# Magnetic confinement of massless Dirac fermions in graphene

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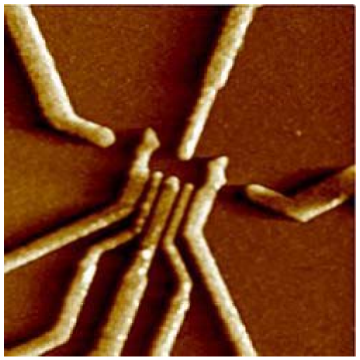
# Mesoscopic structures on 2-DEGS:

Semiconducting heterostructures

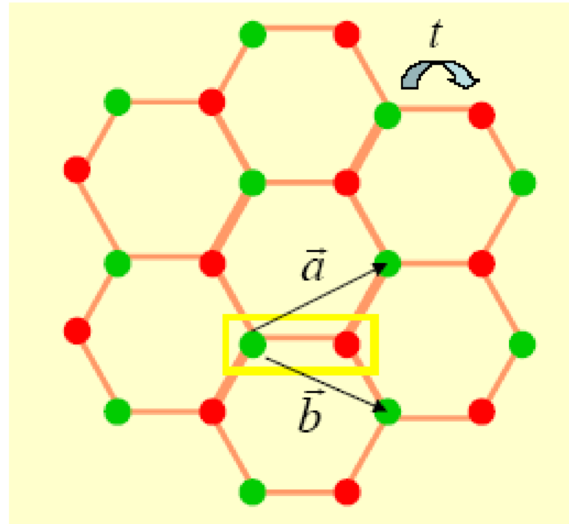


From Kavli Institute Delft

electrostatic fields by gating

$$H = \frac{\vec{p}^{\,2}}{2m^*} + V(\vec{r})$$

Graphene



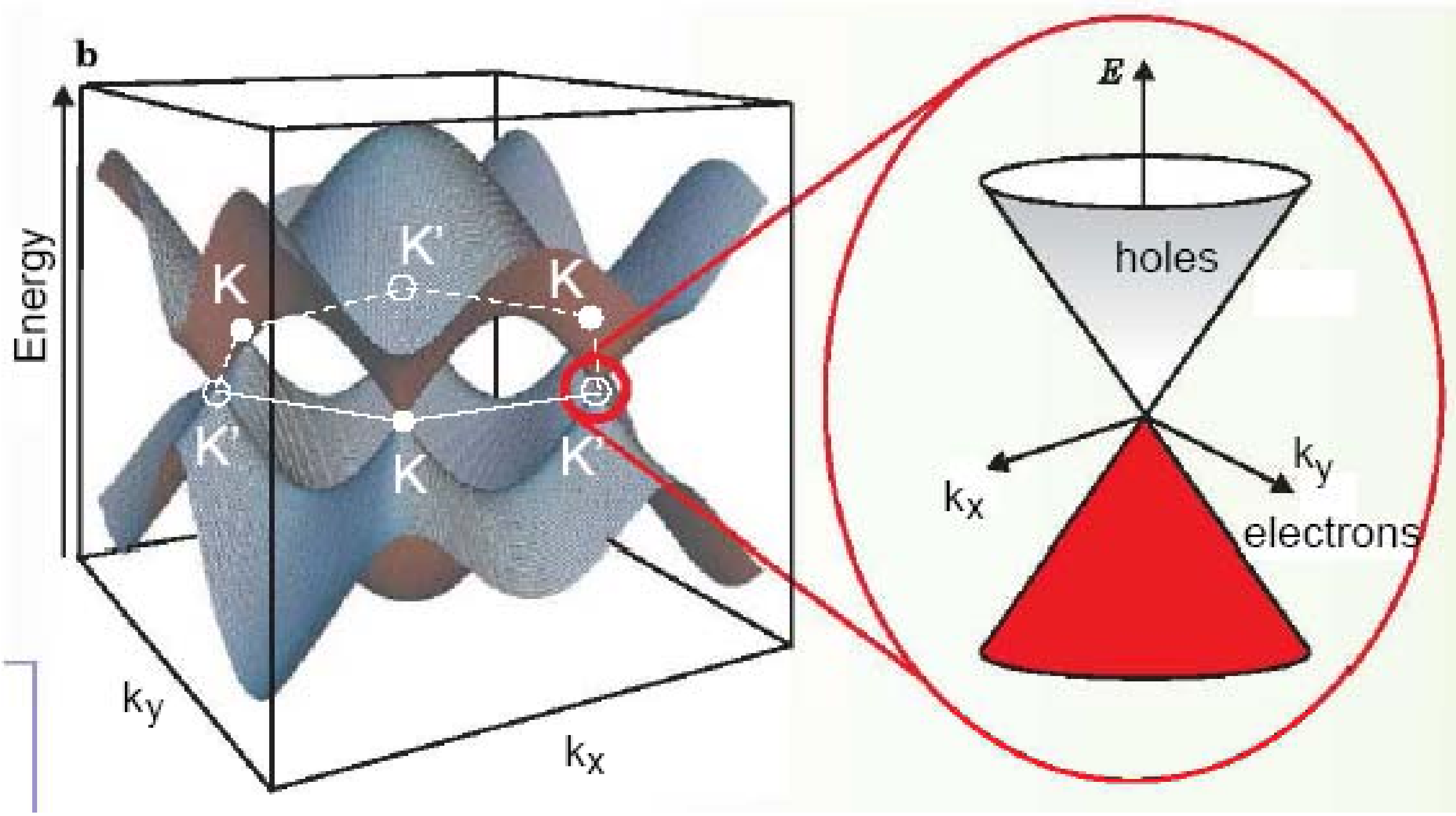

cf. M.I. Katsnelson, cond-mat/0612534

Klein tunneling !

O. Klein, Z. Phys. **53**, 157 (1929)

