Science in High Magnetic Fields

FACULTY OF SCIENCE, MATHEMATICS AND COMPUTING SCIENCE

DPG-Frühjahrstagung Regensburg, 29.3.2007, HL 37.4

News from the Quantum Hall Effect in Graphene

Uli Zeitler

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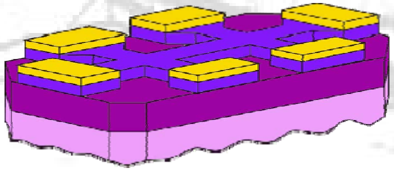
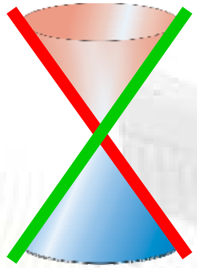
Institute for Molecules and Materials, Radboud University Nijmegen

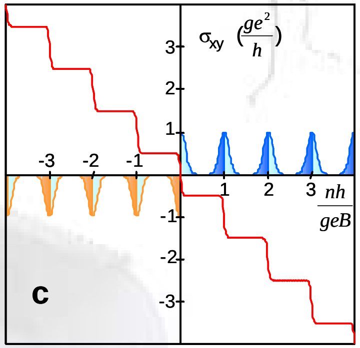
Contents

- ➢ Introduction
  - ➢Graphene devices
  - ➢Quantum Hall effect

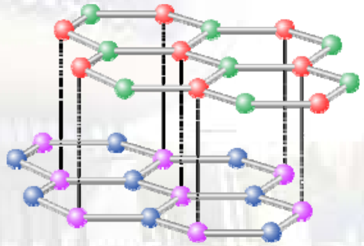


- ➢ Single-layer graphene
  - ➢Bandstructure of graphene
  - ➢Half-integer QHE
  - ➢Room-temperature QHE

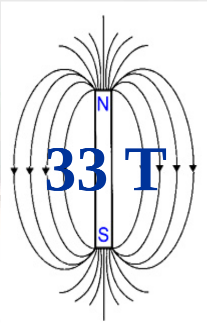




- ➢ Bilayer graphene
  - ➢Bandstucture of massive chiral Dirac fermions
  - ➢A new type of integer QHE

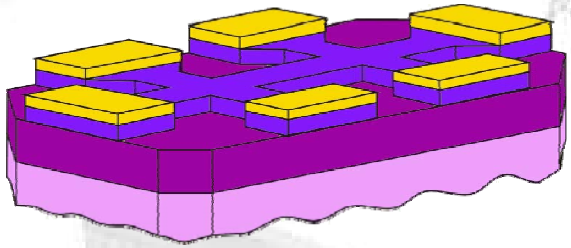


- ➢ Conclusions
- ➢ Commercial



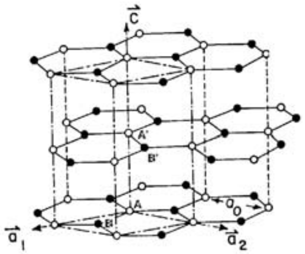
Introduction: Graphene devices

- **Introduction**
  - **Graphene devices**
    - Quantum Hall effect
- Single-layer graphene
  - Bandstructure of graphene
  - Half-integer QHE
  - Room-temperature QHE & activation gaps
- Bilayer graphene
  - Bandstructure of massive chiral Dirac fermions
  - A new type of inteher QHE
- Conclusions

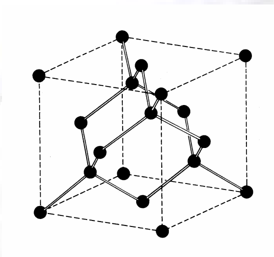


Carbon allotropes

3D

Graphite

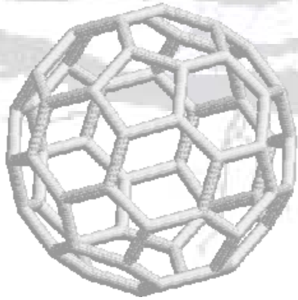
Diamond

2D

Graphene

Novoselov et al., 2004

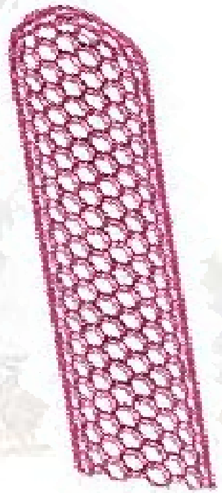
0D



Fullerenes

Curl, Kroto, Smalley 1985
Nobel prize 1996

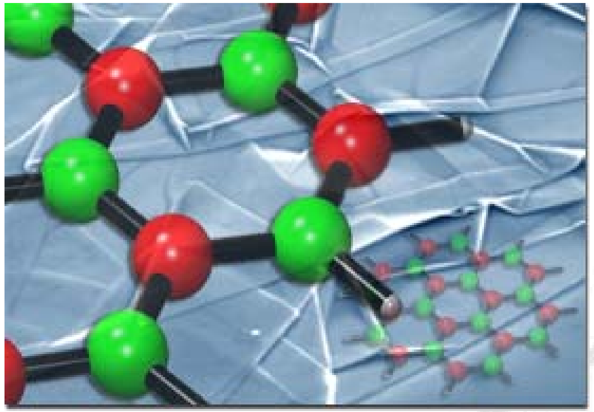
1D



Carbon Nanotubes

Multi-wall 1991
Single-wall 1993

Graphene



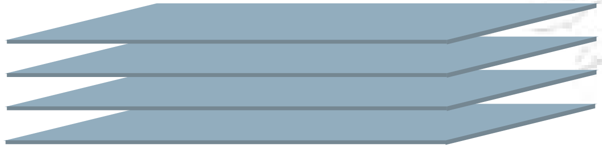
- mono-atomic layer of carbon
- first truly 2D system
  (that should not exist !!)
- unique elecronique properties
  (zero-gap semiconductor with
  massless chiral Dirac fermions)

A monolayer of graphite
on SIMOX is visible
under an **optical microscope**

Other truly 2D systems

Novoselov et al., PNAS 2005

Making graphene devices



- Mechanical cleavage of natural graphite

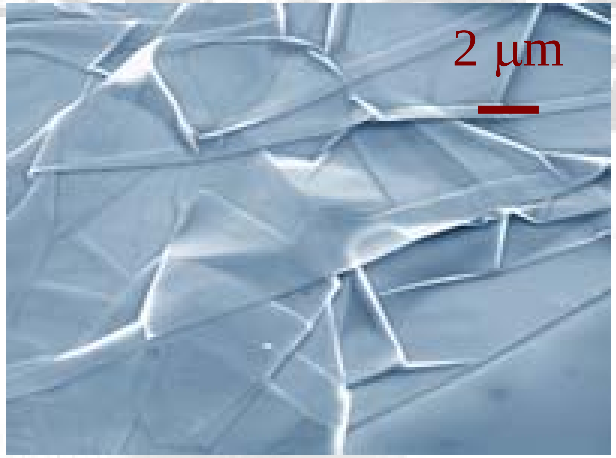
  "peel off down to a single layer"

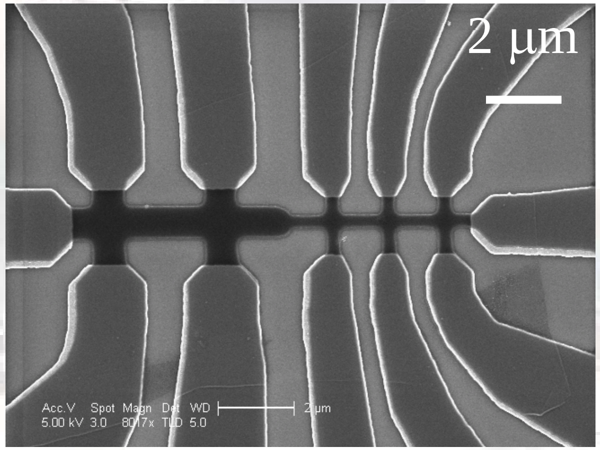
- Identification of single-layer flakes under ***optical microscope***

- Contacting with Au contacts

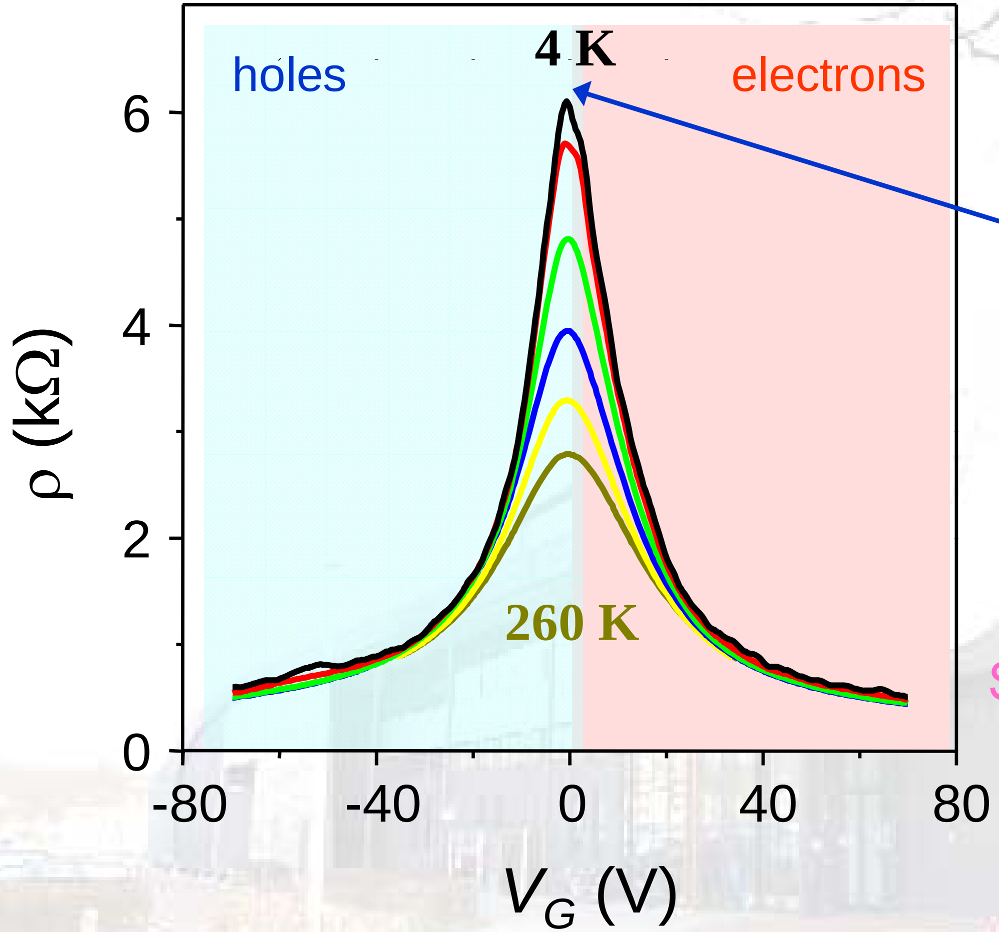
- Mesa etching

Novoselov et al., Science 2004



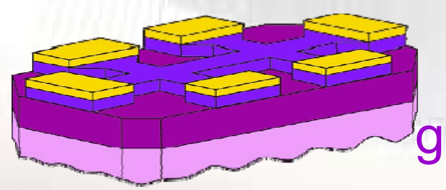


Ambipolar field-effect transistor



Field effect doping in a MOSFET

Saturating at ($h/4e^2$)
in charge-neutrality point



Novoselov et al.,
Science 2004, PNAS 2005

conducting without any charge carriers !!!

