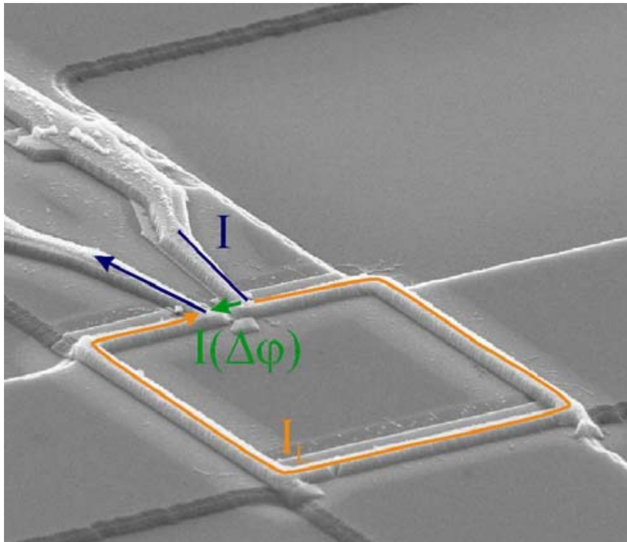
Mesoscopic superconductivity – a primer

C. S. – data from J. Bentner and M. Füchsle

- Supercurrent and flux-quantization in superconducting loops
- Measurement of the current-phase-relation (CPR) in unkonventional Josephson contacts
- Andreev-bound states and the proximity effect
  - Higher harmonics in the CPR of SNS-contacts
  - Generation of higher harmonics by microwaves



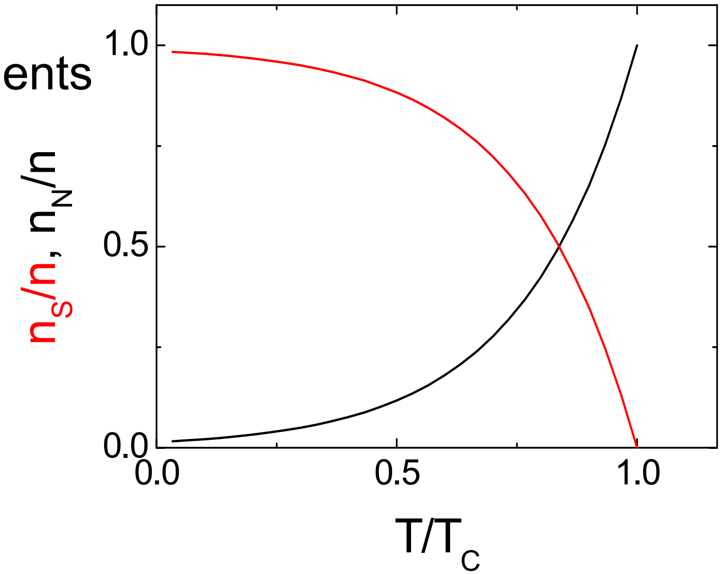
Superconductivity in the two-fluid model:

Below a critical temperature T_C the electron liquid becomes instable in many liquids

simplest model assuption:

electron system decomposes into two components

- normal component
- superfluid component



The superfluid component is responsible for the unusual electric properties of the SL.

Can the normal component be composed from ‚normal' electrons?

No, there is a gap in the spectrum of single particle excitations, electron/hole mixing….

Eventually both components are formed by ‚the same' electrons !

What drives a supercurrent ?

Ginzburg & Landau: describe superconducting phase in terms of a *macroscopic* wave function ψ with $n_S = |\psi|^2$

non-linear Schrödinger-equation

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2\tilde{m}}\left(\frac{\hbar}{i}\vec{\nabla} - 2e\vec{A}\right)^2\psi = 0 \quad \left(1^{\text{st}}\ \text{GL - Eq.}\right)$$

supercurrent driven by *phase*-gradients

$$\vec{j}_S = 2e|\psi|^2\frac{1}{\tilde{m}}\left(\hbar\vec{\nabla}\varphi(\vec{r}) - 2e\vec{A}\right) \quad \left(2^{\text{nd}}\ \text{GL - Eq.}\right)$$

$$\Rightarrow \quad \text{rot}\,\vec{j}_S = \frac{\left(2e|\psi|\right)^2}{\tilde{m}}\,\text{rot}\,\vec{A} = \frac{1}{\mu_0\lambda^2(T)}\vec{B} \qquad \left(2^{\text{nd}}\ \text{London - Eq.}\right),$$

where $\lambda(T) = \left(\frac{\mu_0\tilde{m}}{4e^2|\psi|^2}\right)^{\frac{1}{2}}$ magnetic penetration depth