$$H = \vec{p} \cdot \vec{\tau} + V(\vec{r})$$

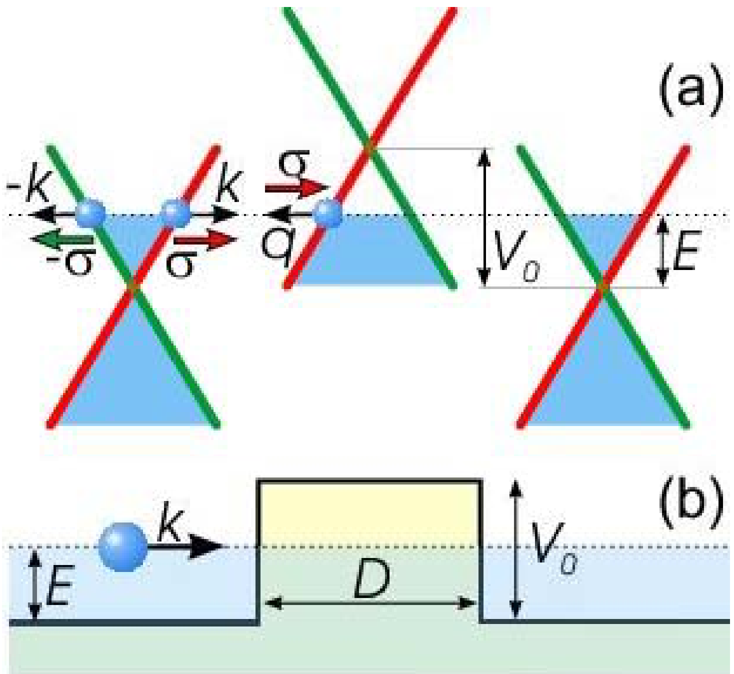
acting on 2-component wave functions



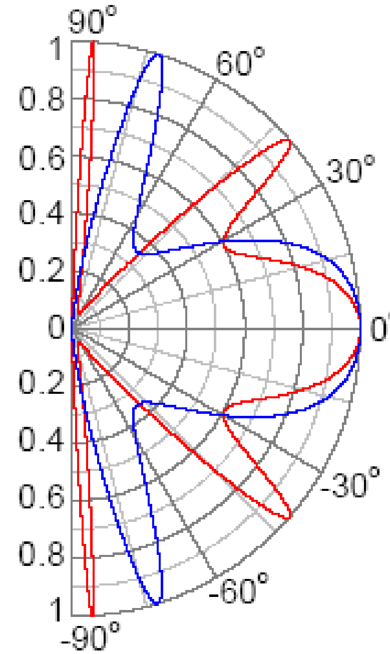

Stationary Dirac-Weyl equation at long wave length

$$\begin{pmatrix} 0 & p_x - \mathrm{i}p_y \\ p_x + \mathrm{i}p_y & 0 \end{pmatrix} \begin{pmatrix} \phi(x,y) \\ \chi(x,y) \end{pmatrix} = E \begin{pmatrix} \phi(x,y) \\ \chi(x,y) \end{pmatrix}$$

# Klein tunneling




Katsnelson, Novoselov, Geim,
Nature Physics **2**, 620 (2006)




$T(\varphi=0) = 1$ !