Introduction: Quantum Hall effect

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  - **Quantum Hall effect**
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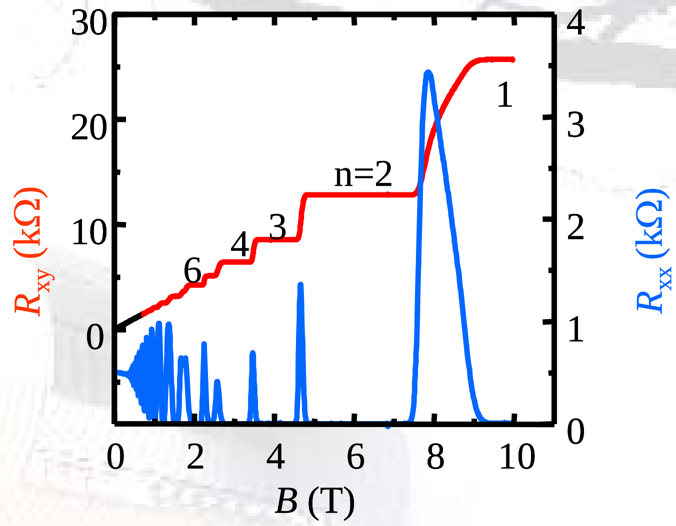
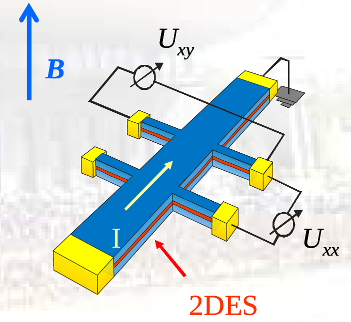
- Bilayer graphene
  - Bandstructure of massive chiral Dirac fermions
  - A new type of integer
- Conclusions

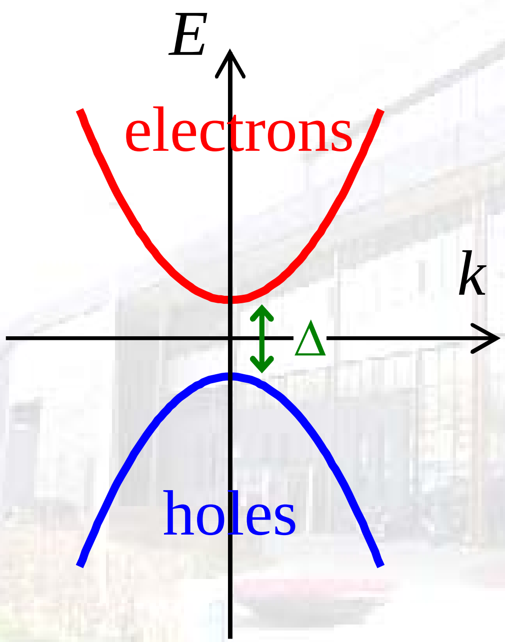




Landau levels

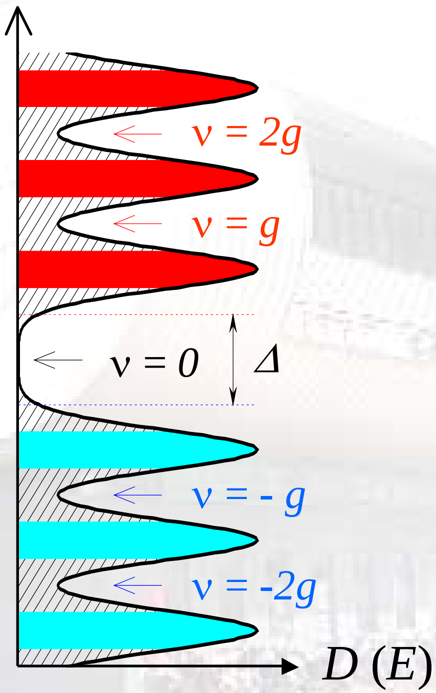
Parabolic Dispersion

$$E^{(e/h)} = \pm\left(\frac{\hbar^2 k^2}{2m^*} + \frac{\Delta}{2}\right)$$



Landau quantisation

$$E_N = \pm\left[\hbar\omega_c(N + \tfrac{1}{2}) + \Delta/2\right]$$



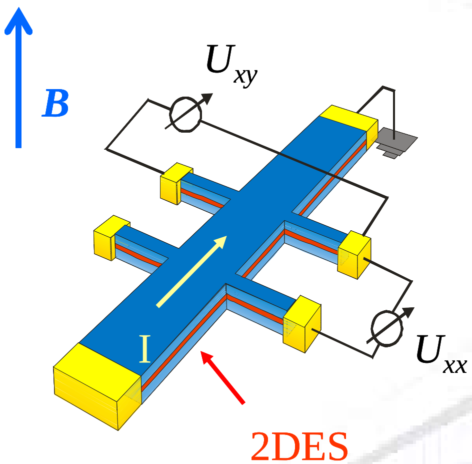
Level-degeneracy:

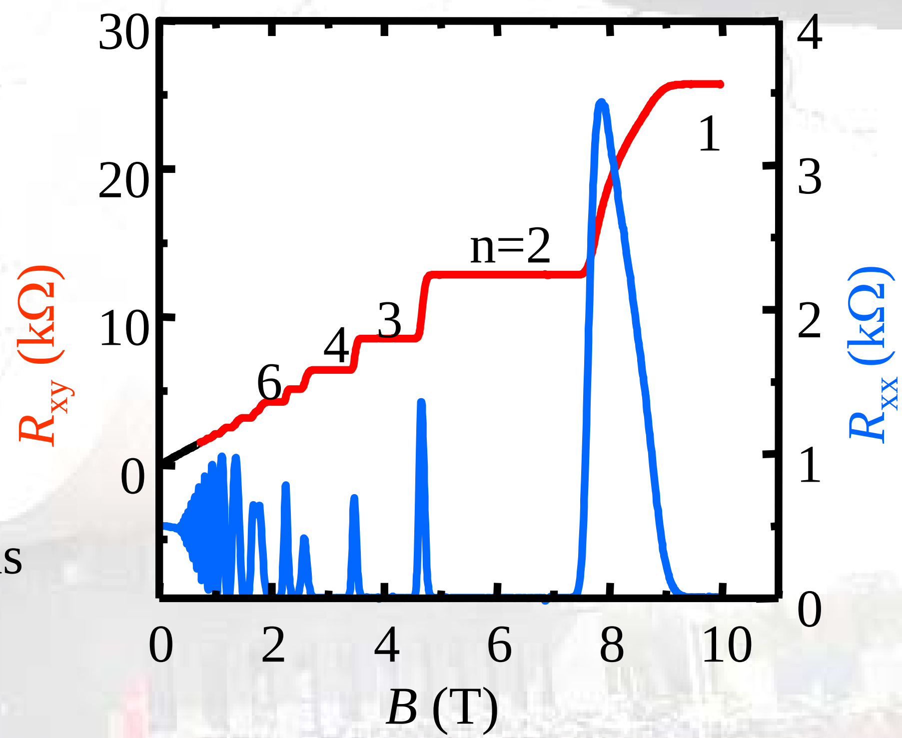
$$\boxed{n_L = g_v g_s \frac{eB}{h} = 4\frac{eB}{h}}$$

independent on dispersion !

