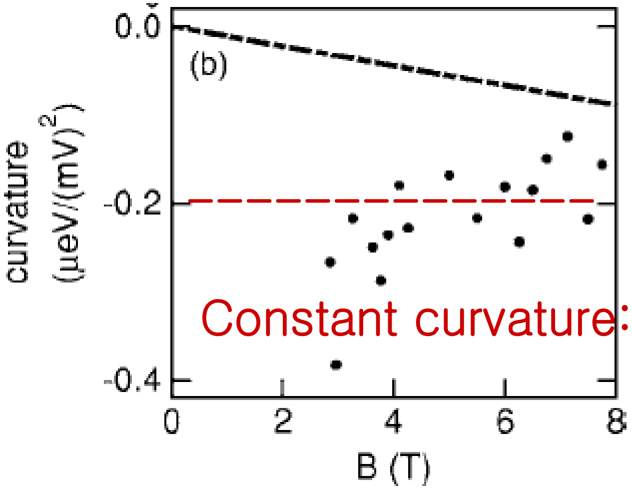


$B_C \approx 2.4T$

(2)

Parabolic





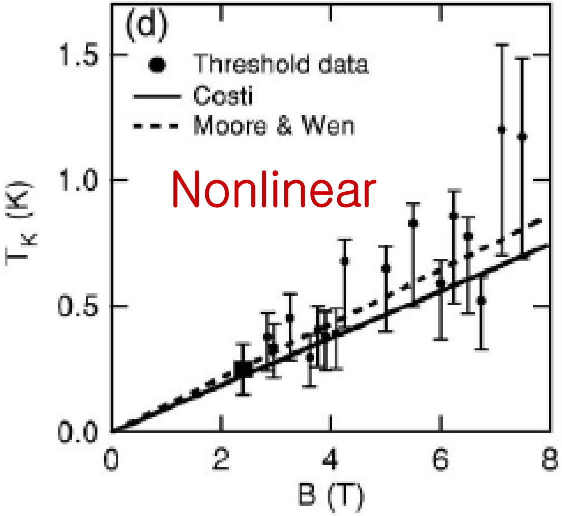
Constant curvature: −0.2

(3)

$$\Delta_K = a - b\ln T_K$$

$$b_{exp} \approx 12\mu eV$$

(4)



Nonlinear

unexplained by existing theories!

Why?

Origin of Theoretical Difficulties

(1) Nonequilibrium:

- **Nonequilibrium physics is poorly understood: No unifying theory like ensemble theory exists.**

- **Many of existing concepts, scaling, RG, etc. may not applicable.**

- **Nonequilibrium situations are different from system to system.**

(2) Strong Correlation:

- **Nonperturbative approach is required.**

- **Successful theories, BA, NRG, are static theories that are irrelevant in treating nonequilibrium situation.**

Existing Nonequilibrium Transport Theory & its Difficulty

Nonequilibrium Green's Function Method by

Keldysh or Kadanoff-Baym formulation:

$$J = \frac{e}{2\hbar}\int \frac{d\omega}{2\pi}\{i[\Gamma^{L}(\omega)-\Gamma^{R}(\omega)]\mathbf{G}^{<}_{dd}(\omega) \\ -2[f_L(\omega)\Gamma^{L}(\omega)-f_R(\omega)\Gamma^{R}(\omega)]\,\mathrm{Im}\mathbf{G}^{+}_{dd}(\omega)\}$$

Meir-Wingreen (1992)

The difficulty in Meir-Wingreen's Formula is to obtain $G^{<}_{dd\uparrow}(\omega)$ & $G^{+}_{dd\uparrow}(\omega)$ at $V \neq 0$

Expecting functional form of $G^{+}_{dd\uparrow}(\omega)|_{v\neq 0}$:

$$G^{+}_{dd\uparrow}(\omega) = f(\langle n_{d\downarrow}\rangle, \langle \rightarrow_{\downarrow}\rangle, \langle \leftarrow_{\downarrow}\rangle)|_{v\neq 0}$$

Since $(\langle \rightarrow_{\downarrow}\rangle, \langle \leftarrow_{\downarrow}\rangle)_{noneq}$ cannot be obtained by static theories, NRG or BA,

We need a new conceptual and theoretical tool!

For Nonequilibrium ➔ Dynamical Theory

For Strong Correlation ➔ Nonperturbative Theory

⇓

★ **New approach:**

Nonperturbative Dynamical Theory

- Searching for a NDT -

JH & W. Woo, cond-mat/0701765

Quantum Mechanics Revisited

Schrödinger vs. Heisenberg (I)

Schrödinger Eq.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t)=H\Psi(x,t)$$

Formal Solutions:

$$\Psi(x,t)=e^{-iHt/\hbar}\Psi(0)$$

$$=\Psi(0)+({}^{-it}\!/\!_{\hbar})H\Psi(0)+\tfrac{1}{2!}({}^{-it}\!/\!_{\hbar})^2H^2\Psi(0)$$

$$+\cdots$$

$$=\sum_n a_n(t)u_n(x)$$

$\{u_n\}$: complete set of a Hilbert space

Eigenfunction (static bases) Expansion

Heisenberg Eq.

$$i\hbar\frac{\partial}{\partial t}A(t)=-[H,A(t)]=-\mathbf{L}\,A(t)$$

$$A(t)=e^{i\mathbf{L}t/\hbar}A(0)$$

$$=A(0)+({}^{it}\!/\!_{\hbar})[H,A]+\tfrac{1}{2!}({}^{it}\!/\!_{\hbar})^2[H,[H,A]]$$

$$+\cdots$$

$$=\sum_n a_n(t)e_n$$

$\{e_n\}$: complete set of a Liouville space (operator space)

Expansion by dynamical bases

Dynamical Bases

Example: Anderson model

$$H=\Sigma_{k\sigma}\ \varepsilon_k c^\dagger_{k\sigma}c_{k\sigma}+\Sigma_{k\sigma}(V_{kd}c^\dagger_{d\sigma}c_{k\sigma}+V^*_{kd}c^\dagger_{k\sigma}c_{d\sigma})+Un_{d\sigma}n_{d\text{-}\sigma}$$

$$\boxed{c_{d\uparrow}(t)=c_{d\uparrow}+it[H,c_{d\uparrow}]-(t^2/2)[H,[H,c_{d\uparrow}]]+\cdots}$$

$$\begin{aligned}
&c_{d\uparrow}\\
&[H,c_{d\uparrow}]=Vc_{k\uparrow}-Un_{d\downarrow}c_{d\uparrow}\\
&[H,[H,c_{d\uparrow}]]=V\varepsilon_l c_{l\uparrow}-2VUn_{d\downarrow}c_{k\uparrow}+U^2n^2_{d\downarrow}c_{d\uparrow}-U\,j^-_{d\downarrow}c_{d\uparrow}\\
&[H,[H,[H,c_{d\uparrow}]]]=V\varepsilon^2_m c_{m\uparrow}-3V\varepsilon_l\,U\,n_{d\downarrow}c_{l\uparrow}+2VU^2\,n^2_{d\downarrow}c_{k\uparrow}-U^3n_{d\downarrow}c_{d\uparrow}\\
&\qquad\qquad -VU\,j^-_{d\downarrow}c_{k\uparrow}+U^2\,j^-_{d\downarrow}n_{d\downarrow}c_{d\uparrow}+iU^2\,j^+_{d\downarrow}n_{d\uparrow}c_{d\uparrow}\\
&[H,[H,[H,[H,c_{d\uparrow}]]]]=\cdots+iVU^2\,j^+_{d\downarrow}n_{d\uparrow}c_{k\uparrow}-iU^3\,j^+_{d\downarrow}n_{d\uparrow}n_{d\downarrow}c_{d\uparrow}+\cdots\\
&\quad\vdots
\end{aligned}$$

Schrödinger vs. Heisenberg (II)

Dynamical variable: $\Psi(t) \Leftarrow\!\Rightarrow A(t)$

Driving operator: $H \Leftarrow\!\Rightarrow \mathbf{L}=[H,A]$

Basis vectors: $\{u_n\} \Leftarrow\!\Rightarrow \{e_n\}$

- Characteristics of Basis Vectors:

  $\{u_n\}$: Static Bases

  $\{e_n\}$: Dynamical Bases

- Resolvent Green's Function:

  $G^{\pm}_{ij}(\omega)=<u_i|(\omega\pm i\delta-\hat{H})^{-1}|u_j> \Leftarrow\!\Rightarrow$

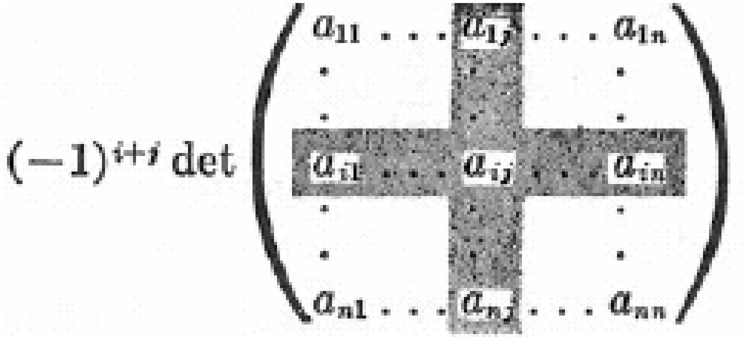
  $G^{\pm}_{\mathbf{ij}}(\omega)=<\psi_0^N|\{c_{\mathbf{i}},(\omega\pm i\delta-\mathbf{L})^{-1}c_{\mathbf{j}}\dagger\}|\psi_0^N>$ (Fulde's book)

$$iG^{+}_{ij\sigma}(\omega)=\int_0^{\infty}\langle\{c_{i\sigma}(t),c^{\dagger}_{j\sigma}\}\rangle e^{i\omega t-\eta t}dt=\langle c_{j\sigma}|(z\mathbf{I}+i\mathbf{L})^{-1}|c_{i\sigma}\rangle$$

$$\boxed{iG^{\pm}_{ij}(\omega)=(\text{cofactor of }\mathbf{M}_{ij})[\det\mathbf{M}]^{-1},\ z=-i\omega\pm\eta}$$

where $\mathbf{M}_{ij}=z\delta_{ij}-\langle\{i\mathbf{L}e_j,e_i^{\dagger}\}\rangle$, $z=-i\omega+\eta$, $\{e_i\}$: normalized bases set

Cofactor of a_{ij} in det A =



$$(-1)^{i+j}\det\begin{pmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n}\\ \vdots & & & & \vdots\\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in}\\ \vdots & & & & \vdots\\ a_{n1} & \cdots & a_{nj} & \cdots & a_{nn}\end{pmatrix}$$

★ Essence of NDT: Constructing dynamical bases $\{e_j\}$

Then, Constructing Matrix M ➔ Matrix Reduction ➔ Calculating M^{-1} is straightforward

The Paradigm of Nonperturbative Dynamical Theory

Picture: Heisenberg instead of Shrödinger

Space: Liouville instead of Hilbert

Bases: Dynamical instead of Static

*Simplification: Bases instead of Hamiltonian

▼

Construct the resolvent Green's function matrix,
and transform ∞×∞ matrix into n×n

▼

Matrix inversion for $\mathbf{G}^{+}_{dd\uparrow}(\omega)|_{\mathbf{v}\neq\mathbf{0}}$

We will try a simplest system first.

Strongly Correlated System: Quantum Impurities

Nonequilibrium: Steady-State

⇓

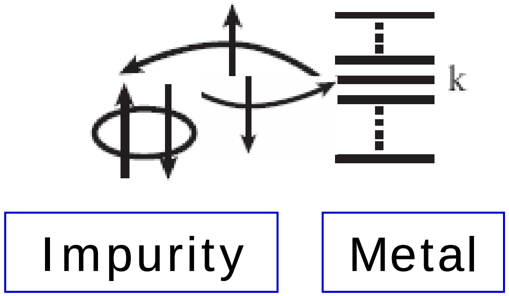
Single Electron Transistor with a Quantum Dot

We apply the NDT to Equilibrium Kondo Problem, then go to Non-Equilibrium case

Example: Single Impurity Anderson Model

$$\hat{H} = \sum_{\sigma} \epsilon_d c^\dagger_{d\sigma} c_{d\sigma} + \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$
$$+ \sum_{k,\sigma} (V_{kd} c^\dagger_{d\sigma} c_{k\sigma} + V^*_{kd} c^\dagger_{k\sigma} c_{d\sigma}) + U n_{d\uparrow} n_{d\downarrow}$$

Dynamical process in the large-U regime



Bases $\{e_i\}$:

Metal: $\{c_{k\uparrow}\}$, $k = 1, 2, \cdots, \infty$

Metal-Impurity (U): $\{\delta n_{d\downarrow} c_{k\uparrow}\}$, $\boxed{\{\delta j^-_{d\downarrow} c_{k\uparrow}\}}$, $\boxed{\{\delta j^+_{d\downarrow} c_{k\uparrow}\}}$

$k = 1, 2, \cdots, \infty$

Impurity-Metal (no U): $\{c_{d\uparrow}, \delta j^-_{d\downarrow} c_{d\uparrow}, \delta j^+_{d\downarrow} c_{d\uparrow}\}$

$c_{d\uparrow}$ is orthogonal to all other bases!