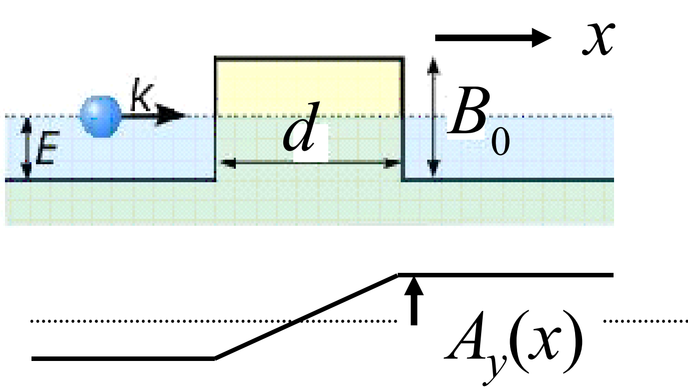
$$V_0 = \begin{cases} 200 \text{ meV} & \text{(red)} \\ 285 \text{ meV} & \text{(blue)} \end{cases}$$

Silvestrov, Efetov, PRL **98**, 016802 (2007)
Pereira, Peeters, Vasilopoulos, cond-mat/0610237
Pereira, Mlinar, Peeters, Vasilopoulos, PRB **74**, 045424 (2006)
Chaplik, Tudorovskiy, cond-mat/0610705
Prada, San-Jose, Wunsch, Guinea, cond-mat/0611189
Chen, Apalkov, Chakraborty, cond-mat/0611457

➡ **Quasi bound states**
**(Scattering resonances)**

# **Solution: use magnetostatic field design !**

$$\vec{p} \rightarrow \vec{p} - \frac{e}{c}\vec{A}$$




First example: magnetic barrier

$$\vec{B}(x,y) = B_0 \Theta(d - |x|)\vec{e}_z$$

$$p_x = E\cos\varphi \qquad p_y = E\sin\varphi + d/l_\mathrm{B}^2$$

$$\lambda_\mathrm{F} \gg \lambda_s \gg a$$

$$l_\mathrm{B} = \sqrt{c/eB_0} \qquad \sin\varphi' = \frac{2d}{El_\mathrm{B}^2} + \sin\varphi \Rightarrow T(\varphi) = |\,t\,|^2 = 0$$

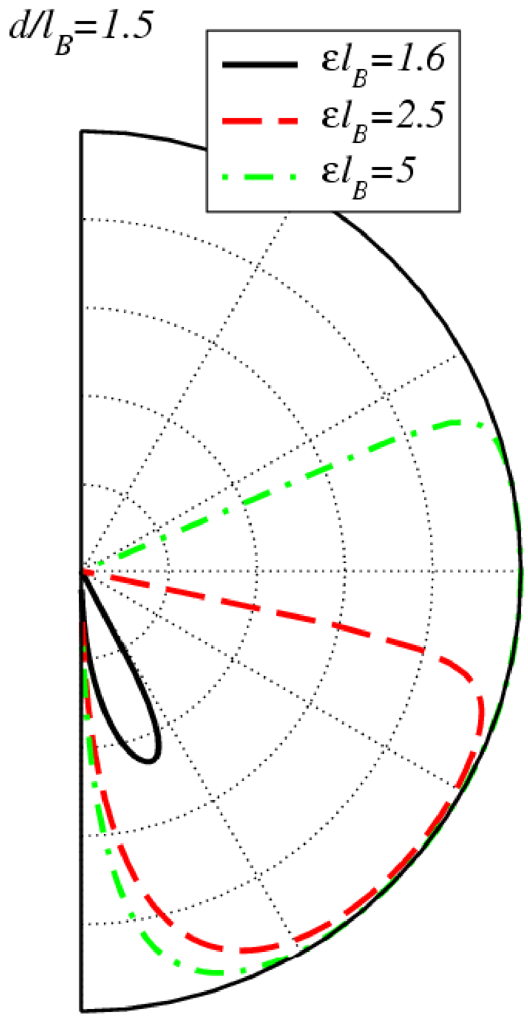
$$\Psi_I(x) = \begin{pmatrix} 1 \\ \mathrm{e}^{\mathrm{i}\varphi} \end{pmatrix} \mathrm{e}^{\mathrm{i}p_x x} + r \begin{pmatrix} 1 \\ -\mathrm{e}^{-\mathrm{i}\varphi} \end{pmatrix} \mathrm{e}^{-\mathrm{i}p_x x}$$