localised and ***extended*** states

Magnetotransport





sweep magnetic field

⇒ successively depopulate Landau levels

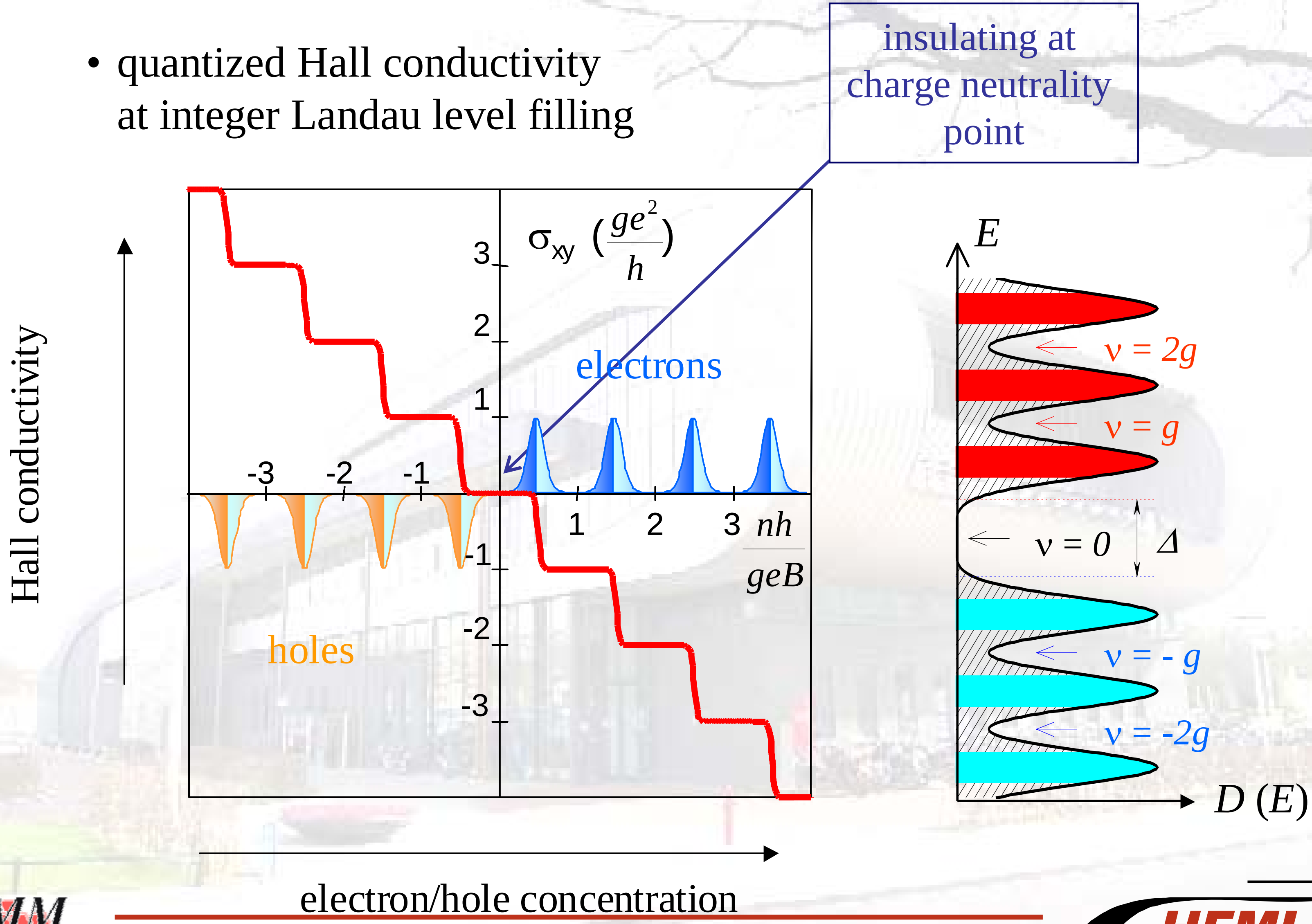
⇒ $1/B$-periodic oscillations in R_{xx}

⇒ quantized plateaus in $R_{xy} = (h/ne^2)$

⇒ $R_{xx} \rightarrow 0$ in quantum Hall plateaus

Conventional quantum Hall effect

- quantized Hall conductivity at integer Landau level filling



Single-layer graphene

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- **Single-layer graphene**
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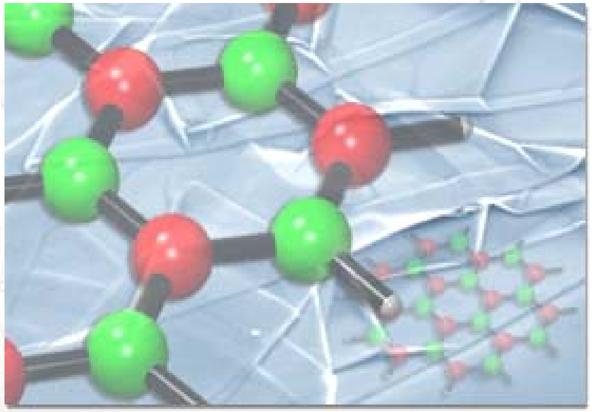
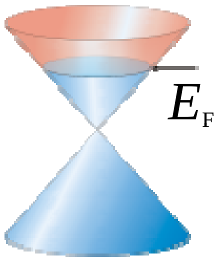
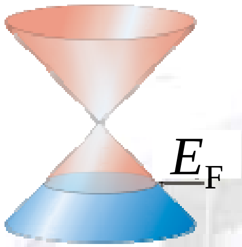


Bandstructure graphene

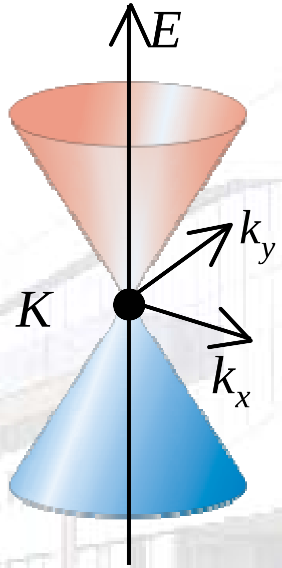
n-doped





p-doped

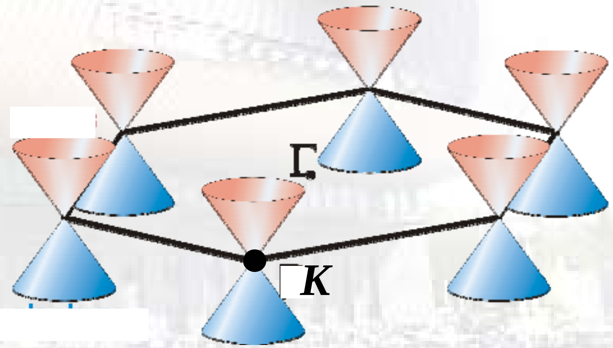
$$E(k) = \pm c\,\hbar\,|k|$$



Fermi energy
exactly on Dirac point

$$E = c\,\hbar k \qquad c \approx 10^6\ \mathrm{m/s}$$

zero-gap semiconductor
linear dispersion
(relativistic particles)



Wallace, 1947

Dirac equation in a magnetic field

<u>Schrödinger</u>

$$\left(\frac{1}{2m^*}(\hat{\mathbf{p}}\text{-}e\mathbf{A})^2\right)\psi = E\,\psi$$

$$\boxed{E_N = \frac{\hbar eB}{m^*}(N + \tfrac{1}{2})}$$

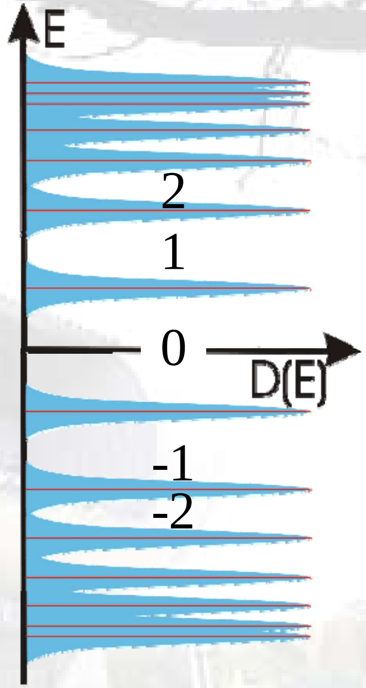
<u>Dirac</u>

$$\left(c\,(\hat{\mathbf{p}}\text{-}e\mathbf{A})\right)\psi = E\,\psi$$

$$\left(c^2\,(\hat{\mathbf{p}}\text{-}e\mathbf{A})^2\right)\psi = E^2\,\psi$$

$$E_N^2 = 2c\,\hbar eB\,(N + \tfrac{1}{2})$$

$$\boxed{E_N = \pm\sqrt{2c\,\hbar eB\,(N + \tfrac{1}{2})}}$$



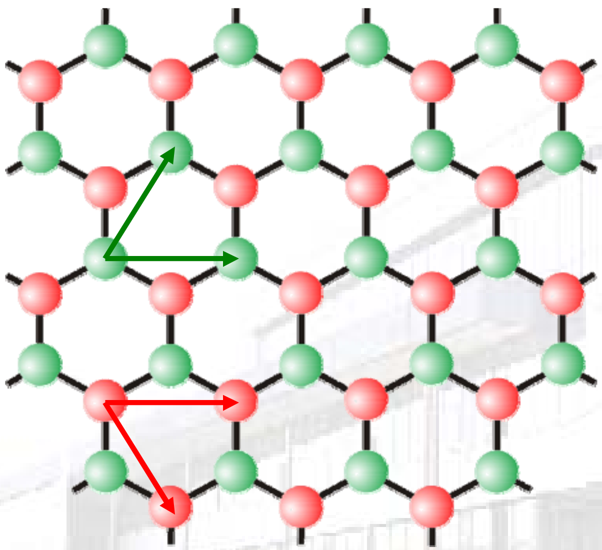
integer QHE !

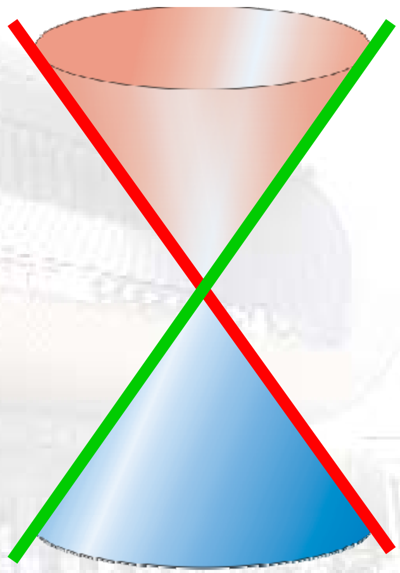
Chirality

honeycomb lattice:

2 triagonal sublattices (+/ -)

$\psi = \sin\varphi \;|+\rangle + \cos\varphi \;|-\rangle$





Chiral superposition of sublattice wave functions (pseudospin)

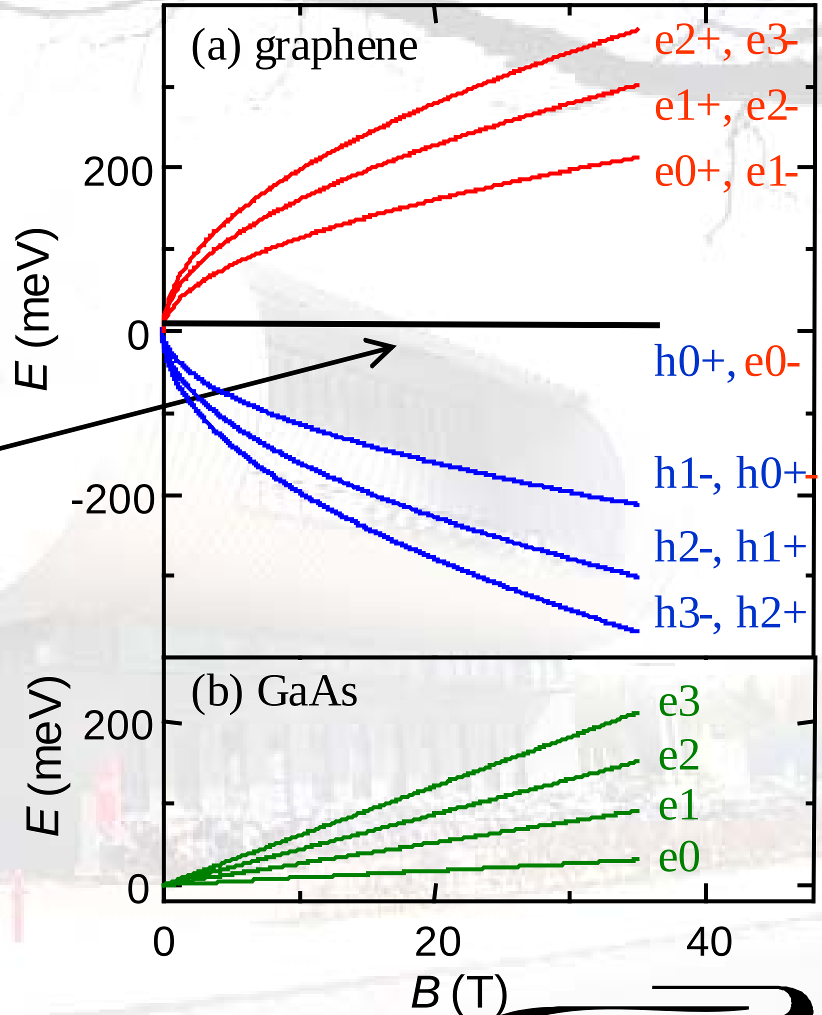
Chiral Dirac fermions in a magnetic field