Then we obtain the correct projection $<\{c_{d\sigma}(t), c^\dagger_{d\sigma}\}>$ for the Green's function

Dynamical Bases

Example: Anderson model

$$H=\Sigma_{k\sigma}\ \varepsilon_k c^\dagger_{k\sigma}c_{k\sigma}+\Sigma_{k\sigma}(V_{kd}c^\dagger_{d\sigma}c_{k\sigma}+V^*_{kd}c^\dagger_{k\sigma}c_{d\sigma})+Un_{d\sigma}n_{d\text{-}\sigma}$$

$$\boxed{c_{d\uparrow}(t)=c_{d\uparrow}+it[H,c_{d\uparrow}]-(t^2/2)[H,[H,c_{d\uparrow}]]+\cdots}$$

$$c_{d\uparrow}$$

$$[H,c_{d\uparrow}]=Vc_{k\uparrow}-Un_{d\downarrow}c_{d\uparrow}$$

$$[H,[H,c_{d\uparrow}]]=V\varepsilon_l c_{l\uparrow}-2VUn_{d\downarrow}c_{k\uparrow}+U^2n^2_{d\downarrow}c_{d\uparrow}-U\,j^-_{d\downarrow}c_{d\uparrow}$$

$$\begin{aligned}[H,[H,[H,c_{d\uparrow}]]]=&V\varepsilon^2_m c_{m\uparrow}-3V\varepsilon_l U\,n_{d\downarrow}c_{l\uparrow}+2VU^2\,n^2_{d\downarrow}c_{k\uparrow}-U^3n_{d\downarrow}c_{d\uparrow}\\&-VU\,j^-_{d\downarrow}c_{k\uparrow}+U^2\,j^-_{d\downarrow}n_{d\downarrow}c_{d\uparrow}+iU^2\,j^+_{d\downarrow}n_{d\uparrow}c_{d\uparrow}\end{aligned}$$

$$[H,[H,[H,[H,c_{d\uparrow}]]]]=\cdots+iVU^2\,j^+_{d\downarrow}n_{d\uparrow}c_{k\uparrow}-iU^3\,j^+_{d\downarrow}n_{d\uparrow}n_{d\downarrow}c_{d\uparrow}+\cdots$$

$$\vdots$$

Constructing Matrix M: $M_{ij}=z\delta_{ij}-\langle\{i\mathbf{L}\hat{e}_j,\hat{e}_i^\dagger\}\rangle$

Reduced Liouville Space

M_{SIAM}:

$$
\begin{array}{c|cc|ccc}
\text{Bases:} & \{c_{k\uparrow}\} & \{\delta n_{d\downarrow}c_{k\uparrow}\} & c_{d\uparrow} & \delta j^-_{d\downarrow}c_{d\uparrow} & \delta j^+_{d\downarrow}c_{d\uparrow} \\
\hline
\{c_{k\uparrow}\} & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} \; M_{kk} & 0 & \begin{pmatrix} iV_{kd} \\ \vdots \\ iV_{kd} \end{pmatrix} & 0 \quad M_{dk} & 0 \\
\{\delta n_{d\downarrow}c_{k\uparrow}\} & 0 & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} & 0 & \begin{pmatrix} \xi_d^- V_{kd} \\ \vdots \\ \xi_d^- V_{kd} \end{pmatrix} & \begin{pmatrix} \xi_d^+ V_{kd} \\ \vdots \\ \xi_d^+ V_{kd} \end{pmatrix} \\
\hline
c_{d\uparrow} & (iV^*_{kd}\cdots iV^*_{kd}) & 0 & \tilde{z} & \zeta_d^- U/2 & \zeta_d^+ U/2 \\
\delta j^-_{d\downarrow}c_{d\uparrow} & 0 & (-\xi_d^{-*}V^*_{kd}\cdots-\xi_d^{-*}V^*_{kd}) & -\zeta_d^- U/2 & \tilde{z} & \gamma \\
\delta j^+_{d\downarrow}c_{d\uparrow} & 0 & M_{kd} \; (-\xi_d^{+*}V^*_{kd}\cdots-\xi_d^{+*}V^*_{kd}) & -\zeta_d^+ U/2 & -\gamma \quad M_{dd} & \tilde{z}
\end{array}
$$

where
$\tilde{z}=z+i(\varepsilon_d+\langle n_{d\downarrow}\rangle U)$

$$\xi_d^{\pm}=(i/2)\frac{\langle[n_{d\downarrow,}j^{\pm}_{d\downarrow}](1-2n_{d\uparrow})\rangle+(1-2\langle n_{d\downarrow}\rangle)\langle j^{\pm}_{d\downarrow}\rangle}{\sqrt{\langle(\delta n_{d\downarrow})^2\rangle}\sqrt{\langle(\delta j^{\pm}_{d\downarrow})^2\rangle}}=\frac{\zeta_d^{\pm}}{2\sqrt{\langle(\delta n_{d\downarrow})^2\rangle}}$$

$$\gamma=\langle\sum_k V^*_{kd}\,c_{k\uparrow}c^+_{d\uparrow}j^-_{d\downarrow}j^+_{d\downarrow}\rangle/[\sqrt{(\delta j^-_{d\downarrow})^2}\sqrt{(\delta j^+_{d\downarrow})^2}]$$

Matrix Reduction by Löwdin's partitioning technique

P.O. Löwdin, J. Math. Phys. 3, 969 (1962)

Mujica et al., J. Chem. Phys. 101, 6849 (1994)

$$\mathrm{M}_{\mathrm{SIAM}} = \begin{pmatrix} \mathrm{M}_{kk} & \mathrm{M}_{dk} \\ -\mathrm{M}^{*}_{dk} & \mathrm{M}_{dd} \end{pmatrix},$$

$$\mathrm{M}_{\mathrm{SIAM}}\mathrm{C} = 0, \qquad \mathbf{C} = \begin{pmatrix} \mathbf{C}_k \\ \mathbf{C}_d \end{pmatrix} \qquad \begin{pmatrix} \mathrm{M}_{kk} & \mathrm{M}_{dk} \\ -\mathrm{M}^{*}_{dk} & \mathrm{M}_{dd} \end{pmatrix}\begin{pmatrix} \mathbf{C}_k \\ \mathbf{C}_d \end{pmatrix} = \mathbf{0}$$

Equation for $\mathrm{C}_{\mathbf{d}}$:

$$(\mathrm{M}_{dd} - \mathrm{M}_{kd}\mathrm{M}_{kk}^{-1}\mathrm{M}_{dk})\mathrm{C}_d \equiv \widetilde{\mathrm{M}}_{dd}\mathrm{C}_d = 0$$

Obtaining $\mathrm{M}_{\mathbf{kk}}{}^{\mathbf{-1}}$ is possible when $\mathrm{M}_{\mathbf{kk}}$ is block diagonal

Reduced M-Matrix for the Symmetric Anderson Model:

$$\widetilde{\mathbf{M}}_{dd} = \begin{pmatrix} -i\omega + i\Sigma_0 & U\zeta_d/2 & U\zeta_d/2 \\ -U\zeta_d/2 & -i\omega + i\xi_d^2\Sigma_0 & \gamma + i\xi_d^2\Sigma_0 \\ -U\zeta_d/2 & -\gamma + i\xi_d^2\Sigma_0 & -i\omega + i\xi_d^2\Sigma_0 \end{pmatrix}$$

$$\boxed{\xi_d^{\mp} = \xi_d = \zeta_d}$$

Retarded Green's Function: $iG^{+}_{dd}(\omega) = (\mathrm{adj}\ \widetilde{\mathbf{M}}_{dd})_{11}[\det \widetilde{\mathbf{M}}_{dd}]^{-1}$

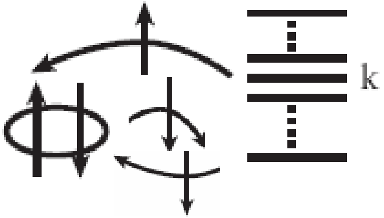
- ♦ $\zeta_{\mathbf{d}}$ governs the positions of incoherent peak
- ♦ γ governs the width of coherent peak
- ♦ At the atomic limit, $|\mathrm{Re}(\zeta_{\mathbf{d}})| = 1/\sqrt{2}$

Spectral weight at $\omega = 0$: $Z_S = [1 + (U^2/4\gamma^2)]^{-1}$ or from $\mathrm{Re}\Sigma(\omega)$

$$\widetilde{\gamma} = \langle \sum_k V_{kd}^{*}\, c_{k\uparrow} c_{d\uparrow}^{+} j_{d\downarrow}^{-} j_{d\downarrow}^{+} \rangle :$$

$$j_{d\downarrow}^{-} = i\sum_{\mathbf{k}} (V_{\mathbf{k}d}\underline{c_{\mathbf{k}\downarrow}^{+} c_{d\downarrow}} - V_{\mathbf{k}d}^{*} c_{d\downarrow}^{+} c_{\mathbf{k}\downarrow}),$$

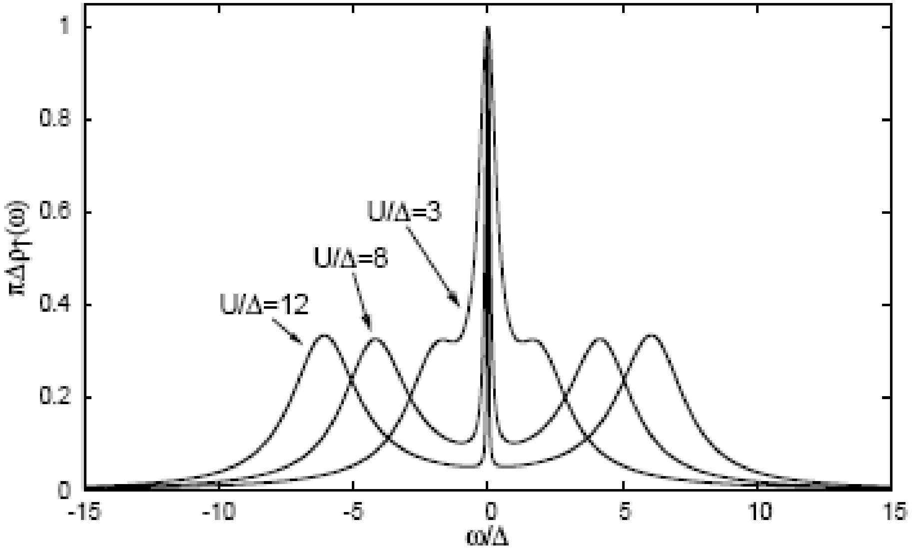
$$j_{d\downarrow}^{+} = \sum_{\mathbf{k}} (V_{\mathbf{k}d} c_{\mathbf{k}\downarrow}^{+} c_{d\downarrow} + V_{\mathbf{k}d}^{*}\underline{c_{d\downarrow}^{+} c_{\mathbf{k}\downarrow}})$$



Kondo process

Spectral Densities

$$\rho_{\uparrow}(\omega) = -(1/\pi)\ \mathrm{Im}G^{+}_{dd\uparrow}(\omega)$$



$$\gamma = (U/2)\sqrt{\frac{Z_S}{1-Z_S}},\ Z_S = Z_S^{\mathrm{BA}}$$

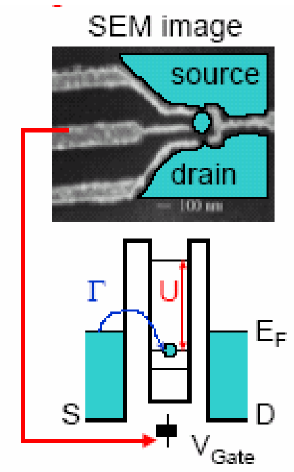
$$Z_S^{\mathrm{BA}} = (4/\pi)\sqrt{U/\Gamma}\ e^{-\frac{\pi U}{4\Gamma}}$$

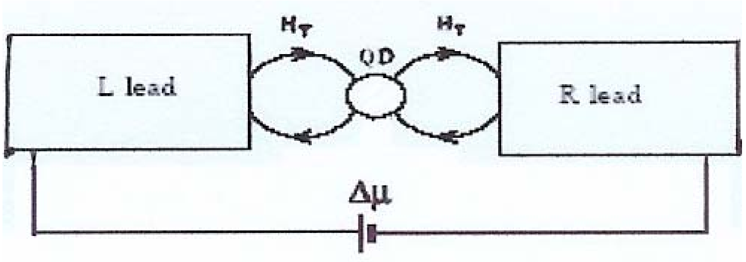
Unfortunately, γ is hard to obtain directly.
We borrow Z_S from the static theory, BA.

Since the validity of the NDT has been checked for the SIAM in equilibrium, we now go to the nonequilibrium Kondo problem.

- Nonequilibrium Kondo -

Single Electron Transistor under Bias





$$H = H_L + H_R + H_T + H_{QD}$$

$$H_{L,R} = \Sigma_{k\sigma}\ \varepsilon_k c^{\dagger L,R}_{k\sigma} c^{L,R}_{k\sigma}$$

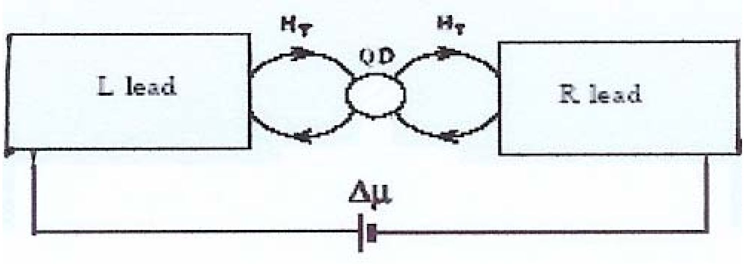
$$H_T = \Sigma_{k\sigma}(V_{kd} c^{\dagger}_{d\sigma} c^{L,R}_{k\sigma} + V^*{}_{kd} c^{\dagger L,R}_{k\sigma} c_{d\sigma})$$

$$H_{QD} = \Sigma_{\sigma}\varepsilon_d c^{\dagger}_{d\sigma} c_{d\sigma} + U\Sigma_{\sigma} n_{d\sigma} n_{d-\sigma}$$

$$\boxed{\varepsilon_d \Rightarrow \varepsilon_d \mp g|\mu_B|B \text{ when } \mathbf{B} = B\hat{z} \text{ is applied.}}$$

The simplest system of nonequil.+ strong correl.