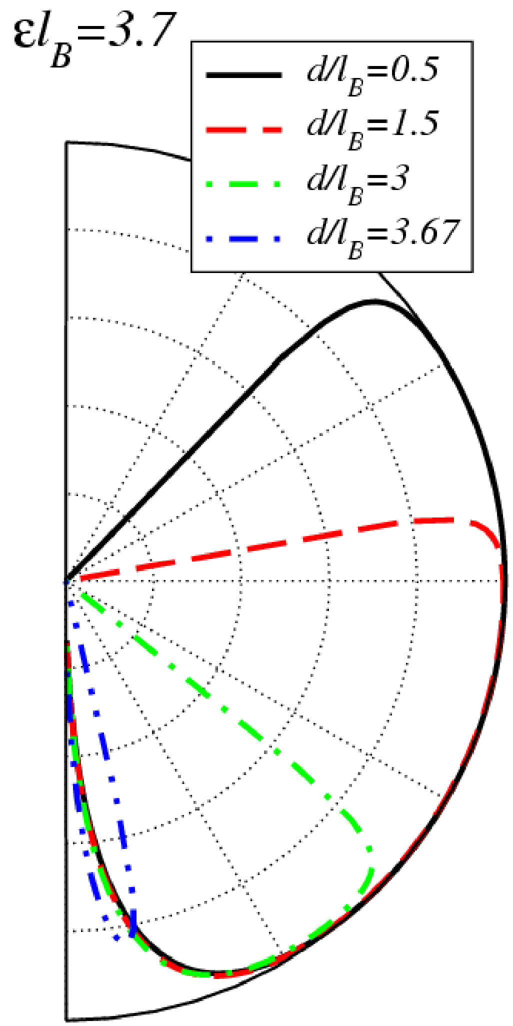
for $El_\mathrm{B} \leq \dfrac{d}{l_\mathrm{B}}$

$$\Psi_{II}(x) = \sum_{\pm} c_\pm \begin{pmatrix} D_{(El_\mathrm{B})^2/2-1}(\pm\sqrt{2}(x/l_\mathrm{B} + p_y l_\mathrm{B})) \\ \pm\mathrm{i}\frac{\sqrt{2}}{El_\mathrm{B}} D_{(El_\mathrm{B})^2/2}(\pm\sqrt{2}(x/l_\mathrm{B} + p_y l_\mathrm{B})) \end{pmatrix}$$

$$\Psi_{III}(x) = t\sqrt{p_x/p'_x} \begin{pmatrix} 1 \\ \mathrm{e}^{\mathrm{i}\varphi'} \end{pmatrix} \mathrm{e}^{\mathrm{i}p'_x x}$$

# Transmission probability T(φ)




$T(\varphi)$ can be controlled !

different energies $\varepsilon$ different barrier widths $d$

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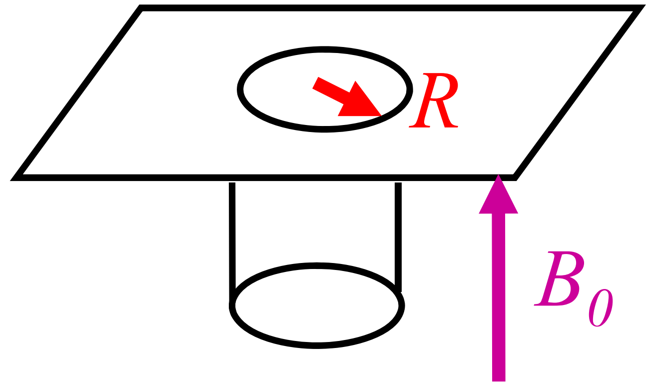
# Second example: magnetic hard wall quantum dot

$$\vec{B}(r) = B_0 \Theta(r - R)\vec{e}_z$$

$$\vec{A}(r,\varphi) = \frac{1}{2l_B^2}\left(r - \frac{R^2}{r}\right)\Theta(r-R)\vec{e}_\varphi$$




$$\psi(r,\varphi) \propto \begin{pmatrix} e^{im\varphi} & \phi_m(r) \\ e^{i(m+1)\varphi} & \chi_m(r) \end{pmatrix}$$

$$\phi_m(r<R) \sim J_m(Er)$$

$$\phi_m(r>R) \sim \Psi(1 + \tilde{m}\Theta(\tilde{m}) - \frac{E^2}{2}, 1 + |\tilde{m}|, \frac{r^2}{2})$$