relativistic Landau levels

$$E_{N,\pm} = \pm\sqrt{2\hbar c^2 eB(N + \tfrac{1}{2} \pm \tfrac{1}{2})}$$

chirality index

Landau level **at zero energy**
shared between electrons and holes

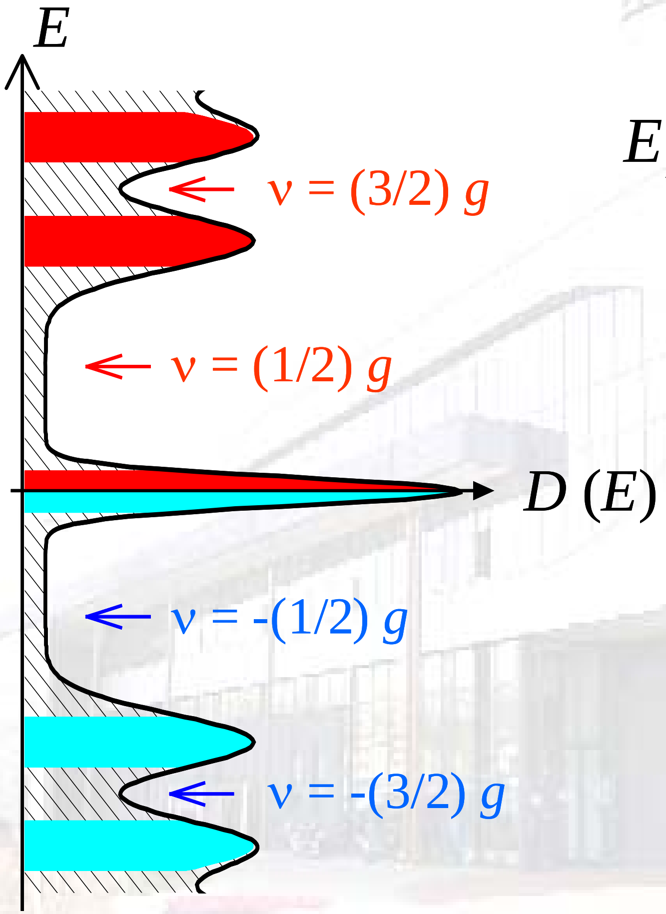


traditional Landau levels

$$E_N = \pm\hbar\omega_c(N + \tfrac{1}{2})$$

Landau levels in graphene

electrons



ν =0 →

holes

$$E_{N,\pm} = \pm\sqrt{2\hbar c^2 eB(N + \tfrac{1}{2} \pm \tfrac{1}{2})}$$

Level-degeneracy:

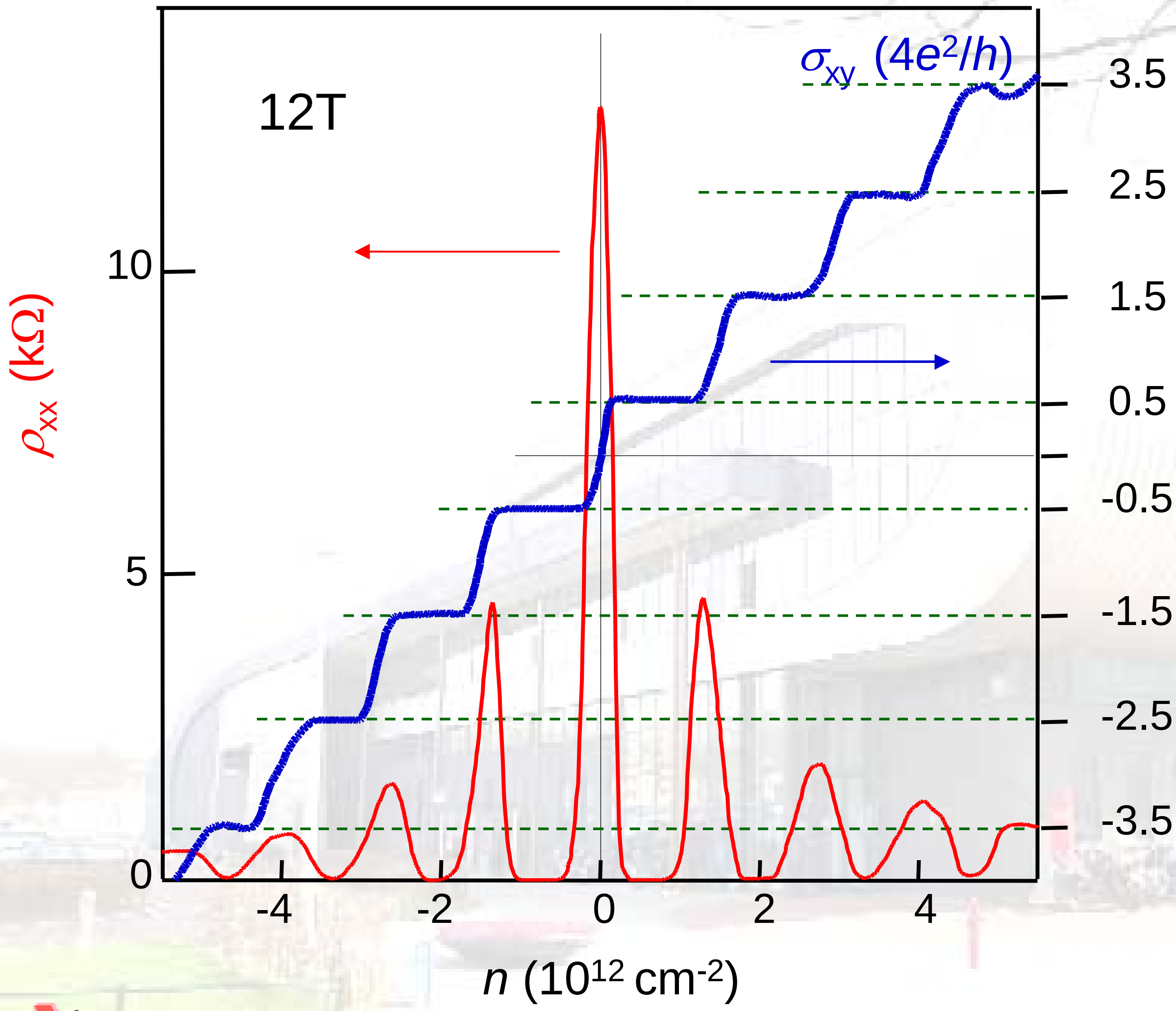
$$\boxed{n_L = g_v g_s \frac{eB}{h} = 4\frac{eB}{h}}$$

ν = 0 Landau level **at zero energy**

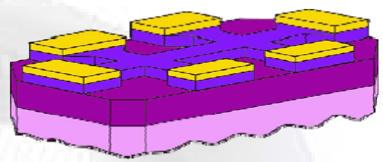
half-filled with electrons and holes

⇒ half-integer QHE

Half-integer quantum Hall effect



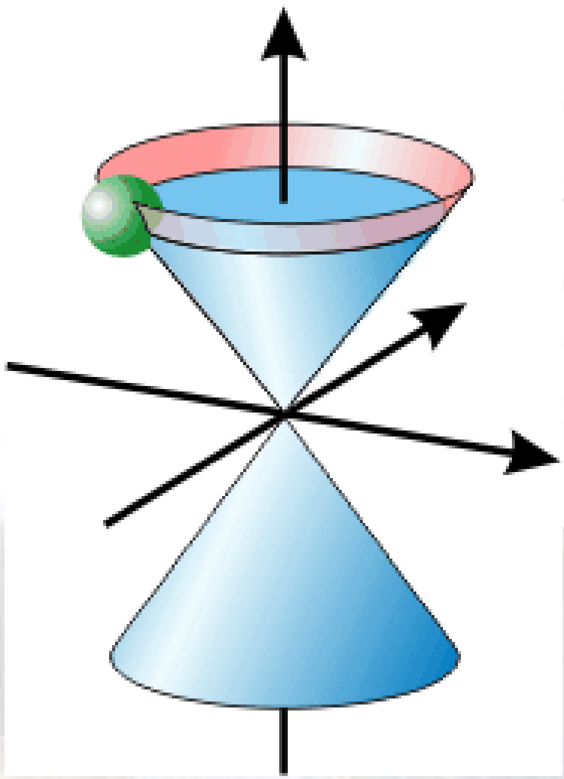
$$\sigma_{xy} = \frac{4e^2}{h}(N + \tfrac{1}{2})$$