Bases for the Anderson model with 2 reservoirs



$S_k^L = \{c_{k\uparrow}^L\}$, where $k = 1, 2, \cdots, \infty$

Indexes *L* & *R*: Leads

$S_n^L = \{\delta n_{d\downarrow} c_{k\uparrow}^L\}$,

$S_d = \{\delta j_{d\downarrow}^{-L} c_{d\uparrow}, \delta j_{d\downarrow}^{+L} c_{d\uparrow}, c_{d\uparrow}, \delta j_{d\downarrow}^{+R} c_{d\uparrow}, \delta j_{d\downarrow}^{-R} c_{d\uparrow}\}$,

$S_n^R = \{c_{k\uparrow}^R\}$

$S_k^R = \{\delta n_{d\downarrow} c_{k\uparrow}^R\}$,

$$\mathrm{M}_{\ell d\ell} = \begin{pmatrix} \mathrm{M}_{LL} & \mathrm{M}_{dL} & 0 \\ \mathrm{M}_{Ld} & \mathrm{M}_{dd} & \mathrm{M}_{Rd} \\ 0 & \mathrm{M}_{dR} & \mathrm{M}_{RR} \end{pmatrix}, \quad \mathbf{M}_{\mathbf{dd}} : 5 \times 5 \text{ matrix}$$

Constructing Matrix M: $\boxed{M_{ij}=z\delta_{ij}-<\{i\mathbf{L}\hat{e}_j,\hat{e}_i^\dagger\}>}$

Reduced Liouville Space

Bases:

$$
\begin{array}{c|cc|ccccc|cc}
 & \boxed{c^L_{k\uparrow}} & \boxed{\delta n_{d\downarrow}c^L_{k\uparrow}} & \delta j^{-L}_{d\downarrow}c_{d\uparrow} & \delta j^{+L}_{d\downarrow}c_{d\uparrow} & c_{d\uparrow} & \delta j^{+R}_{d\downarrow}c_{d\uparrow} & \delta j^{-R}_{d\downarrow}c_{d\uparrow} & \boxed{\delta n_{d\downarrow}c^R_{k\uparrow}} & \boxed{c^R_{k\uparrow}} \\
\hline
\boxed{c^L_{k\uparrow}} & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} & M_{LL} \quad 0 & 0 & 0 & \begin{pmatrix} iV_{kd} \\ \vdots \\ iV_{kd} \end{pmatrix} & 0 & 0 & 0 & \\
\boxed{\delta n_{d\downarrow}c^L_{k\uparrow}} & 0 & \begin{pmatrix} z+i\varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & z+i\varepsilon_\infty \end{pmatrix} & \begin{pmatrix} \xi_d^- V_{kd} \\ \vdots \\ \xi_d^- V_{kd} \end{pmatrix} & \begin{pmatrix} \xi_d^+ V_{kd} \\ \vdots \\ \xi_d^+ V_{kd} \end{pmatrix} & M_{Ld} \quad 0 & \begin{pmatrix} \xi_d^+ V_{kd} \\ \vdots \\ \xi_d^+ V_{kd} \end{pmatrix} & \begin{pmatrix} \xi_d^- V_{kd} \\ \vdots \\ \xi_d^- V_{kd} \end{pmatrix} & & \\
\hline
\delta j^{-L}_{d\downarrow}c_{d\uparrow} & 0 & (-\xi_d^{-*}V_{kd}^* \cdots -\xi_d^{-*}V_{kd}^*) & & & & & & & \\
\delta j^{+L}_{d\downarrow}c_{d\uparrow} & 0 & (-\xi_d^{+*}V_{kd}^* \cdots -\xi_d^{+*}V_{kd}^*) & & & M_{dd}\ (5X5) & & & M_{dR}\ (\infty X5) & \\
c_{d\uparrow} & (iV_{kd}^* \cdots iV_{kd}^*) & M_{dL} \quad 0 & & & & & & & \\
\delta j^{+R}_{d\downarrow}c_{d\uparrow} & 0 & (-\xi_d^{+*}V_{kd}^* \cdots -\xi_d^{+*}V_{kd}^*) & & & & & & & \\
\delta j^{-R}_{d\downarrow}c_{d\uparrow} & 0 & (-\xi_d^{-*}V_{kd}^* \cdots -\xi_d^{-*}V_{kd}^*) & & & & & & & \\
\hline
\boxed{\delta n_{d\downarrow}c^R_{k\uparrow}} & 0 & & & & M_{Rd}(5X\infty) & & & M_{RR}\ (\infty X\infty) & \\
\boxed{c^R_{k\uparrow}} & & & & & & & & &
\end{array}
$$

Matrix Reduction procedure:

$$\mathbf{M}_{\ell d\ell} = \begin{pmatrix} \mathrm{M}_{LL} & \mathrm{M}_{dL} & 0 \\ \mathrm{M}_{Ld} & \mathrm{M}_{dd} & \mathrm{M}_{Rd} \\ 0 & \mathrm{M}_{dR} & \mathrm{M}_{RR} \end{pmatrix}, \quad \rightarrow \quad \begin{pmatrix} \mathrm{M}_{LL} & \mathrm{M}_{dL} & 0 \\ \mathrm{M}_{Ld} & \mathrm{M}_{dd} & \mathrm{M}_{Rd} \\ 0 & \mathrm{M}_{dR} & \mathrm{M}_{RR} \end{pmatrix} \begin{pmatrix} \mathbf{C}_k^L \\ \mathbf{C}_d \\ \mathbf{C}_k^R \end{pmatrix} = \mathbf{0}$$

$$(\mathrm{M}_{dd} - \mathrm{M}_{Ld}\mathrm{M}_{LL}^{-1}\mathrm{M}_{dL} - \mathrm{M}_{\mathrm{Rd}}\mathrm{M}_{RR}^{-1}\mathrm{M}_{dR})\mathbf{C}_d \equiv \widetilde{\mathbf{M}}_{dd}\mathbf{C}_d = \mathbf{0}$$

$$\widetilde{\mathbf{M}}_{dd} = \mathrm{M}_{dd} - \mathrm{M}_{Ld}\mathrm{M}_{LL}^{-1}\mathrm{M}_{dL} - \mathrm{M}_{Rd}\mathrm{M}_{RR}^{-1}\mathrm{M}_{dR}$$

$$\Downarrow$$

Retarded Green's function: $iG_{\mathrm{dd}}^{+}(\omega) = (adj\,\widetilde{\mathbf{M}}_{\mathrm{dd}})_{33}[\det \widetilde{\mathbf{M}}_{\mathrm{dd}}]^{-1}$

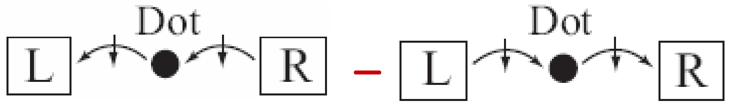
$$\widetilde{\mathbf{M}}_{dd}=\begin{bmatrix} -i\omega-\alpha_B & \gamma_{LL} & -U^{L}_{J^-} & -\gamma_{LR} & -\gamma_{J^-} \\ -\gamma_{LL} & -i\omega-\alpha_B & -U^{L}_{J^+} & -\gamma_{J^+} & -\gamma_{LR} \\ U^{L*}_{J^-} & U^{L*}_{J^+} & -i\omega-\alpha_B & U^{R*}_{J^+} & U^{R*}_{J^-} \\ \gamma_{LR} & \gamma_{J^+} & -U^{R}_{J^+} & -i\omega-\alpha_B & -\gamma_{RR} \\ \gamma_{J^-} & \gamma_{LR} & -U^{R}_{J^-} & \gamma_{RR} & -i\omega-\alpha_B \end{bmatrix}$$

with additional $i\beta_{ij}[\Sigma^L_{ij}(\omega)+\Sigma^R_{ij}(\omega)]$ except U−elements,

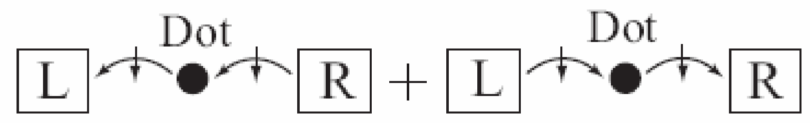
where $\Sigma^L_{ij}(\omega)=\Sigma^R_{ij}(\omega)=\Sigma_0(\omega)$, $\alpha_B=i[\varepsilon_d+U\langle n_{d\downarrow}\rangle-g|\mu_B|B]+0^+$

Consider the atomic limit ($\Sigma_0(\omega)=0$) with information

$$\tilde{\gamma}_{J^\mp}=\langle\sum_k(V^{L*}_{kd}c^{L}_{k\uparrow}+V^{R*}_{kd}c^{R}_{k\uparrow})c^{+}_{d\uparrow}[j^{\mp L}_{d\downarrow},j^{\mp R}_{d\downarrow}]\rangle$$



$$\tilde{\gamma}_{LR}=\langle\sum_k(V^{L*}_{kd}c^{L}_{k\uparrow}+V^{R*}_{kd}c^{R}_{k\uparrow})c^{+}_{d\uparrow}[j^{-L}_{d\downarrow},j^{+R}_{d\downarrow}]\rangle$$



$$\tilde{\gamma}_{LL(RR)}=\langle\sum_k(V^{L*}_{kd}c^{L}_{k\uparrow}+V^{R*}_{kd}c^{R}_{k\uparrow})c^{+}_{d\uparrow}[j^{-L,R}_{d\downarrow},j^{+L,R}_{d\downarrow}]\rangle$$

Resonant Levels & Their Spectral Weights

– Atomic limit analysis –

(1) Zeros of det $\widetilde{\mathbf{M}}_{\mathrm{dd}}$: $\omega = 0, \ \pm[\gamma_{LL}^2 + (\gamma_{LR} - \gamma_{J^\pm})^2 + O(U^{-2})]^{1/2}$,

and $\pm U/2$ at large $-U$

(2) Spectralweightat $\omega = 0$:

$$Z_S(0) = \left[1 + \frac{U^2\{\gamma_{LL}^2 + (\gamma_{LR} - \gamma_{J^\pm})^2\}}{4[\gamma_{LL}{}^4 + (\gamma_{LR}{}^2 - \gamma_{J^\pm}{}^2)(2\gamma_{LL}{}^2 + \gamma_{LR}{}^2 - \gamma_{J^\pm}{}^2)]}\right]^{-1}$$

In theKondoregime,

at $\gamma_{J^\pm} = 0, \ Z_S(0) = (4\gamma_{LL}^2/U^2) + (4\gamma_{LR}^2/U^2)$, andat $\gamma_{J^\pm} = \gamma_{LR}, \ Z_S(0) = 4\gamma_{LL}{}^2/U^2$

zero bias

saturated bias

(3) Spectral weight of new levels : $Z_S^{new} = \dfrac{\gamma_{LL}{}^2 + (\gamma_{LR} + \gamma_{J^\pm})^2 - Z_S(U^2/4)}{(U^2/4) - [\gamma_{LL}{}^2 + (\gamma_{LR} - \gamma_{J^\pm})^2]}$

at $\gamma_{J^\pm} = 0$, $\boxed{Z_S^{new} = 0}$, and at $\gamma_{J^\pm} = \gamma_{LR}$, $Z_S^{new} = \boxed{16\gamma_{LR}{}^2/U^2}$

zero bias

saturated bias

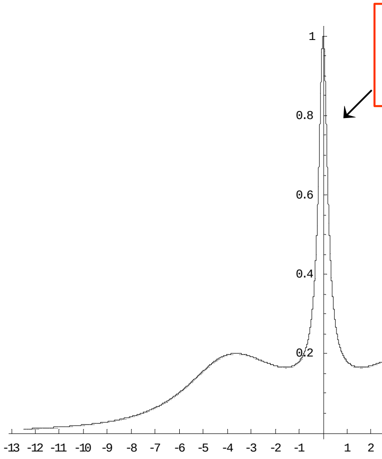
– summary of the atomic limit analysis –

(1) 5 zeros of $\det \widetilde{\mathbf{M}}_{\mathrm{dd}}$ imply that three resonant levels ($\omega = 0, \ \pm \gamma_{\mathrm{LL}}$) with two U - peaks exist in $\rho_{\mathrm{d}\uparrow}(\omega)$.

(2) Additional two resonant levels are activated only under bias : No bias no weight.

(3) Their spectral weights come from both central and side peaks as bias increases.

* In equil. $\rho_{d\uparrow}(\omega)$ has a single coherence peak at the Fermi level.