degenerate hypergeometric function

$$\tilde{m} := m - \frac{R^2}{2l_B^2}$$

angular momentum

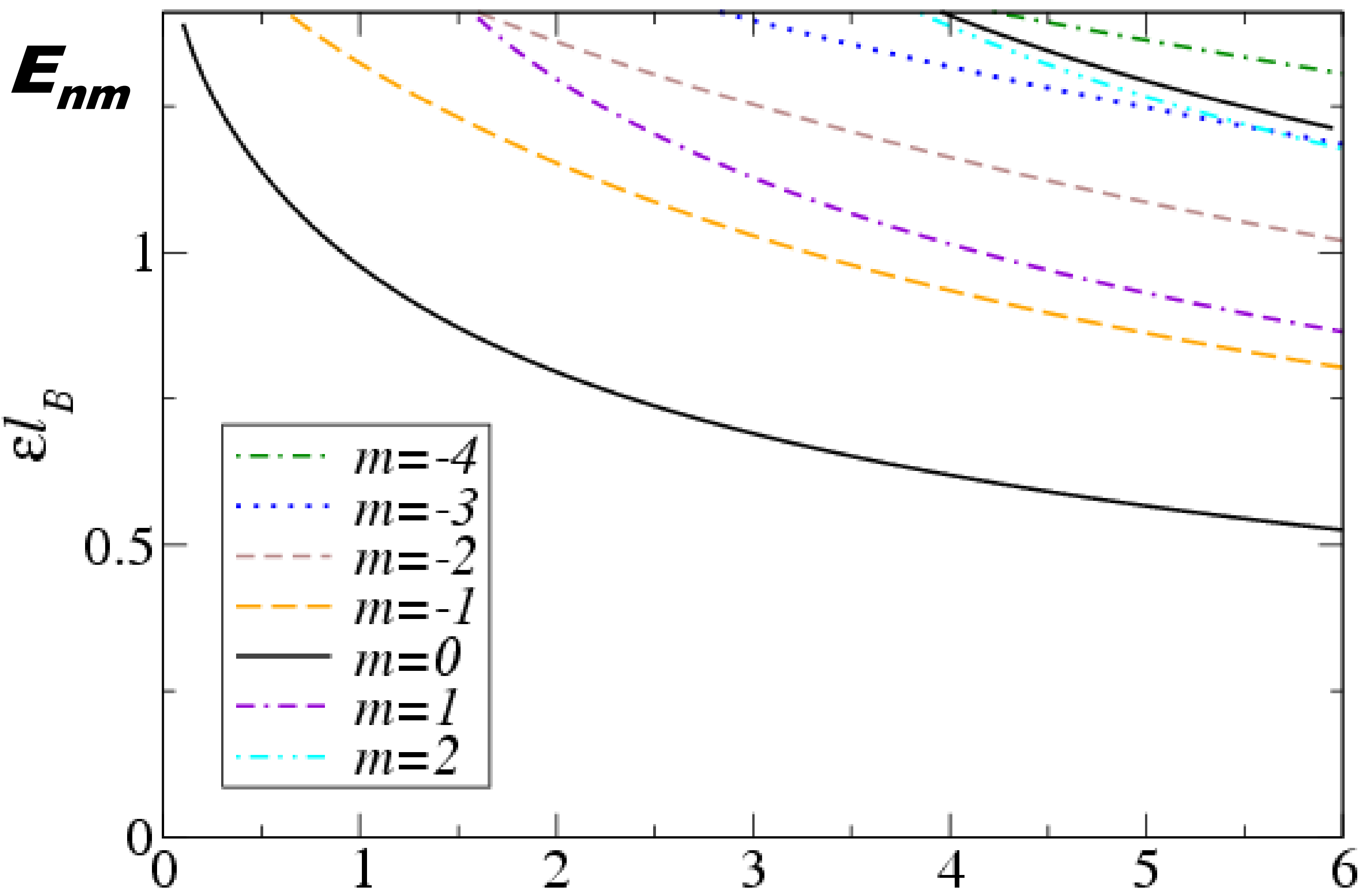
missing flux due to the dot

$$\chi_m(r \lessgtr R) : \text{ similarly}$$

Matching conditions: $\psi(R_-,\varphi) = \psi(R_+,\varphi)$

➡ **eigenenergies** $E_{nm}$

# (Unbroadened) energy levels




missing flux $R^2/2l_B^2$

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# Summary:

**main message:**

Use *inhomogeneous* magnetic fields for designing mesoscopic graphene devices (constrictions, quantum point contacts, quantum dots, artificial atoms, ...) !

- magnetic barrier

- magnetic hard wall quantum dot