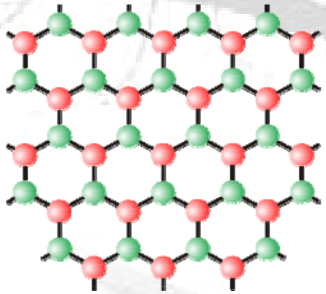


Novoselov *et al*, Nature 2005
Zhang *et al*, Nature 2005

Berry's phase

$$\psi = \sin\varphi\ |+\rangle + \cos\varphi\ |-\rangle$$

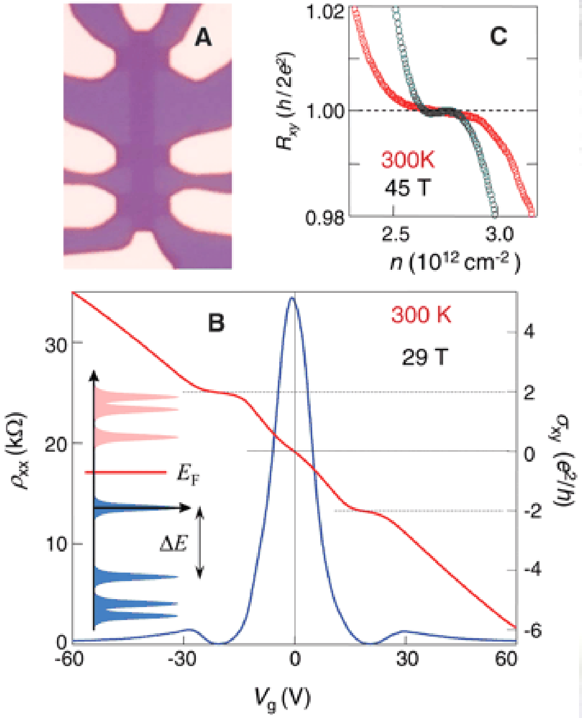




Chiral superposition of sublattice wave functions

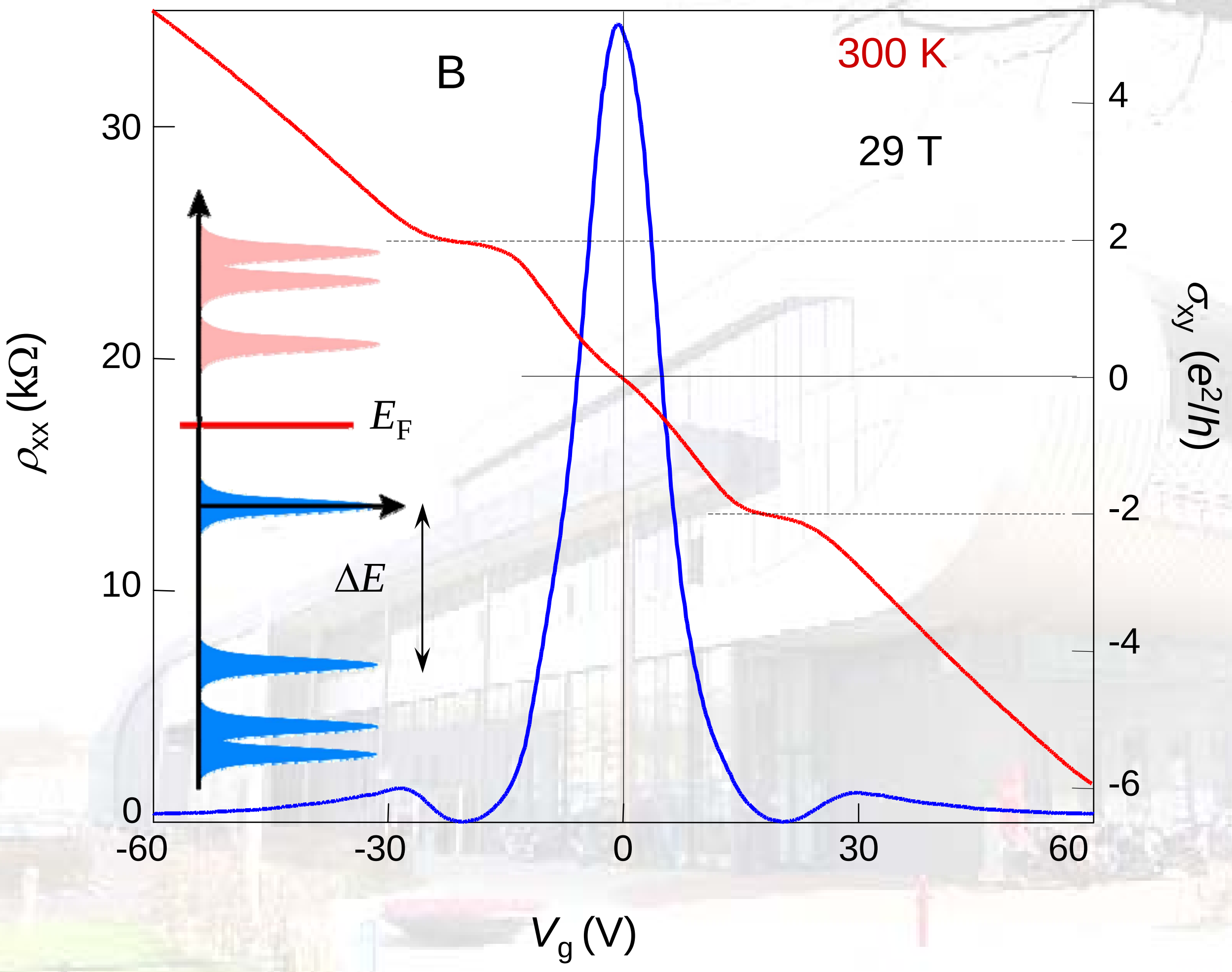
$\Rightarrow$ Topological Berry's phase π

Room Temperature QHE



K.S. Novoselov *et al.*, Science 315, 1379 (2007).

Room Temperature QHE at 30 T

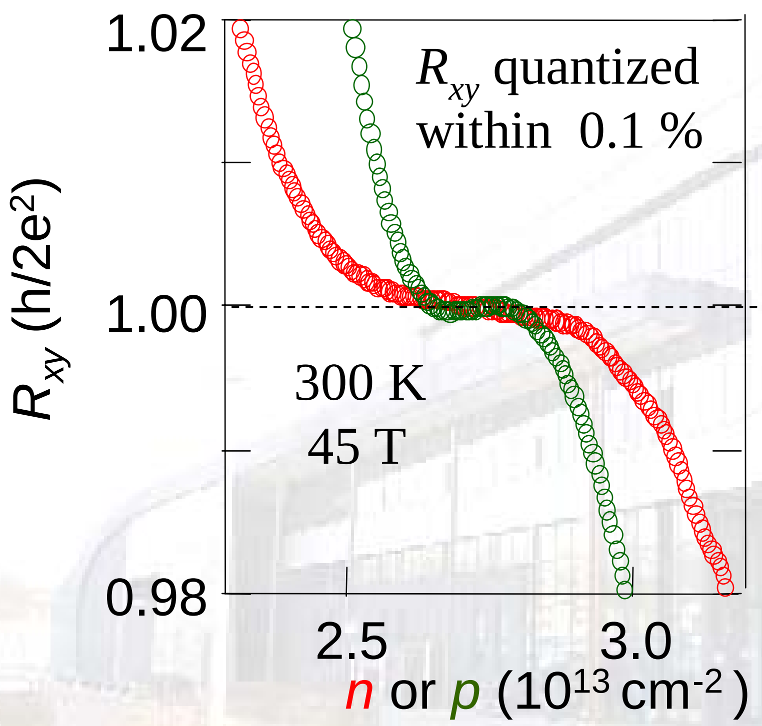


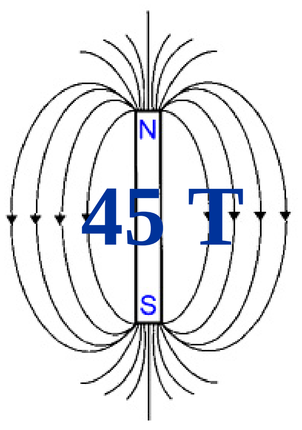
previously only below 30 K !

Novosolov *et al.*, Science 2007

Room Temperature QHE at 45 T

MAGNET LAB
NATIONAL HIGH MAGNETIC FIELD LABORATORY
FLORIDA STATE UNIVERSITY - LOS ALAMOS NATIONAL LABORATORY - UNIVERSITY OF FLORIDA





With:

Z. Jiang,* Y. Zhang, H. L. Stormer,

G. Boebinger,* and **P. Kim**,

Columbia University, New York, USA

**National High Magnetic Field Laboratory, Tallahassee, USA*

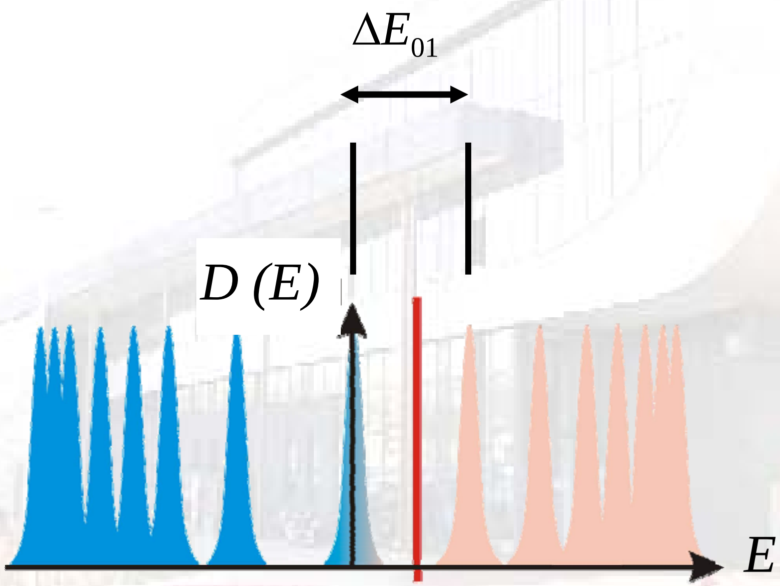
Novosolov *et al.*, Science 2007

Gaps in graphene

$$E_{N,\pm} = \mathrm{sgn}(N)\sqrt{2\hbar c^2 eB\,|\,N + \tfrac{1}{2} \pm \tfrac{1}{2}\,|}$$

$$\Delta E_{kl} = c_* \sqrt{2e\hbar B}\,(\sqrt{l} - \sqrt{k}\,)$$

B = 30 T



$$\Delta E_{01} \approx 2300\,\mathrm{K}$$

... more than just gaps!

- *single-layer graphene:*

$$E_{01} = \pm\sqrt{2\hbar c^2 eB} \approx 2300\,\mathrm{K} \quad \text{at } 30\,\mathrm{T}$$

we do observe room-temperature QHE !