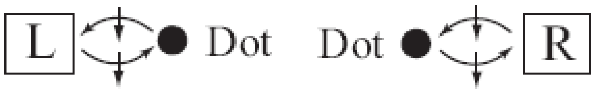
$$\boxed{Z_S = Z_S^{LL} + Z_S^{LR}}$$

$$Z_S^{LL} = 4\gamma_{LL}^2 / U^2$$

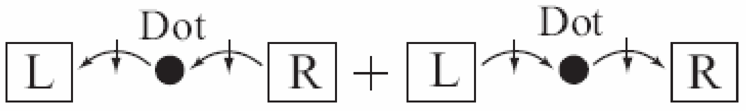
$$Z_S^{LR} = 4\gamma_{LR}^2 / U^2$$

Two types of coherence: $\gamma_{\mathbf{LL}}$, $\gamma_{\mathbf{LR}}$

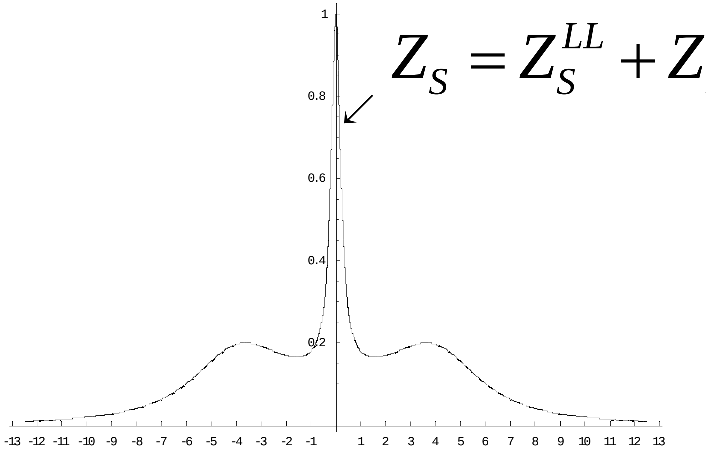
γ_{LL} :



γ_{LR} :

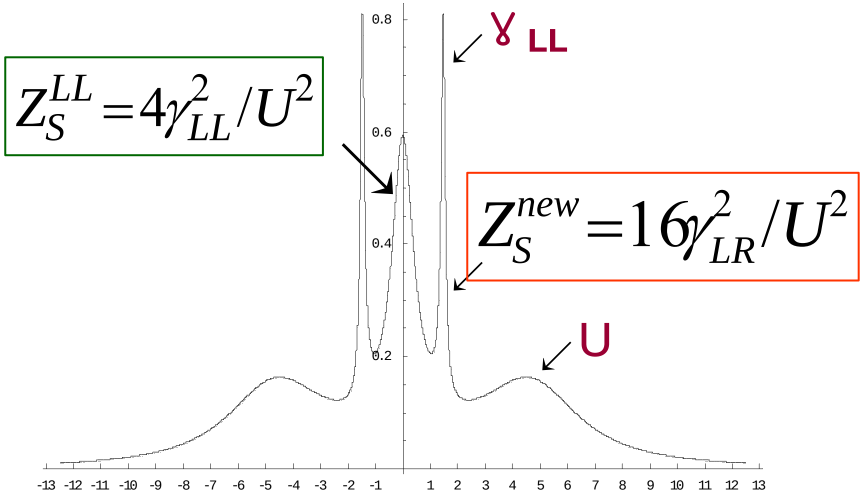


Understanding Nonequilibrium Kondo Phenomena
– qualitative –



$$Z_S = Z_S^{LL} + Z_S^{LR}$$

➔ $V_{sd} \neq 0$



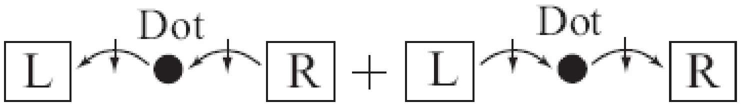
$$Z_S^{LL} = 4\gamma_{LL}^2 / U^2$$

$$Z_S^{new} = 16\gamma_{LR}^2 / U^2$$

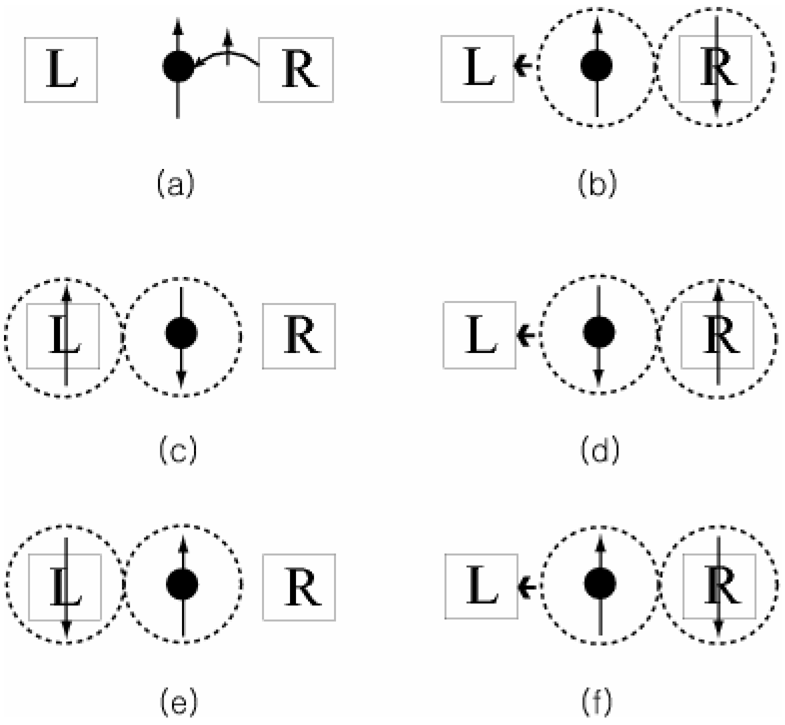
(conceptual view)

The coherent transport through the novel resonant level may be described by γ_{LR}

γ_{LR} :



Coherent Transport in a Single Electron Transistor



(a) (b)

(c) (d)

(e) (f)

(swing)

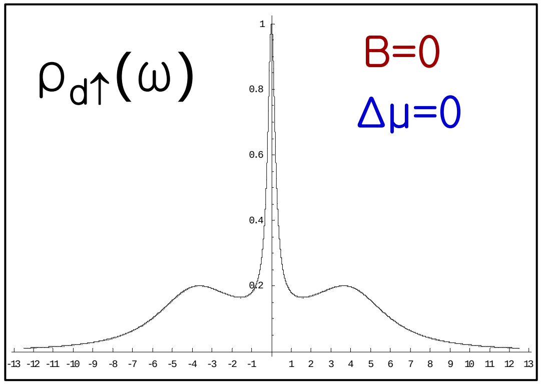
(partner change)

(swing)

(partner change)

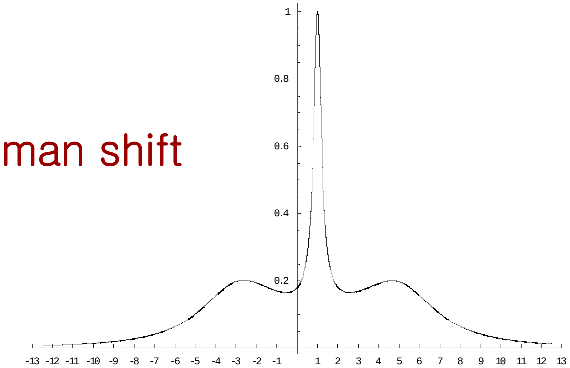
$$\widetilde{\gamma}_{LR} = \langle \sum_{k} V^{*}_{kd} (c^{L}_{k\uparrow} + c^{R}_{k\uparrow}) c^{+}_{d\uparrow} [j^{-L}_{d\downarrow}, j^{+R}_{d\downarrow}] \rangle$$

From equil. to nonequil.: schematic change of $\rho_{d\uparrow}(\omega)$

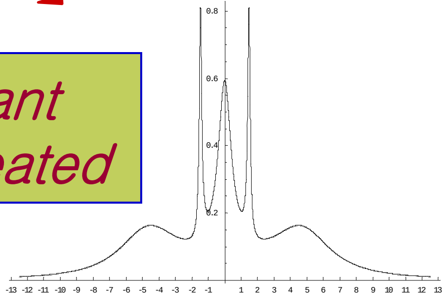


$\xrightarrow{\Delta\mu=0}$ B≠0

Zeeman shift



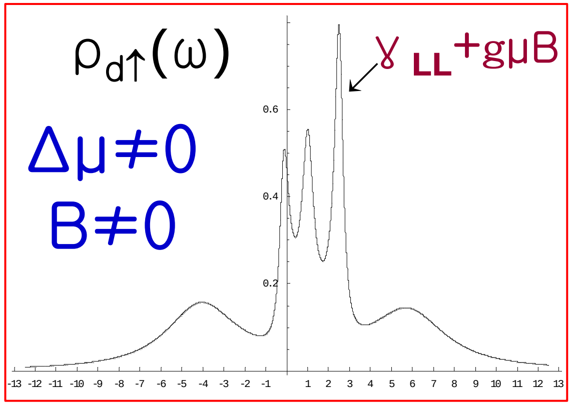
Δμ≠0



new resonant levels are created

↓ Δμ≠0

B≠0



Bias induces side peaks and central peak reduction, and B−field causes Zeeman shift. Both B−field & bias cause asymmetry.

JH & W. Woo, PRL, 99, 196801(2007)

Self-consistent calculation will show this.

Understanding Kondo-Peak Splitting

Our result: Major peak positions at $\gamma^{S}_{LL}+g\mu B$

$$\gamma^{S}_{LL} = U\sqrt{Z^{LL}_{S}}\,/\,2$$

Note that U is not involved in γ^{S}_{LL}: $\langle \sum_{k} (V^{L*}_{kd} c^{L}_{k\uparrow} + V^{R*}_{kd} c^{R}_{k\uparrow}) c^{+}_{d\uparrow} [j^{-L}_{d\downarrow}, j^{+L}_{d\downarrow}] \rangle$

U must come from the averaging process.

$$Z^{\mathrm{SIAM}}_{S} = (4/\pi)\sqrt{U/\Gamma}\; e^{-\frac{\pi U}{4\Gamma}}$$

←

$$U \Rightarrow U/\sqrt{2},\ \Gamma \Rightarrow 2\Gamma$$

Effect of 2 Reservoirs

▼

$$Z_{S} = (4/\pi)\sqrt{U/2\sqrt{2}\Gamma}\; e^{-\frac{\pi U}{8\sqrt{2}\Gamma}}$$

▶

$$U = \frac{8\sqrt{2}\Gamma}{\pi} \ln\left(\frac{4/\pi\sqrt{U\Gamma/2\sqrt{2}}}{Z_{S}\Gamma} \right)$$

$$\therefore \quad \boxed{\gamma_{LL}^{S} = U\sqrt{Z_S^{LL}}/2} \;\blacktriangleright\; \boxed{\gamma_{LL}^{S} = \left(\frac{4\sqrt{2}\Gamma}{\pi}\right)\sqrt{Z_S^{LL}}\,\ln\left(\frac{\tilde{D}}{Z_S\Gamma}\right)}$$

$$\blacktriangledown$$

$$\boxed{\gamma_{LL}^{Asy} = \left(\frac{4\sqrt{2}\Gamma}{\pi}\right)\sqrt{Z_S^{LL}}\,\ln\left(\frac{\tilde{D}}{Z_{Asy}\Gamma}\right)}$$

where $\boxed{Z_{Asy} = Z_S \exp[\chi(\mathrm{V_g} - \mathrm{V_{g,0}})^2] = Z_S(\mathrm{T_K}/\mathrm{T_{K,0}})}$

and $\tilde{D} = Z_S\Gamma\exp[\pi U/(8\sqrt{2}\Gamma)]$

We now express γ^{Asy}_{LL} in terms of measurable quantities, such as $\Delta^0_{K,S}=\gamma^S_{LL}$ and $\mathrm{V_g}$

$$\Delta^0_{\mathrm{K}}(\mathrm{V_g}) = \Delta^0_{\mathrm{K},S} - \frac{8\sqrt{2}\Gamma}{\pi U}\Delta^0_{\mathrm{K},S}\,\chi(\mathrm{V_g} - \mathrm{V_{g,0}})^2$$

$$= \Delta^0_{\mathrm{K},S} - \frac{8\sqrt{2}\Gamma}{\pi U}\Delta^0_{\mathrm{K},S}\,\ln\left(\frac{\mathrm{T_K}}{\mathrm{T_{K,0}}}\right)$$

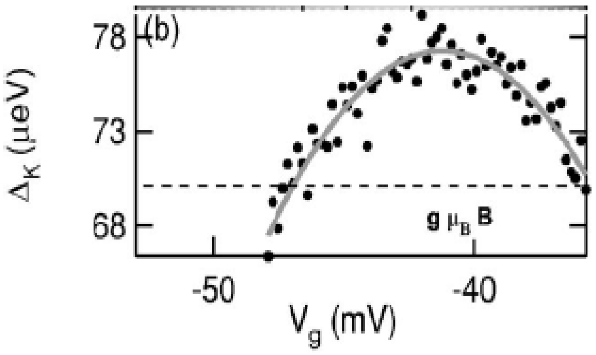
$Exp:\ \Delta^0_{\mathrm{K,S}} = 11\mu\mathrm{eV},\ U = 1.2\mathrm{meV},\ \Gamma = 330\mu\mathrm{eV},$

$\chi = 0.02(\mathrm{mV})^{-2} \Rightarrow$

(1) Curvature: $-(8\sqrt{2}\Gamma/\pi U)\Delta^0_{\mathrm{K},S}\chi = -0.22\mu\mathrm{eV}/(\mathrm{mV})^2$,

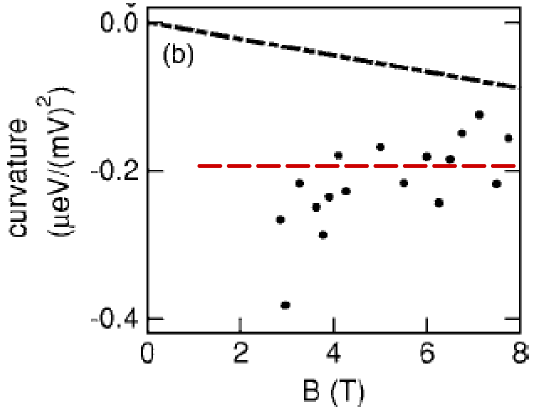
(2) Coefficient of $-\ln T_{\mathrm{K}}$: $(8\sqrt{2}\Gamma/\pi U)\Delta^0_{\mathrm{K},S} = 11\mu\mathrm{eV}$

Perfect agreement!