- *conventional semiconductors:*

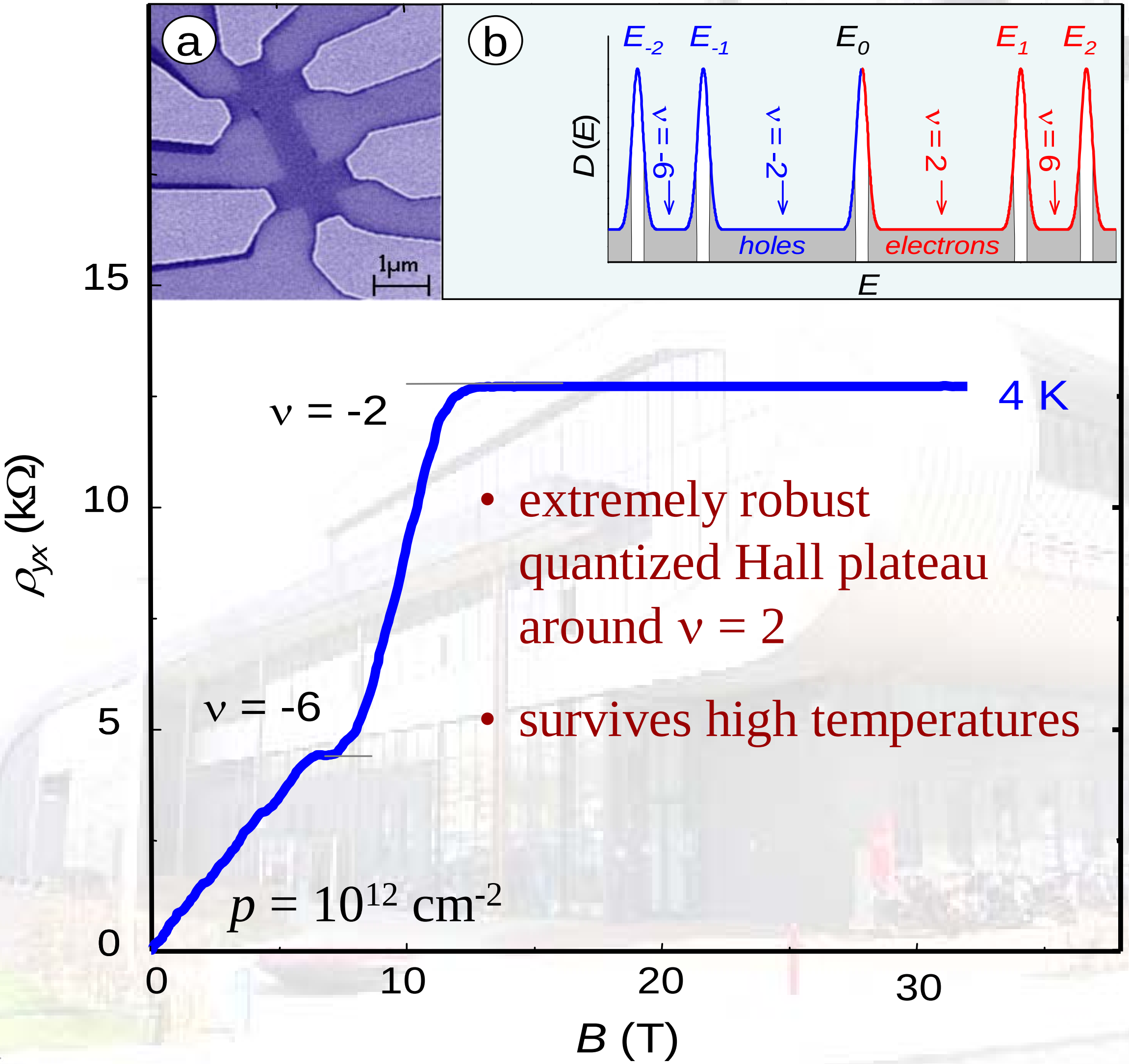
$$\Delta E_{LL} = \frac{\hbar eB}{2m^*} \approx 600\ \mathrm{K} \quad \text{for GaAs}$$ no QHE at 77 K !!

$$\approx 3000\ \mathrm{K} \quad \text{for InSb}$$ no QHE at 300 K !!

- *chirality of edge states ?*
- *mesoscopic samples ?*
- *non-local transport ?*
- ***localization in graphene*** **!?**
- **peculiar zero-energy level !?**
- .....???

$\Rightarrow$ more than just gaps !!

Robustness of the QHE



- extremely robust quantized Hall plateau around ν = 2
- survives high temperatures

Bilayer graphene

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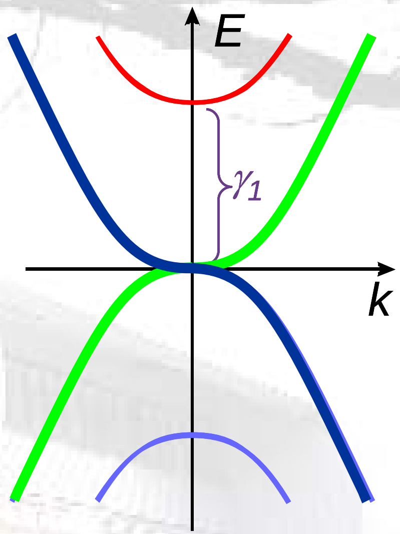
- Single-layer graphene
  - Bandstructure of graphene
  - Half-integer QHE of chiral Dirac fermions
  - Room-temperature QHE & activation gaps

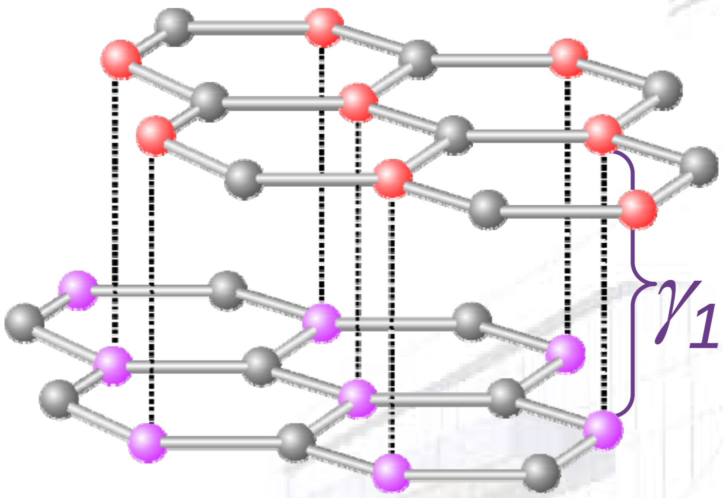


- **Bilayer graphene**
  - **Bandstructure of massive chiral Dirac fermions**
  - A new type of inteher QHE
- Conclusions

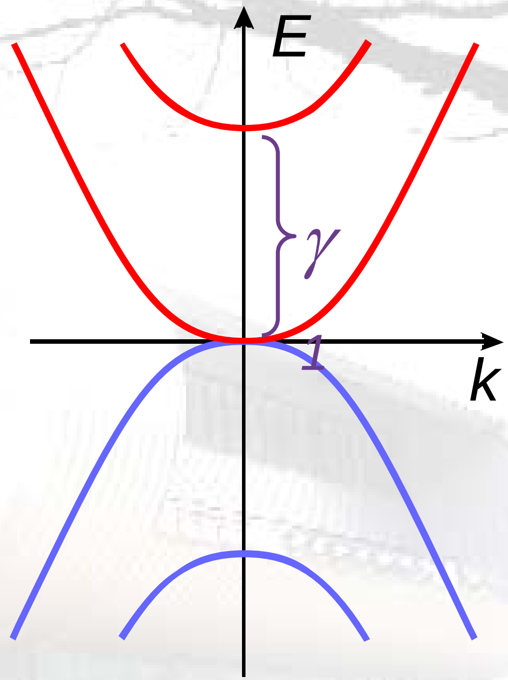
with: Ed McCann and Vladimir Fal'ko
Department of Physics, Lancaster University,UK

Science in High Magnetic Fields

Bandstructure in bilayers



A-B stacking



γ_1: inter-layer coupling

c: particle velocity

$$E(p) = \pm\left(\tfrac{1}{2}\gamma_1 + \sqrt{\tfrac{1}{4}\gamma_1^2 + c^2 p^2}\right)$$

McCann, Fal'ko, PRL **96**, 086805 (2006)

QED in a pencil trace

$$E(p) = \pm \tfrac{1}{2}\gamma_1 \pm \sqrt{\tfrac{1}{4}\gamma_1^2 + v_F^2 p^2}$$

$$E(p) = \pm\left(m_0 c^2 + \sqrt{m_0^2 c^4 + p^2 c^2}\right)$$

inter-layer intra-layer

High energy physics in a pencil trace

$$\left.\begin{array}{r}\text{massive}\\ \text{relativistic}\\ \text{chiral}\end{array}\right\}\ \text{Dirac fermions}$$

A new type of integer QHE

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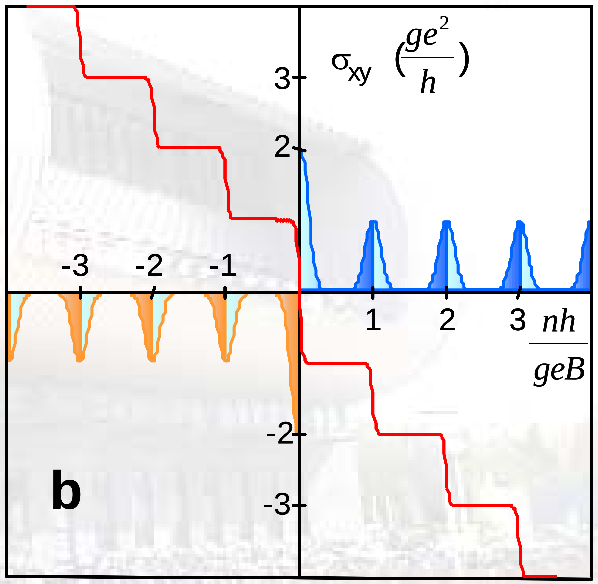
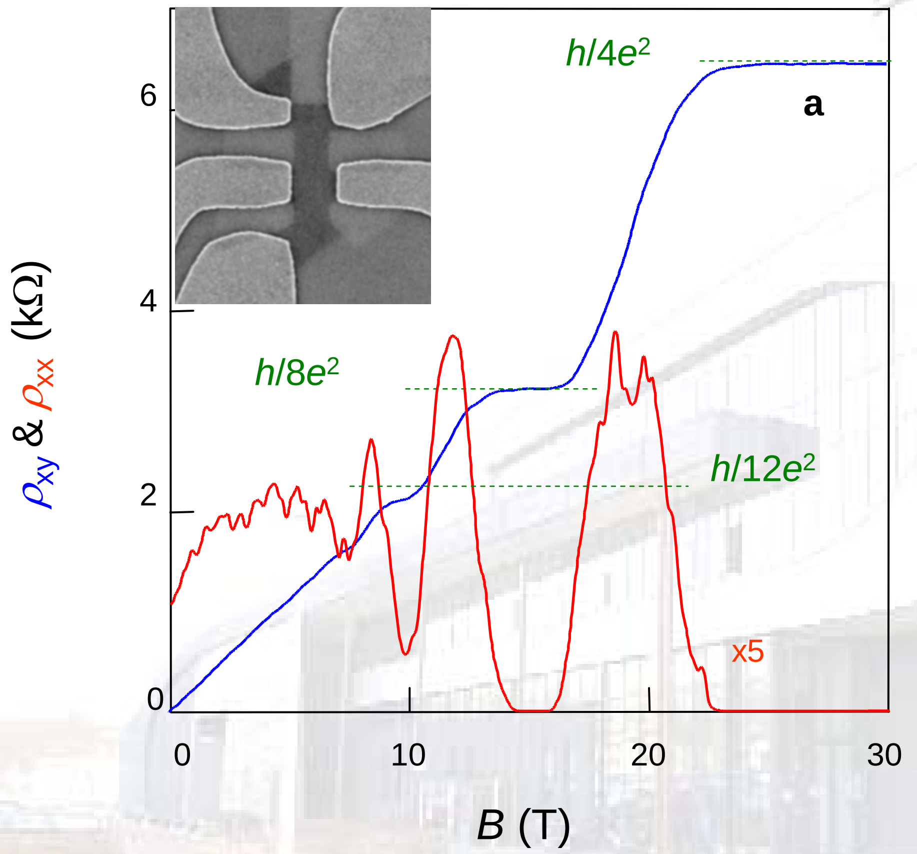
K.S. Novoselov *et al.*, Nature Physics 2, 177 (2006).