Δ^0_{K}= a−b$\ln T_{\mathrm{K}}$

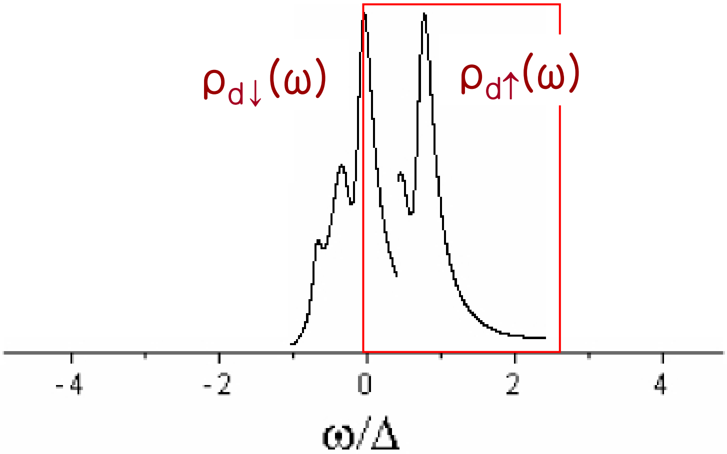
b_{exp}≈12μeV

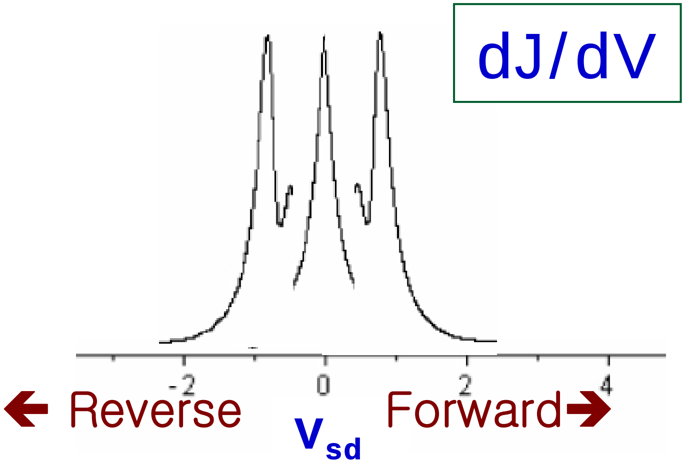


Critical Splitting & Splitting Threshold

Let's see when the peak splits for $\Delta_{K,S}{}^{0}=0.5\Delta$, for example.

(1) Applied field: $g\mu_B B=\Delta_{K,S}{}^{0}=0.5\Delta$

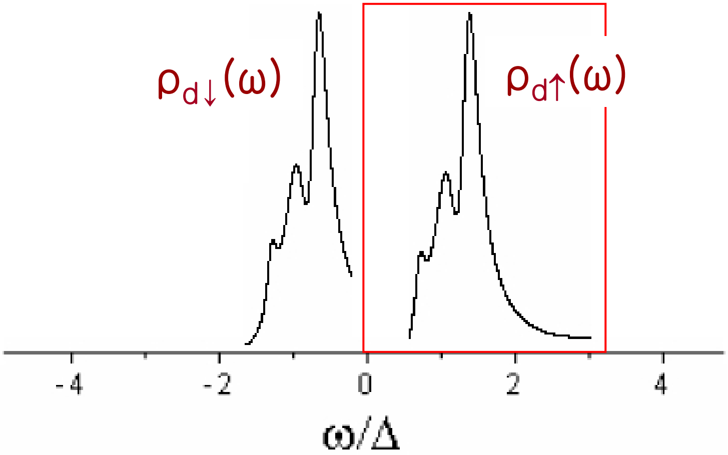


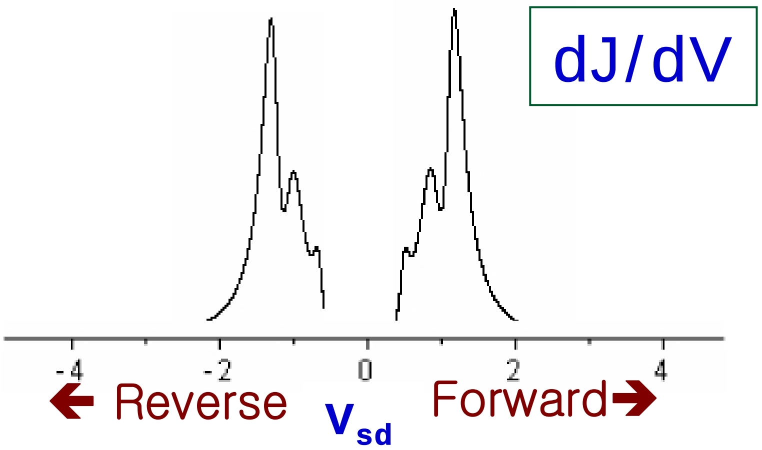


dJ/dV

Not separated

(2) Applied field: $g\mu_B B = 2\Delta_{K,S}^0 = \Delta$





Separated

Condition of separation: Major peak of $\rho_{d\downarrow}(\omega)$ positions $\Delta^0_{K,S}$ below the Fermi level

Critical splitting: $B_C = 2\Delta^0_{K,S} / g\mu_B = 2.4\mathrm{T}$

Threshold Equation:

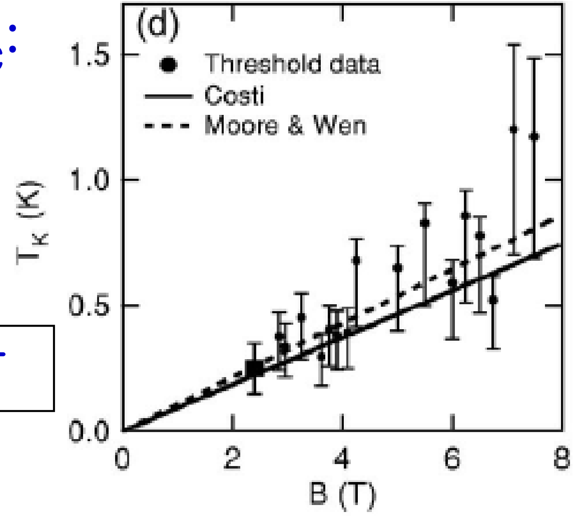
$$\underline{|g|\mu_B B_C + \Delta^0_K(T_{Km})} = 3\Delta^0_{K,S}[1+\underline{(T_{Km}-T_{K,0})/4T_{K,0}}]$$

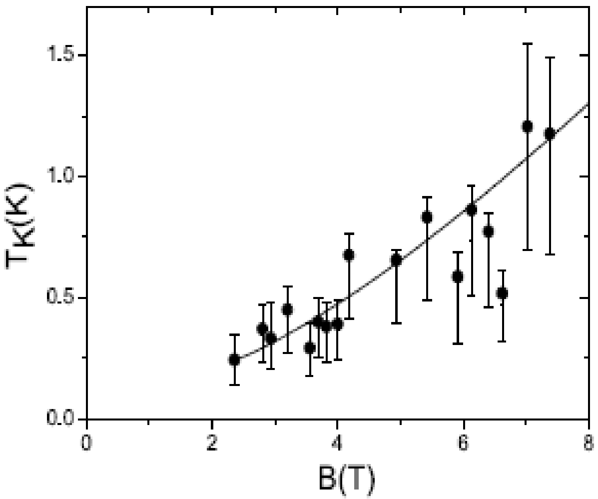
(position of the major peak)

Dispersion Effect by Kondo Temp. (4: phenomenology)

T_{Km} vs. B_C:

$B_C \approx 2.4$T





Constructing Self-Consistent Loop

$$\boxed{iG^{+}_{\mathrm{dd}}(\omega) = (adj\,\widetilde{\mathbf{M}}_{\mathrm{dd}})_{33}[\det \widetilde{\mathbf{M}}_{\mathrm{dd}}]^{-1}}$$

$$\widetilde{\mathbf{M}}_{dd} = \begin{bmatrix} -i\omega-\alpha_B & \gamma_{LL} & -U^{L}_{J^-} & -\gamma_{LR} & -\gamma_{J^-} \\ -\gamma_{LL} & -i\omega-\alpha_B & -U^{L}_{J^+} & -\gamma_{J^+} & -\gamma_{LR} \\ U^{L*}_{J^-} & U^{L*}_{J^+} & -i\omega-\alpha_B & U^{R*}_{J^+} & U^{R*}_{J^-} \\ \gamma_{LR} & \gamma_{J^+} & -U^{R}_{J^+} & -i\omega-\alpha_B & -\gamma_{RR} \\ \gamma_{J^-} & \gamma_{LR} & -U^{R}_{J^-} & \gamma_{RR} & -i\omega-\alpha_B \end{bmatrix}$$

with additional $i\beta_{ij}[\Sigma^{L}_{ij}(\omega)+\Sigma^{R}_{ij}(\omega)]$ except U−elements,

where $\Sigma^{L}_{ij}(\omega)=\Sigma^{R}_{ij}(\omega)= \Sigma_0(\omega)$, $\alpha_B = i[\varepsilon_d + U\langle n_{d\downarrow}\rangle - g\,|\,\mu_B\,|\,B] + 0^{+}$

$$\beta_{ij} = \beta_{ji},\ \beta_{33} = 1 \qquad \beta_{ij} = \frac{1}{4\langle(\delta n_{d\downarrow})^2\rangle}\widetilde{\beta}_{ij}$$

$$\widetilde{\beta}_{22} = \left[\frac{1}{4} + \frac{(1-2\langle n_{d\downarrow}\rangle)^2 \langle j_{d\downarrow}^{+L}\rangle^2}{\langle(\delta j_{d\downarrow}^{+L})^2\rangle}\right] = \widetilde{\beta}_{44} = \widetilde{\beta}_{24} = \widetilde{\beta}_{42},$$

$$\boxed{\langle j_{d\downarrow}^{+L}\rangle = \langle j_{d\downarrow}^{+R}\rangle}$$

$$\vee$$

$$\widetilde{\beta}_{12} = \left[\frac{1}{4} + \frac{(1-2\langle n_{d\downarrow}\rangle)^2 \langle j_{d\downarrow}^{-L}\rangle\langle j_{d\downarrow}^{+L}\rangle}{\sqrt{\langle(\delta j_{d\downarrow}^{-L})^2\rangle}\sqrt{\langle(\delta j_{d\downarrow}^{+L})^2\rangle}}\right] = \widetilde{\beta}_{21} = \widetilde{\beta}_{14} = \widetilde{\beta}_{41}$$

$$\boxed{\langle j_{d\downarrow}^{+L}\rangle > \langle j_{d\downarrow}^{-L}\rangle}$$

$$\vee$$

$$\widetilde{\beta}_{11} = \left[\frac{1}{4} + \frac{(1-2\langle n_{d\downarrow}\rangle)^2 \langle j_{d\downarrow}^{-L}\rangle^2}{\langle(\delta j_{d\downarrow}^{-L})^2\rangle}\right] = \widetilde{\beta}_{55},$$

$$\boxed{\langle j_{d\downarrow}^{+L}\rangle > \langle j_{d\downarrow}^{-L}\rangle}$$

$$\vee$$

$$\widetilde{\beta}_{15} = \left[\frac{1}{4} + \frac{(1-2\langle n_{d\downarrow}\rangle)^2 \langle j_{d\downarrow}^{-L}\rangle\langle j_{d\downarrow}^{-R}\rangle}{\sqrt{\langle(\delta j_{d\downarrow}^{-L})^2\rangle}\sqrt{\langle(\delta j_{d\downarrow}^{-R})^2\rangle}}\right] = \widetilde{\beta}_{51}$$

$$\boxed{\langle j_{d\downarrow}^{-L}\rangle = -\langle j_{d\downarrow}^{-R}\rangle}$$

$$\vee$$

$$\widetilde{\beta}_{25} = \left[\frac{1}{4} + \frac{(1-2\langle n_{d\downarrow}\rangle)^2 \langle j_{d\downarrow}^{+L}\rangle\langle j_{d\downarrow}^{-R}\rangle}{\sqrt{\langle(\delta j_{d\downarrow}^{+L})^2\rangle}\sqrt{\langle(\delta j_{d\downarrow}^{-R})^2\rangle}}\right] = \widetilde{\beta}_{52} = \widetilde{\beta}_{45} = \widetilde{\beta}_{54},$$

$$\boxed{\langle j_{d\downarrow}^{-L}\rangle = -\langle j_{d\downarrow}^{-R}\rangle}$$

$$\widetilde{\mathbf{M}}_{dd} = \begin{bmatrix} -i\omega-\alpha_B & \gamma_{LL} & -U^{L}_{J^-} & -\gamma_{LR} & -\gamma_{J^-} \\ -\gamma_{LL} & -i\omega-\alpha_B & -U^{L}_{J^+} & -\gamma_{J^+} & -\gamma_{LR} \\ U^{L*}_{J^-} & U^{L*}_{J^+} & -i\omega-\alpha_B & U^{R*}_{J^+} & U^{R*}_{J^-} \\ \gamma_{LR} & \gamma_{J^+} & -U^{R}_{J^+} & -i\omega-\alpha_B & -\gamma_{RR} \\ \gamma_{J^-} & \gamma_{LR} & -U^{R}_{J^-} & \gamma_{RR} & -i\omega-\alpha_B \end{bmatrix}$$

$$U^{L,R}_{J^{\pm}} = \frac{U}{2}\left[\frac{i\langle [n_{d\downarrow}, j^{\pm L,R}_{d\downarrow}](1-2n_{d\uparrow})\rangle + i(1-2\langle n_{d\downarrow}\rangle)\langle j^{\pm L,R}_{d\downarrow}\rangle}{\sqrt{\langle(\delta\, j^{\pm L,R}_{d\downarrow})^{2}\rangle}} \right]$$

From the analysis at the atomic limit ➔ $|\mathrm{Re}(U_{J\pm}^{L,R})|=U/4$

$$\widetilde{\mathbf{M}}_{dd} = \begin{bmatrix} -i\omega-\alpha_B & \gamma_{LL} & -U^{L}_{J^-} & -\gamma_{LR} & -\gamma_{J^-} \\ -\gamma_{LL} & -i\omega-\alpha_B & -U^{L}_{J^+} & -\gamma_{J^+} & -\gamma_{LR} \\ U^{L*}_{J^-} & U^{L*}_{J^+} & -i\omega-\alpha_B & U^{R*}_{J^+} & U^{R*}_{J^-} \\ \gamma_{LR} & \gamma_{J^+} & -U^{R}_{J^+} & -i\omega-\alpha_B & -\gamma_{RR} \\ \gamma_{J^-} & \gamma_{LR} & -U^{R}_{J^-} & \gamma_{RR} & -i\omega-\alpha_B \end{bmatrix}$$