QHE in bilayer graphene

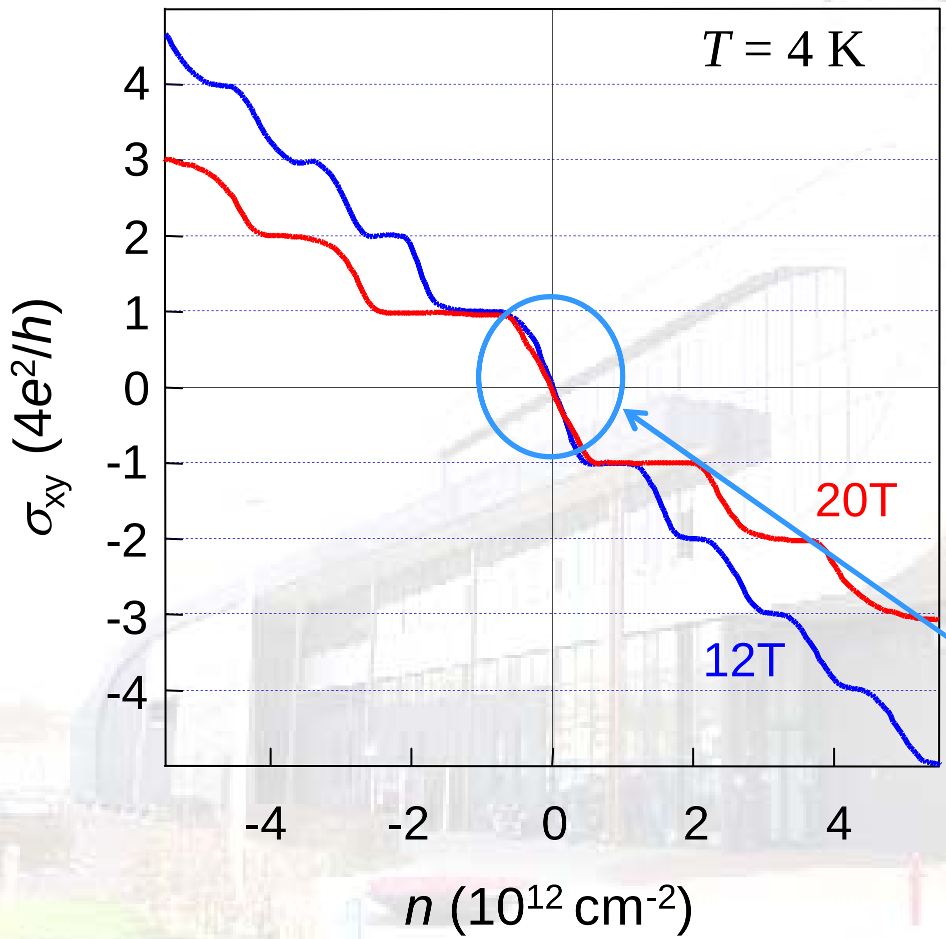


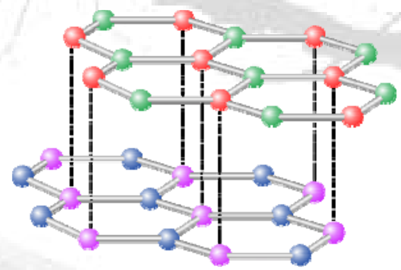
⇒ re-establishment of integer QHE
(four-fold degenerate Landau levels)

⇒ towards a conventional QHE
with a parabolic dispersion ?

Novoselov *et al*, Nature Physics 2006

Unconventional integer QHE





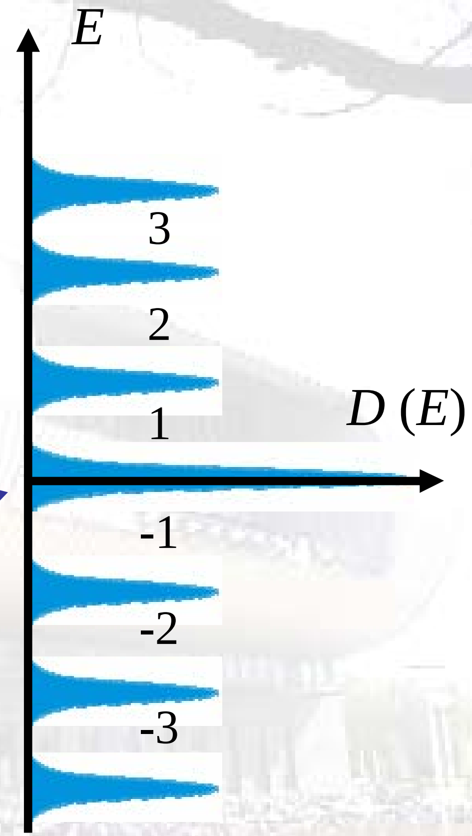
- quantum Hall plateaus
  (of four-fold degenerate Landau levels)
  at integer filling factors
  $N = \pm1, \pm2, \pm3, \ldots$

***N*=0 plateau missing !**

Landau levels in a bilayer

$$E_{LL} = \pm\hbar\omega_c\sqrt{N(N-1)}$$

$$= \begin{cases} 0 & \text{for } N = 0 \\ \pm\hbar\omega_c(N+\tfrac{1}{2}) & \text{for } N \to \infty \end{cases}$$

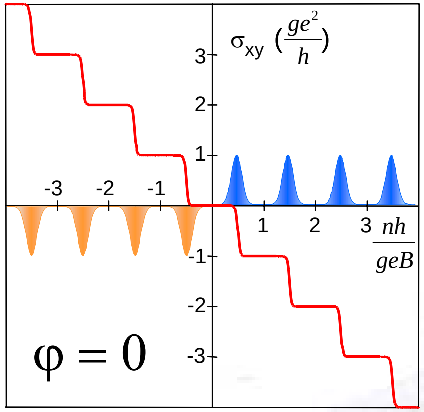


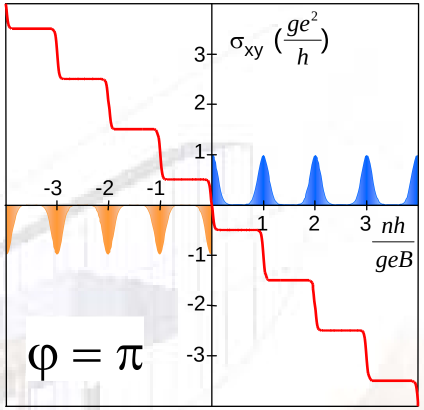
8-fold degenerate Landau level at zero energy shared equally between electrons and holes

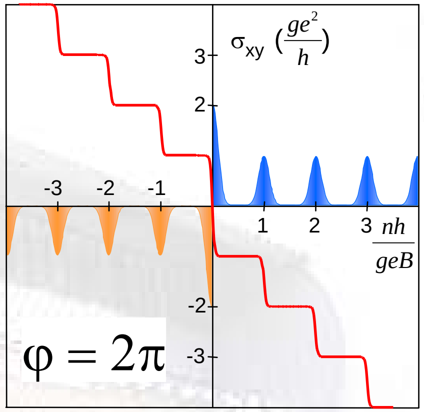
topological (Berry) phase 2π

Three types of integer QHE

conventional integer QHE (semiconductors)	relativistic half-integer QHE (single-layer graphene)	unconventional integer QHE (bilayer graphene)







$$E_{LL} = \pm\hbar\omega_c(n + \tfrac{1}{2})$$

$$E_{LL} = \pm\sqrt{2\hbar c^2 eB(n + \tfrac{1}{2} \pm \tfrac{1}{2})}$$

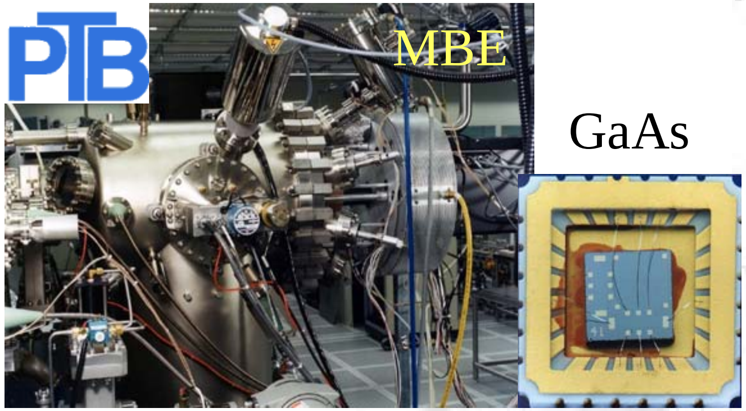
$$E_{LL} = \pm\hbar\omega_c\sqrt{n(n-1)}$$