Unknowns in $\widetilde{\mathbf{M}}_{\mathrm{dd}}$: (1) $\gamma_{LL} = \gamma_{RR}$ and γ_{LR}, (2) $\gamma_{J^-} = \gamma_{J^+}$, (3) $\mathrm{Re}[U^{L,R}_{J^\mp}]$,

(4) $\langle (j^{\mp L}_{d\downarrow})^2 \rangle = \langle (j^{\mp R}_{d\downarrow})^2 \rangle$, (5) $\langle n_{d\downarrow} \rangle$, $\langle j^{-L}_{d\downarrow} \rangle = -\langle j^{-R}_{d\downarrow} \rangle$, $\langle j^{+L}_{d\downarrow} \rangle = \langle j^{+R}_{d\downarrow} \rangle$

(1) $\gamma_{LR} \Leftarrow \gamma_{LL}$, (2) $\gamma_{J^\pm} / \gamma_{LR} \Leftarrow$ model, (3) $\mathrm{Re}[U^{L,R}_{J^\mp}] \Leftarrow U$ - peaks

(4) $\sqrt{\langle (\delta j^{-}_{d\downarrow})^2 \rangle} = \mathrm{a}\langle j^{+}_{d\downarrow} \rangle - 2\mathrm{a}\langle j^{+}_{d\downarrow} n_{d\uparrow} \rangle = \eta \langle j^{+}_{d\downarrow} \rangle, \eta \leq \mathrm{a},$ ← $|\mathrm{Re}(U_{J\pm}{}^{L,R})| = U/2a$

$\sqrt{\langle (\delta j^{+}_{d\downarrow})^2 \rangle} = \sqrt{(\eta^2 - 1)\langle j^{+}_{d\downarrow} \rangle^2 + \langle j^{-}_{d\downarrow} \rangle^2} \Leftarrow \langle (j^{+}_{d\downarrow})^2 \rangle = \langle (j^{-}_{d\downarrow})^2 \rangle$

(5) $\langle n_{d\downarrow} \rangle, \langle j^{-L}_{d\downarrow} \rangle, \langle j^{+L}_{d\downarrow} \rangle \Leftarrow$ self – consistent loop

$$U^{L,R}_{J^{\pm}} = \frac{U}{2}\left[\frac{i\langle [n_{d\downarrow}, j^{\pm L,R}_{d\downarrow}](1-2n_{d\uparrow})\rangle + i(1-2\langle n_{d\downarrow}\rangle)\langle j^{\pm L,R}_{d\downarrow}\rangle}{\sqrt{\langle(\delta\, j^{\pm L,R}_{d\downarrow})^{2}\rangle}}\right]$$

$$\mathrm{Re}[U^{L,R}_{J^{\pm}}] = \frac{U}{2}\frac{i\langle [n_{d\downarrow}, j^{\pm L,R}_{d\downarrow}](1-2n_{d\uparrow})\rangle}{\sqrt{\langle(\delta\, j^{\pm L,R}_{d\downarrow})^{2}\rangle}} = \frac{U}{2\mathrm{a}}$$

$$\Rightarrow \sqrt{\langle(\delta\, j^{\mp L,R}_{d\downarrow})^{2}\rangle} = \mathrm{a}\langle i[n_{d\downarrow}, j^{\mp L,R}_{d\downarrow}](1-2n_{d\uparrow})\rangle$$

Using the relation $i\,[n_{d\downarrow}, j^{\mp L,R}_{d\downarrow}] \approx \pm j^{\pm L,R}_{d\downarrow}$,

$$\sqrt{\langle(\delta\, j^{\mp L,R}_{d\downarrow})^{2}\rangle} = \mathrm{a}\langle \pm j^{\pm L,R}_{d\downarrow}\rangle - 2\mathrm{a}\langle j^{\pm L,R}_{d\downarrow} n_{d\uparrow}\rangle$$

Self-Consistent Loop

$$\langle j_{d\sigma}^{-L}\rangle = -\int_{-\infty}^{\infty}\frac{d\omega}{\pi}[f_L(\omega-\Delta\mu/2)-f_R(\omega+\Delta\mu/2)]\frac{\Gamma^L\Gamma^R}{\Gamma^L+\Gamma^R}\mathrm{Im}G_{dd\sigma}^{+}(\omega)$$

$$= -\langle j_{d\sigma}^{-R}\rangle \Rightarrow J_\sigma^L = \frac{e}{\hbar}\langle j_{d\sigma}^{-L}\rangle,\ \ \Gamma^L = 2\pi\rho(\omega)|V^L|^2$$

M-W I

$$\langle j_{d\sigma}^{+L}\rangle = -\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\left[\frac{f_L(\omega)\Gamma^L + f_R(\omega)\Gamma^R}{2}\right]\mathrm{Re}G_{dd\sigma}^{+}(\omega)=\langle j_{d\sigma}^{+R}\rangle$$

M-W II

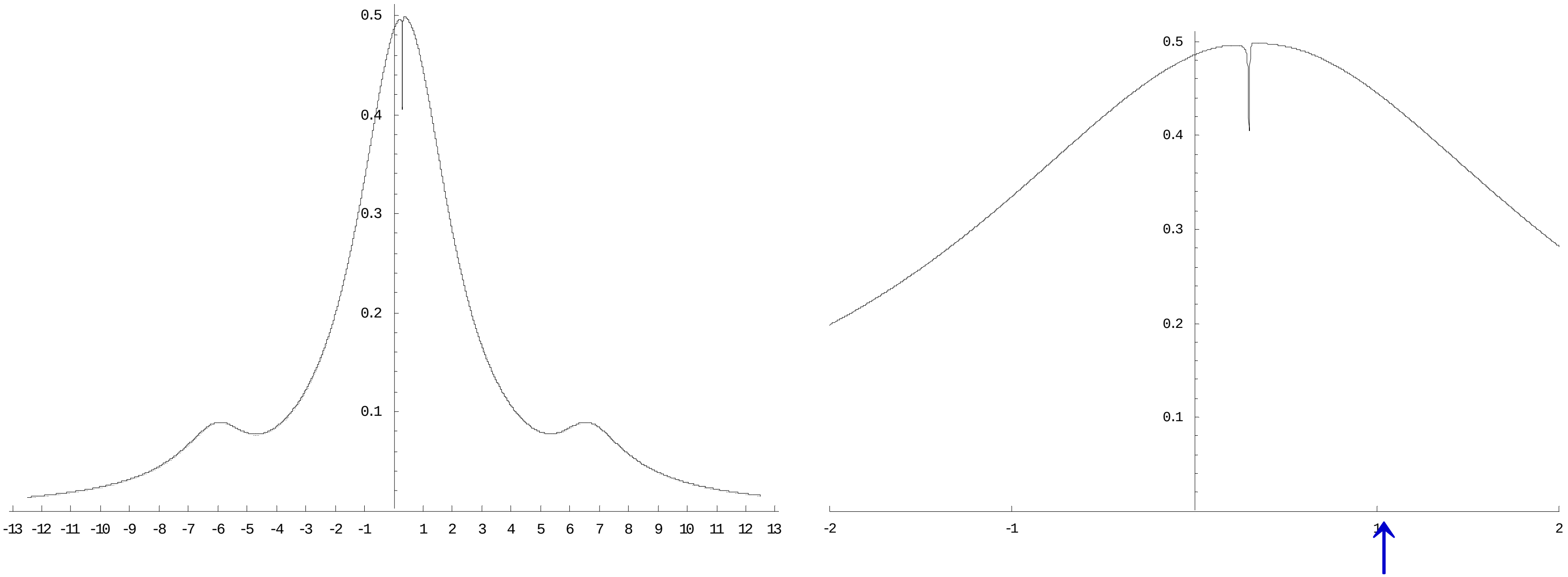
$$\langle n_{d\sigma}\rangle = -i\int\frac{d\omega}{2\pi}G_{dd\sigma}^{<}(\omega) = -\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\left[\frac{f_L(\omega)\Gamma^L + f_R(\omega)\Gamma^R}{\Gamma^L+\Gamma^R}\right]\mathrm{Im}G_{dd\sigma}^{+}(\omega)$$

Definition

$\Gamma^L \propto \Gamma^R$

$$\langle n_{d\sigma}\rangle^{(0)}, \langle j_{d\sigma}^{\mp}\rangle^{(0)} \rightarrow G_{dd\sigma}^{+(0)}(\omega) \rightarrow \langle n_{d\sigma}\rangle^{(1)}, \langle j_{d\sigma}^{\mp}\rangle^{(1)} \rightarrow G_{dd\sigma}^{+(1)}(\omega)$$

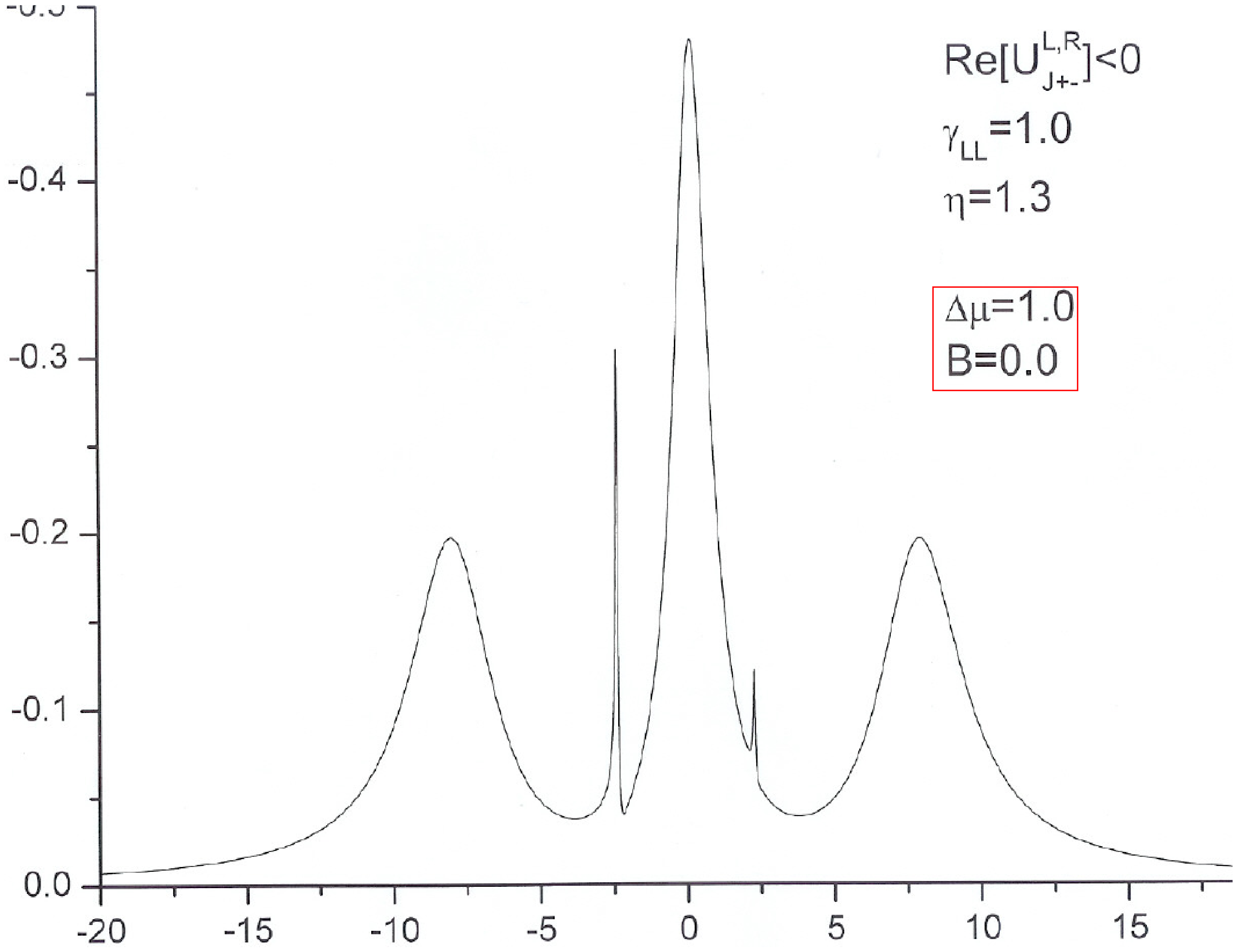
$$\rightarrow \langle n_{d\sigma}\rangle^{(2)}, \langle j_{d\sigma}^{\mp}\rangle^{(2)}$$

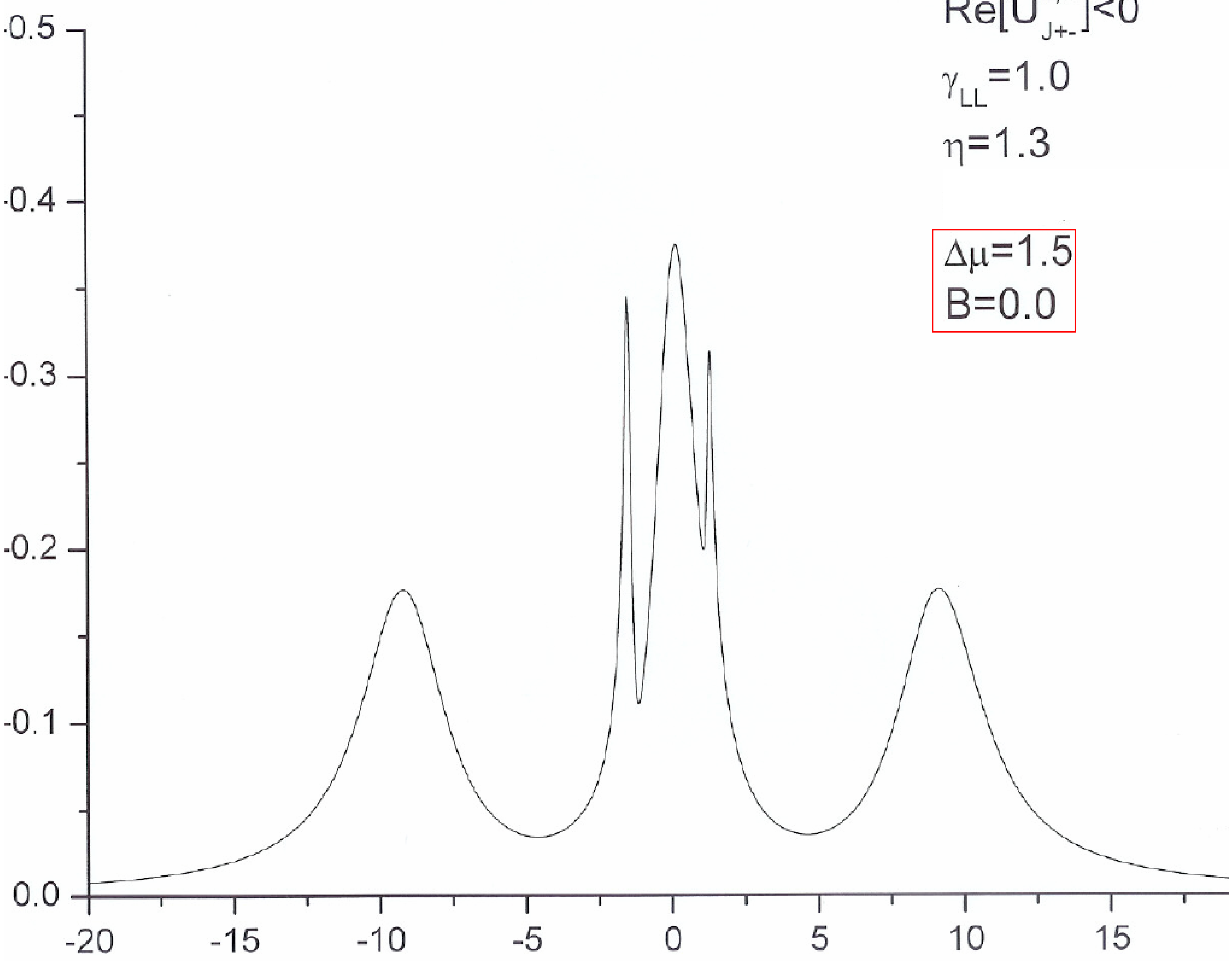


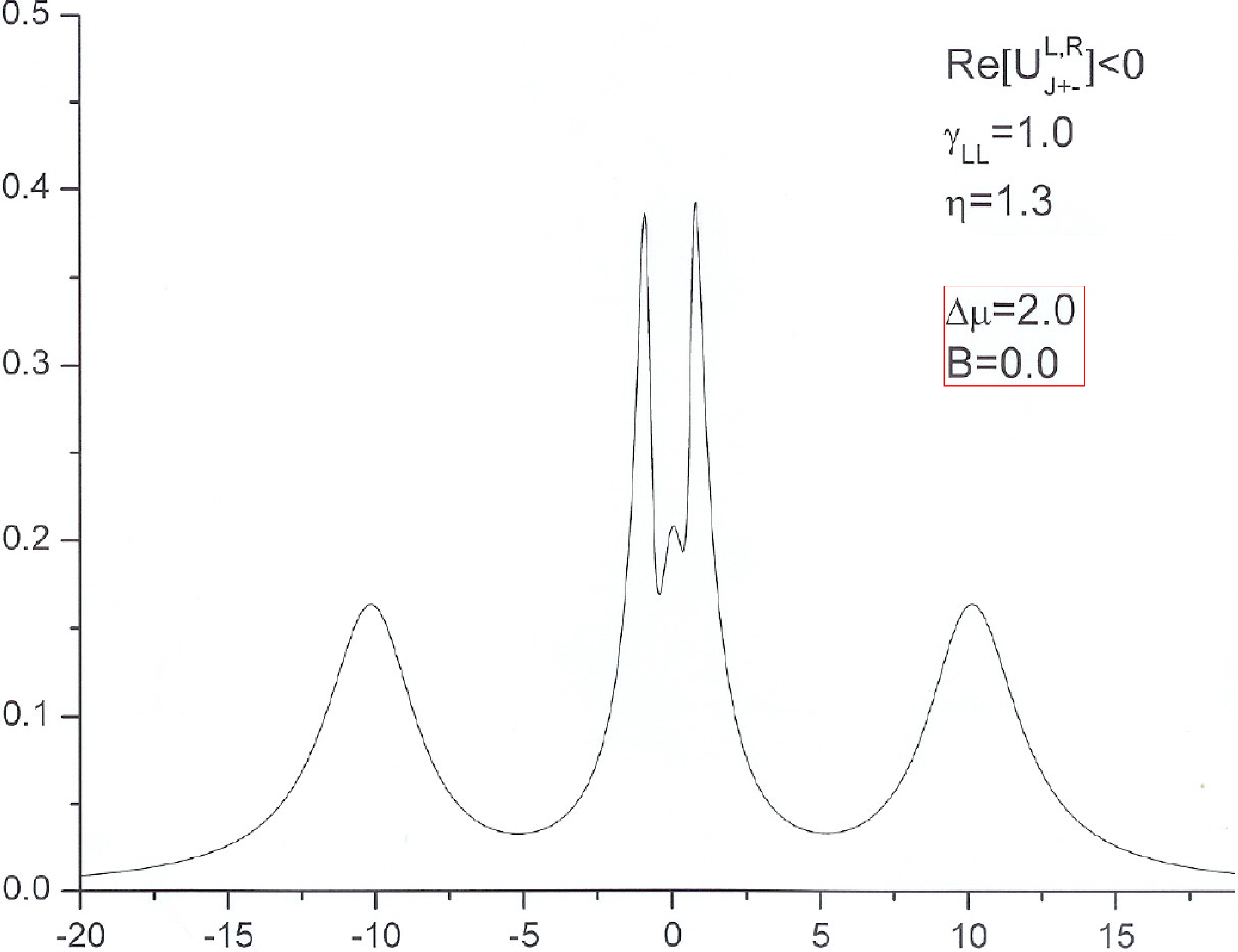
$\omega=1\Delta$

$\Delta\mu=2\Delta$, $B=0.3\Delta$ (5.29 T)

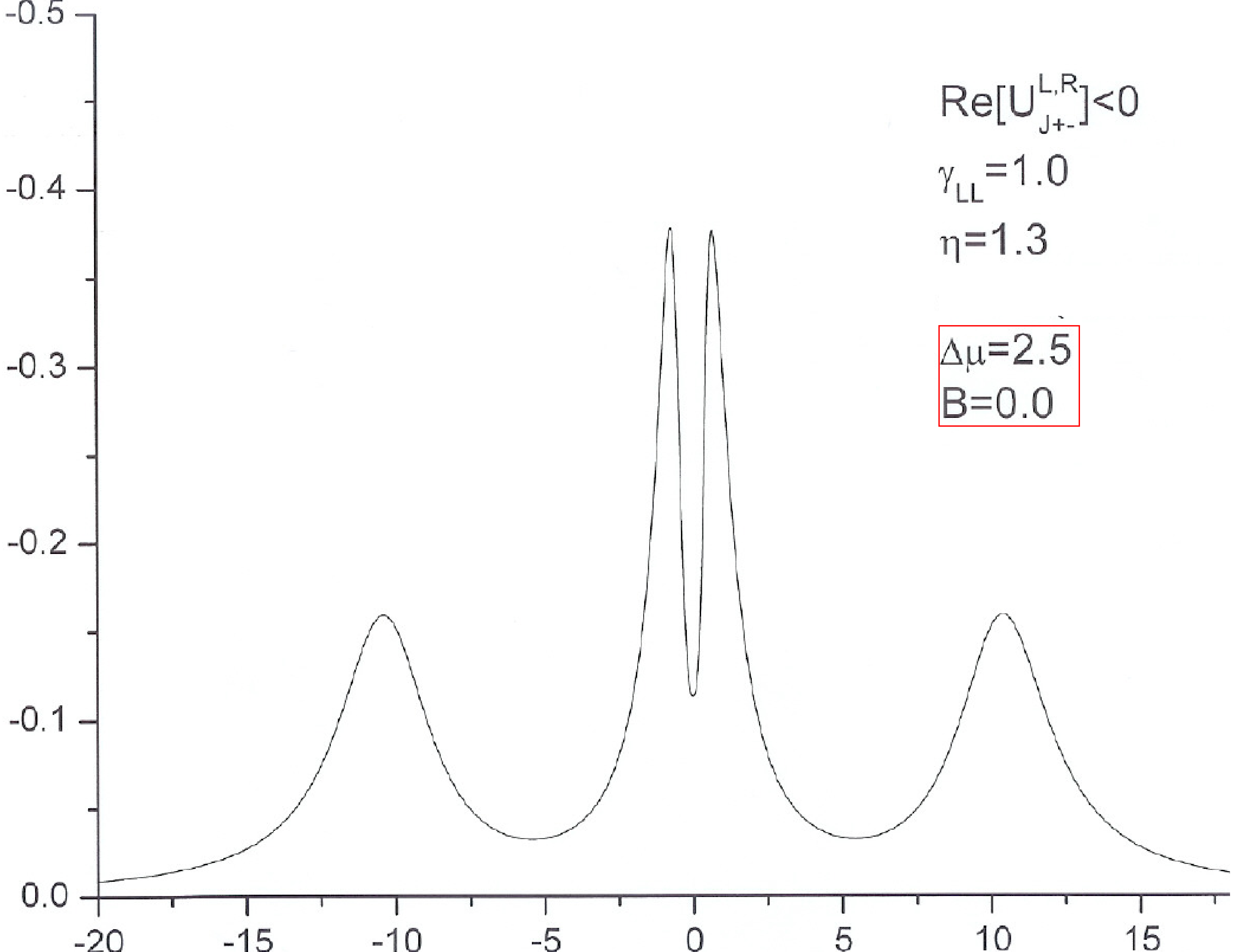
$\gamma_{LL}=0.1\Delta$

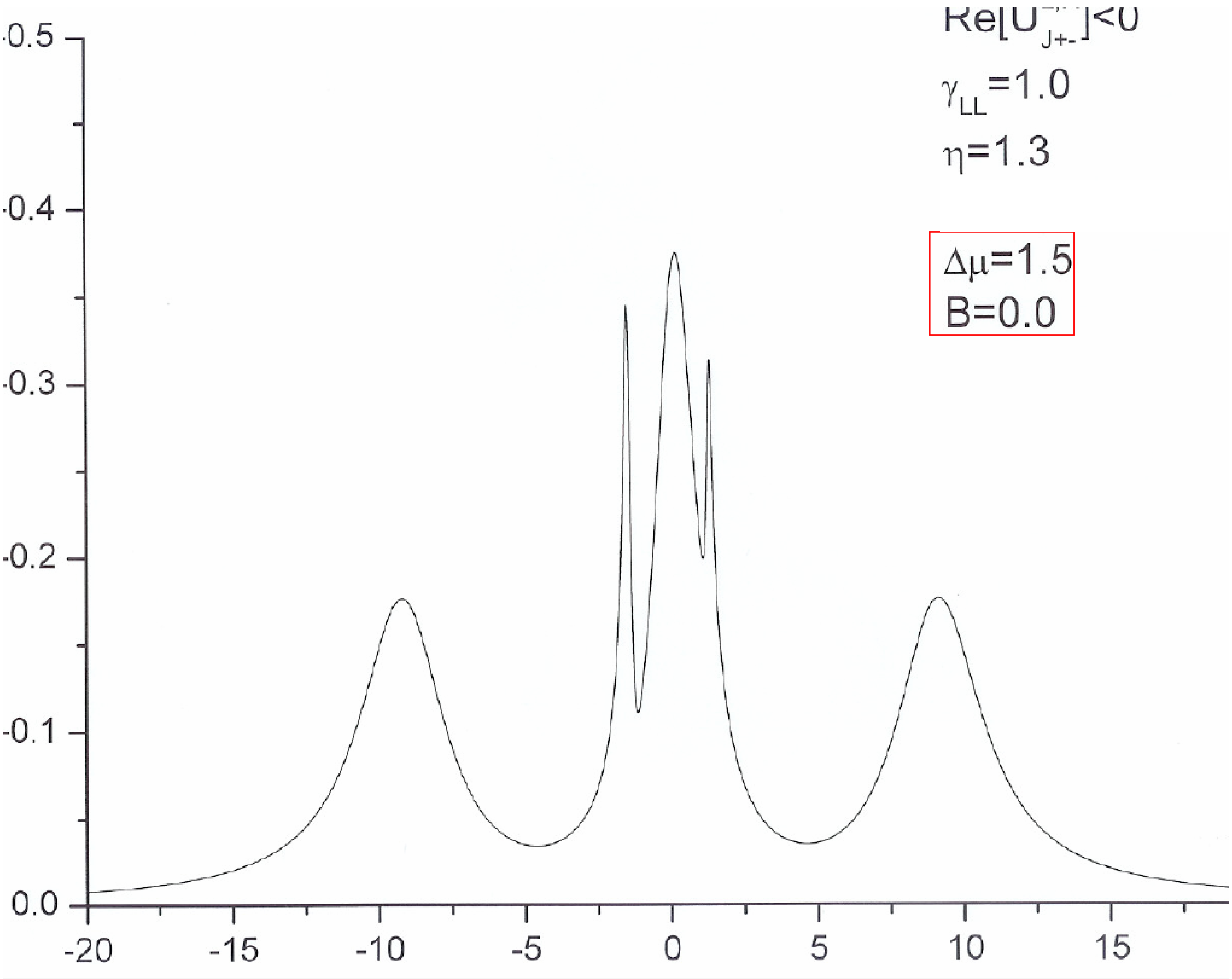

$\mathrm{Re}[U_{J+-}^{L,R}]<0$
$\gamma_{LL}=1.0$
$\eta=1.3$
$\Delta\mu=1.0$
$B=0.0$
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
-20
-15
-10
-5
0
5
10
15