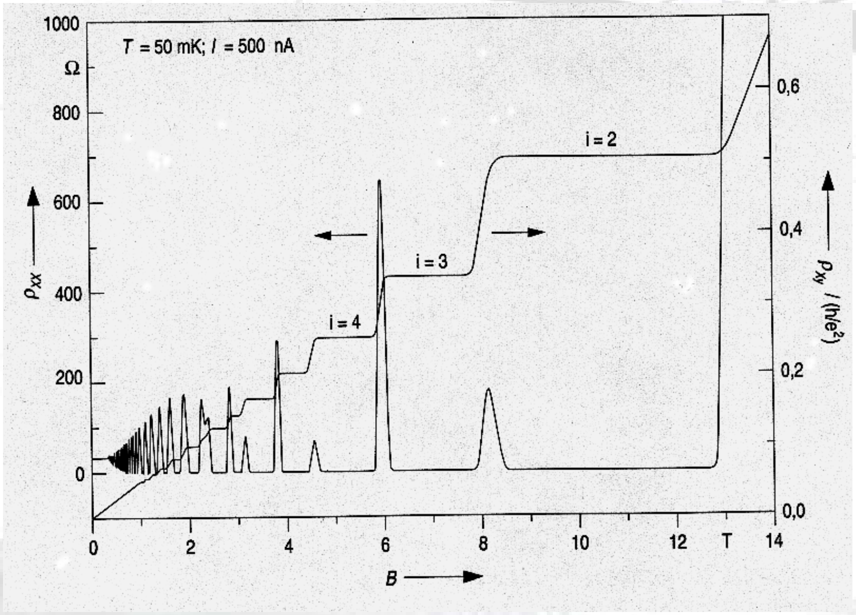
von Klitzing *et al.*
PRL **45**, 494 (1980)

Novoselov *et al.*,
Nature **438**, 197 (2005)
Zhang *et al.*,
Nature **438**, 201 (2005)

Novoselov *et al.*,
Nature Physics 2, 177 (2006).
McCann, Fal'ko,
PRL **96**, 086805 (2006)

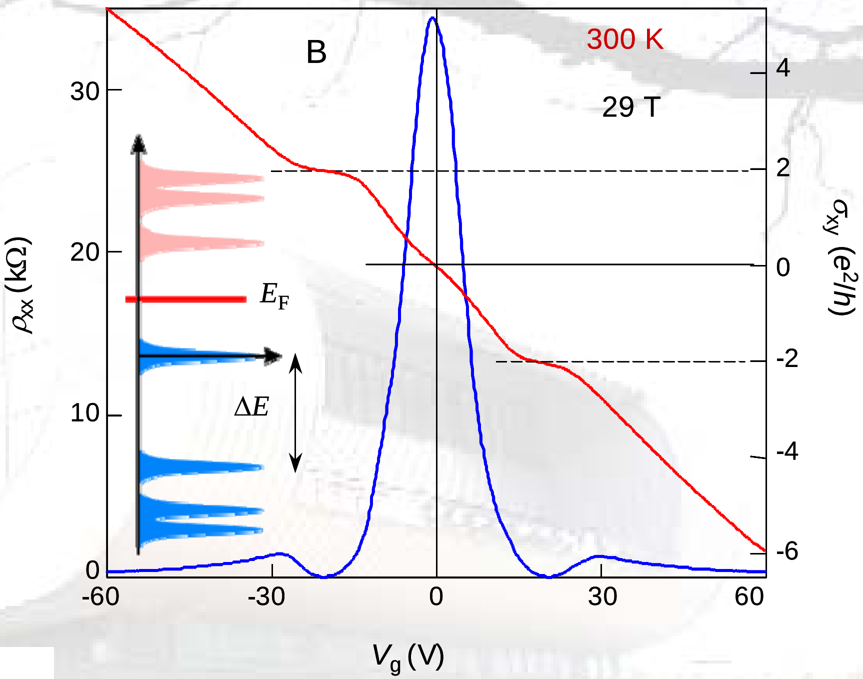
QHE today

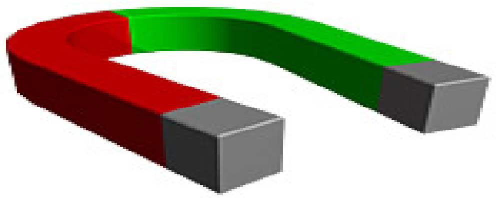




$$R_K = 25812.807\ \Omega$$

Future of QHE



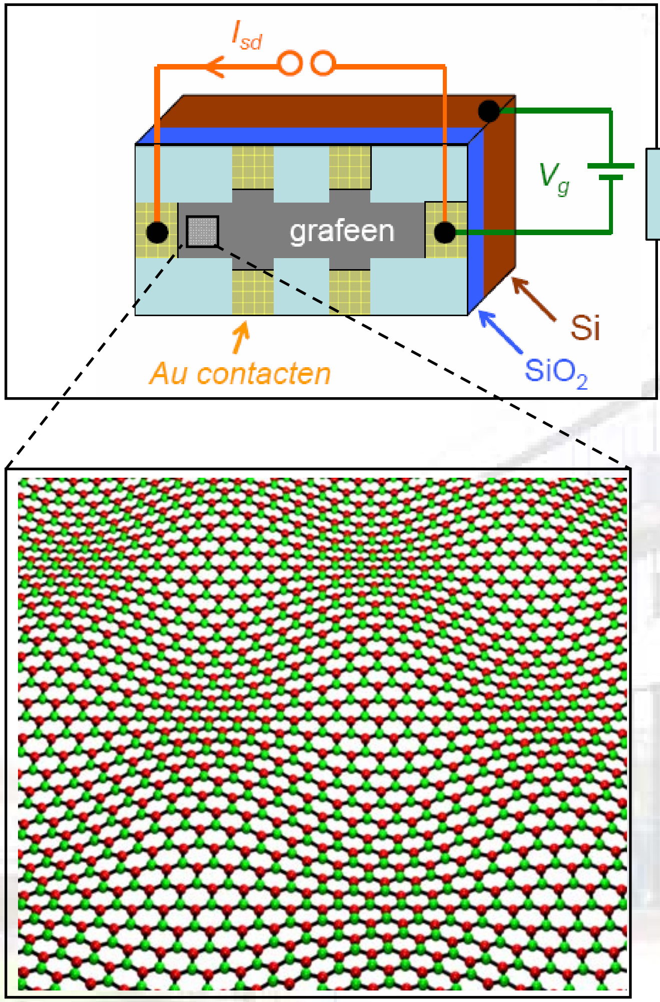


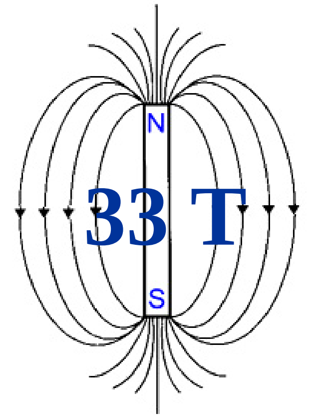
Conclusions

Conclusions

- ***Graphene:*** a real two-dimensional crystal !
- *single-layer graphene:*
  - relativistic electrons with a half-integer QHE
  - room-temperature QHE
  - towards a room-temperature quantum metrology ?
- *bilayer graphene*: a third type of QHE with
  chiral, massive relativistic Dirac fermions
- ***after 25 years QHE*** is still alive
- ***high magnetic fields*** can help a lot

Commercial





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News from the QHE in Graphene

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Jos Giesbers, Jan Kees Maan, and **UZ**
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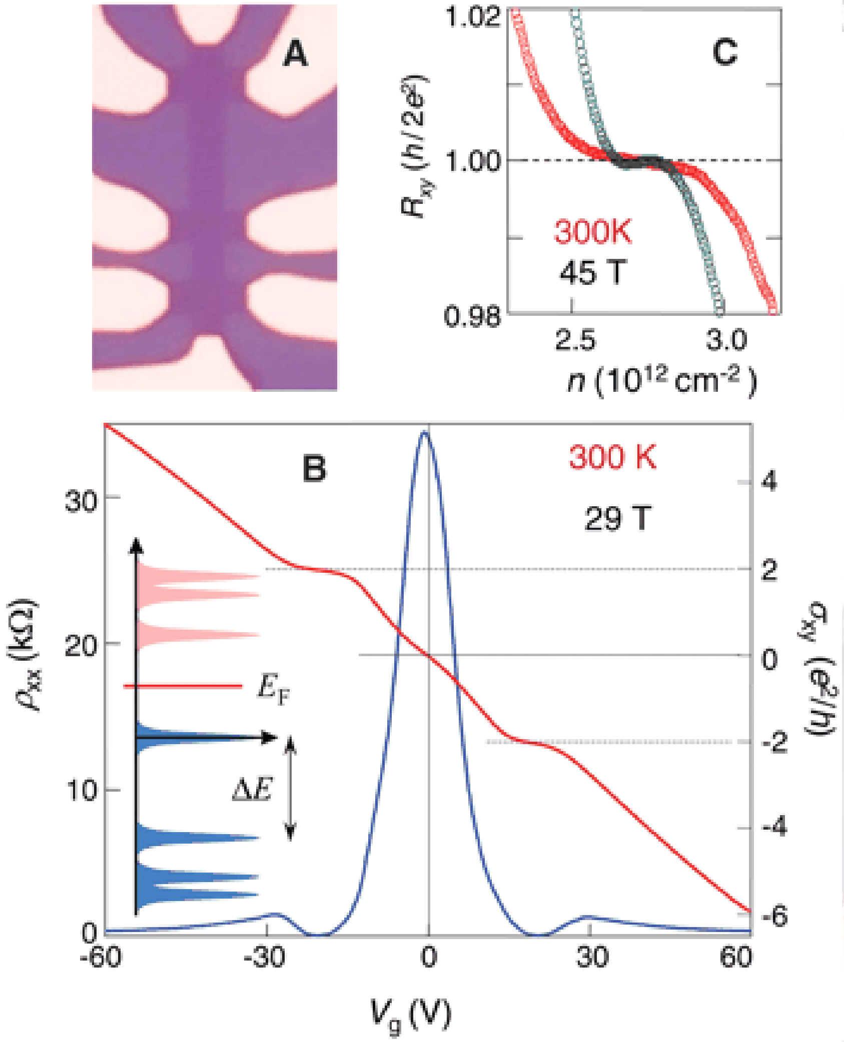
Institute for Molecules and Materials, Radboud University Nijmegen

Review: A.K. Geim and K.S. Novosolov, *The rise of gaphene,* Nature Materials **6**, 183-191 (March 2007).

Room-temperature QHE in graphene



K.S. Novoselov *et al.*, Science 315, 1379 (2007).