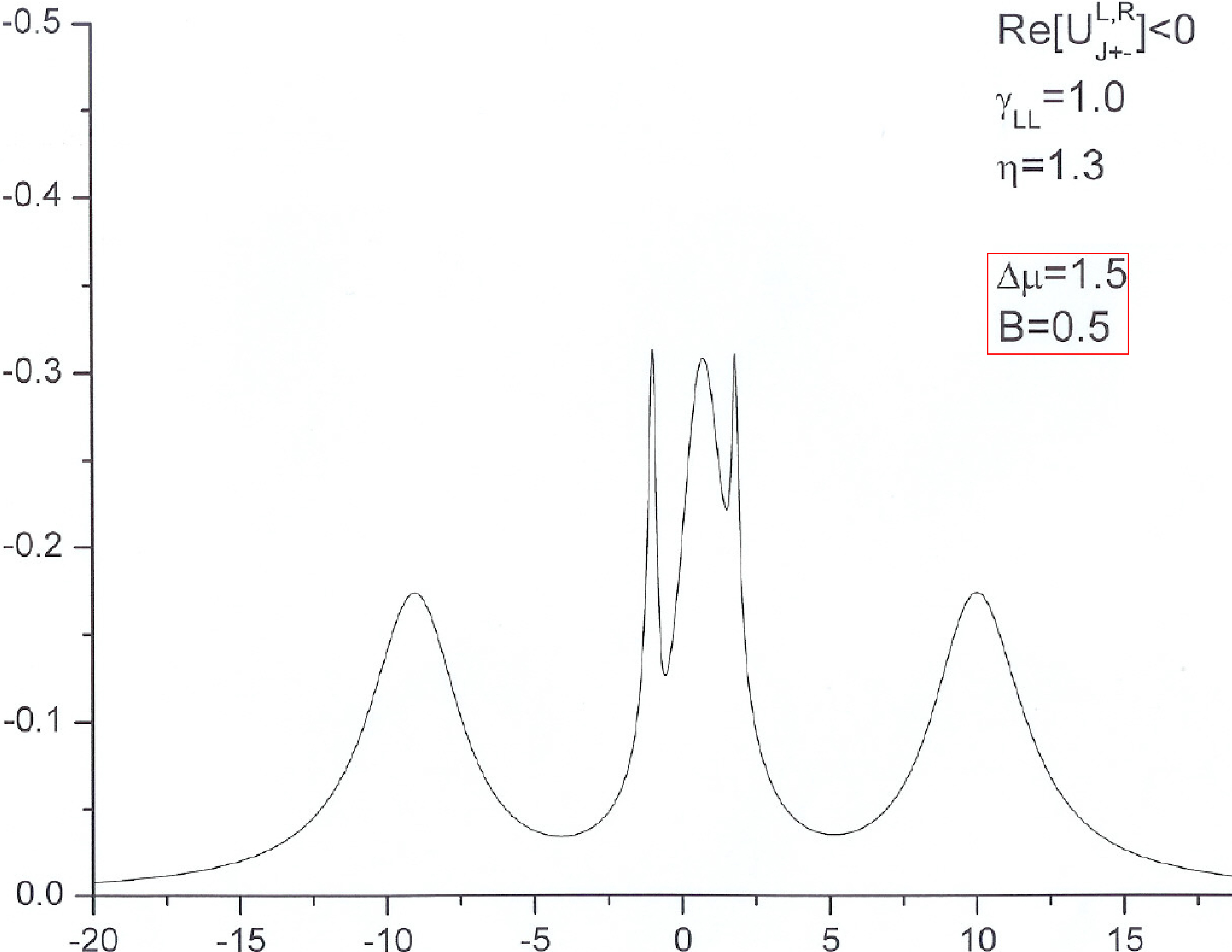

Re[U_{J+-}]<0
γ_{LL}=1.0
η=1.3
Δμ=1.5
B=0.0
0.5
0.4
0.3
0.2
0.1
0.0
-20
-15
-10
-5
0
5
10
15



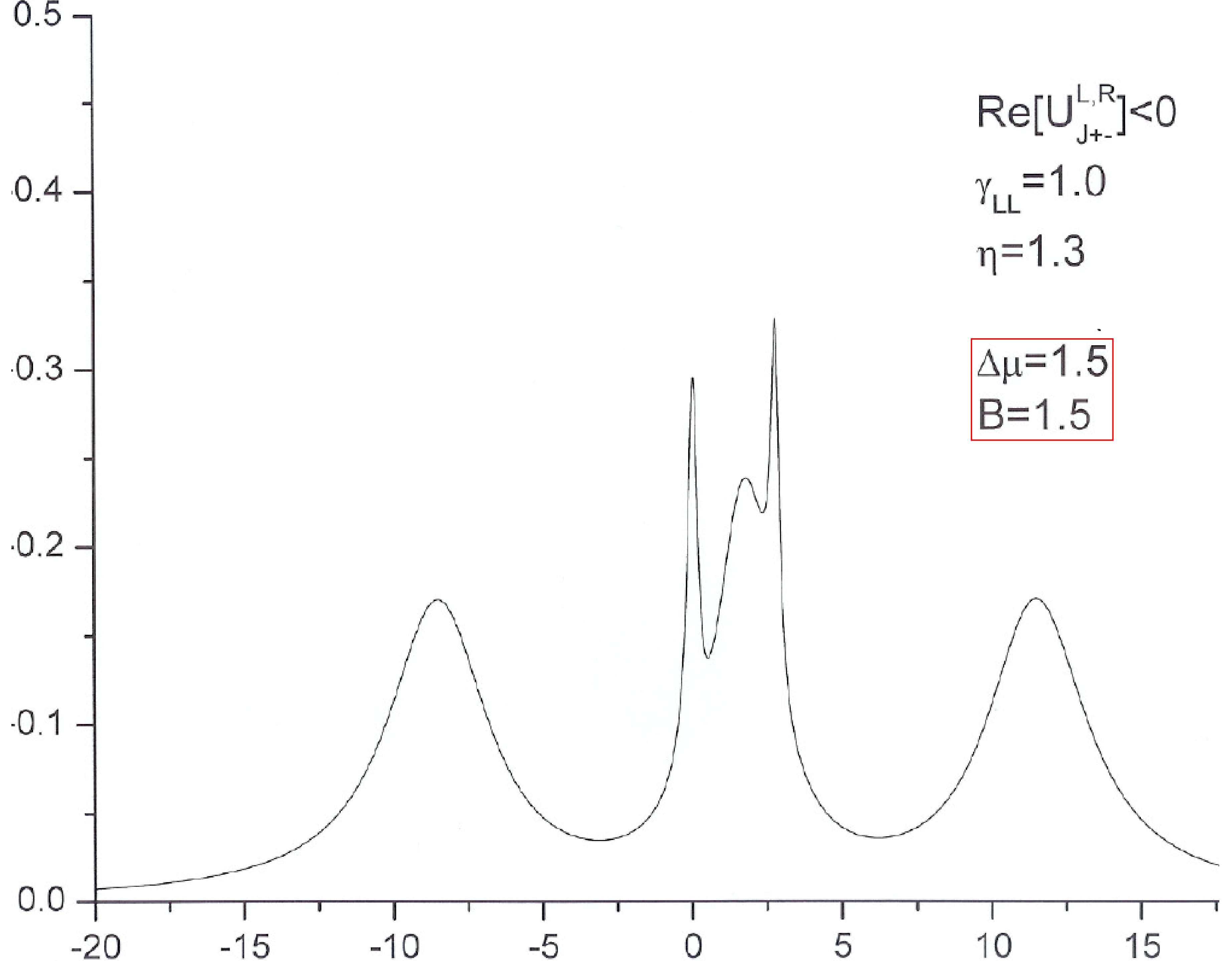
$\mathrm{Re}[U^{L,R}_{J+-}]<0$
$\gamma_{LL}=1.0$
$\eta=1.3$
$\Delta\mu=2.0$
$B=0.0$
0.5
0.4
0.3
0.2
0.1
0.0
-20
-15
-10
-5
0
5
10
15


-0.5
-0.4
-0.3
-0.2
-0.1
0.0
-20
-15
-10
-5
0
5
10
15
$\mathrm{Re}[U^{L,R}_{J+-}]<0$
$\gamma_{LL}=1.0$
$\eta=1.3$
$\Delta\mu=2.5$
$B=0.0$


Re[U$_{J+-}$]<0
γ_{LL}=1.0
η=1.3
$\Delta\mu$=1.5
B=0.0
0.5
0.4
0.3
0.2
0.1
0.0
-20
-15
-10
-5
0
5
10
15

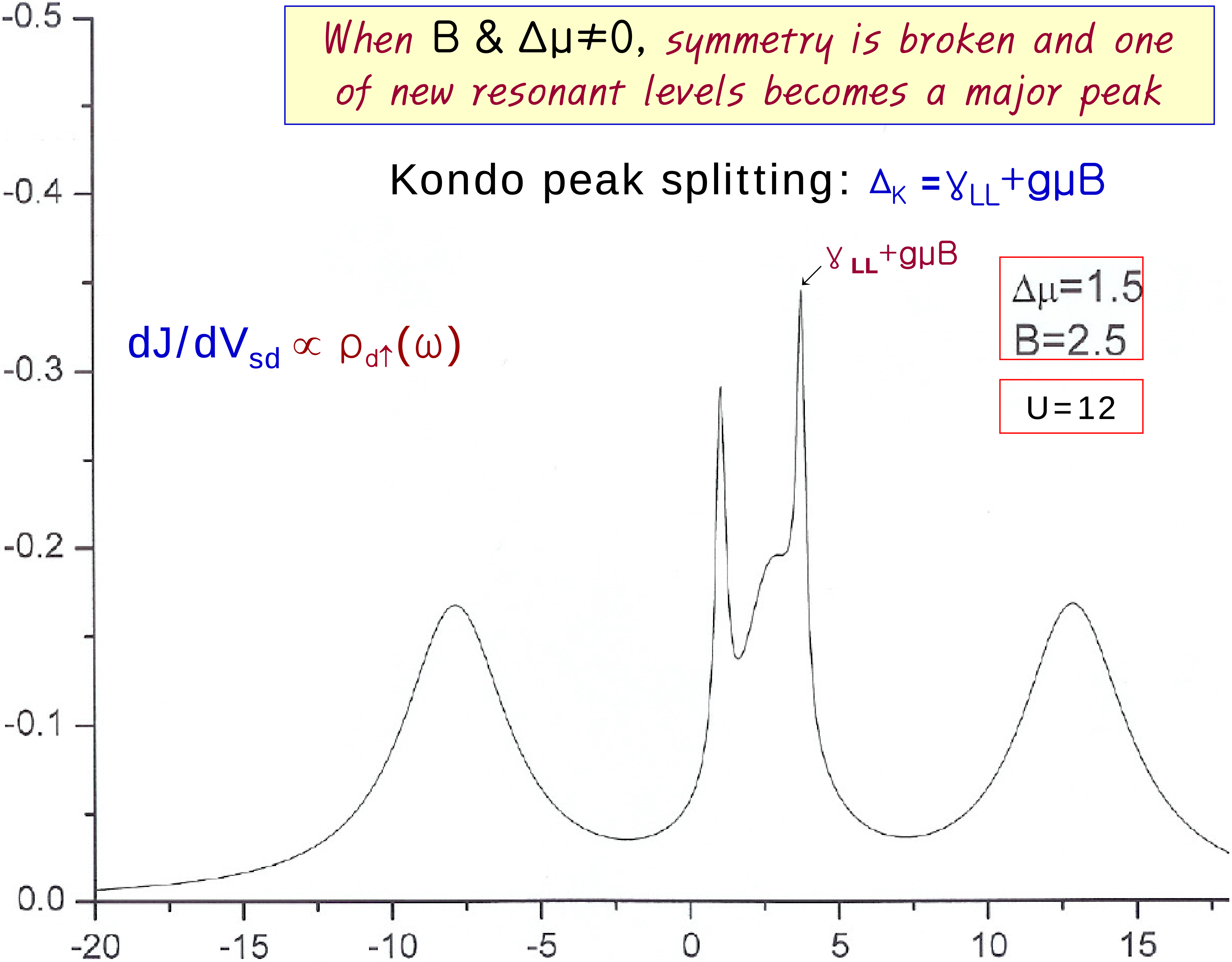


$\mathrm{Re}[U^{L,R}_{J+-}]<0$
$\gamma_{LL}=1.0$
$\eta=1.3$
$\Delta\mu=1.5$
$B=0.5$
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
-20
-15
-10
-5
0
5
10
15


$\mathrm{Re}[U^{L,R}_{J+-}]<0$
$\gamma_{LL}=1.0$
$\eta=1.3$
$\Delta\mu=1.5$
$B=1.5$
0.5
0.4
0.3
0.2
0.1
0.0
-20
-15
-10
-5
0
5
10
15

When B & $\Delta\mu \neq 0$, *symmetry is broken and one of new resonant levels becomes a major peak*

Kondo peak splitting: $\Delta_K = \gamma_{LL} + g\mu B$



Summary

- We saw why new approach is required for treating nonequilibrium + strong correlation.

- We constructed Nonperturbative Dynamical Theory as a new approach.

- We applied the NDT to the Single Electron Transistor under bias.

- We explained experimental results for the Kondo-peak splitting that are unexplained by existing theories.