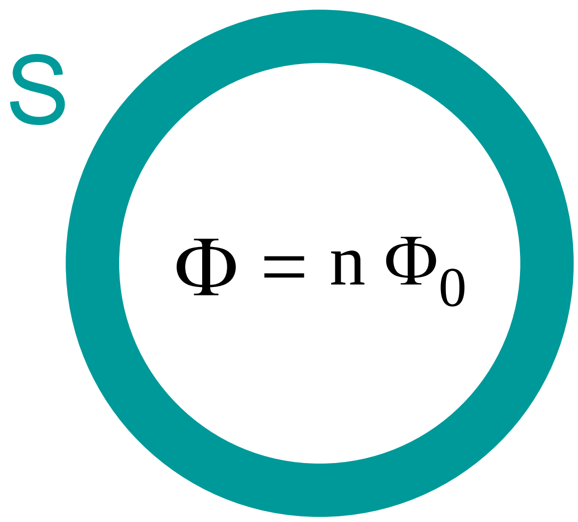
$\Rightarrow$ Meissner effect, flux quantization, Abrikosov vortex lattice…..

flux quantization

circulation of supercurrent:

$$\oint \vec{j}_s\, d\vec{r} = 2e|\psi|^2 \frac{\hbar}{\tilde{m}} \oint \left(\vec{\nabla}\varphi(\vec{r}) - \frac{2e}{\hbar}\vec{A} \right) d\vec{r} \propto 2\pi(n - \Phi/\Phi_0) \stackrel{!}{=} 0$$

with $$\oint \vec{\nabla}\varphi(\vec{r})\, d\vec{r} = 2\pi\, n$$



$$\Rightarrow \quad \boxed{\Phi = n \cdot \Phi_0}$$

where $\Phi_0 = h/2e$: flux quantum

n: integer

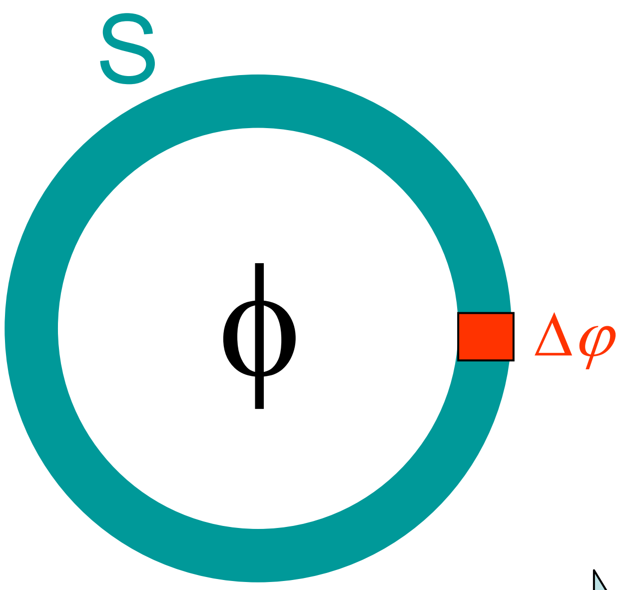
Gauge invariant combination of φ and $\vec{A}$ requires supercurrent response to magnetic field!

insert „weak link“ into loop: Josephson junctions

- weak link (WL) limits supercurrent in loop
- supercurrent through weak link is determined by phase difference
- simplest realization - tunnel junctions:

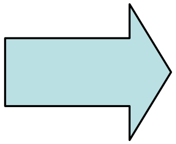
$$j_s = j_c \sin\left(\Delta\varphi - \frac{2e}{\hbar}\int_{WL} \vec{A}\, d\vec{r}\right)$$

current-phase relation (CPR) for tunnel junctions



due to gauge invariance, the phase difference $\Delta\varphi$ across the junction is linked to the total flux in the loop

$$\Delta\varphi - \frac{2e}{\hbar}\int_{WL} \vec{A}\, d\vec{r} = -2\pi\frac{\Phi}{\Phi_0}$$

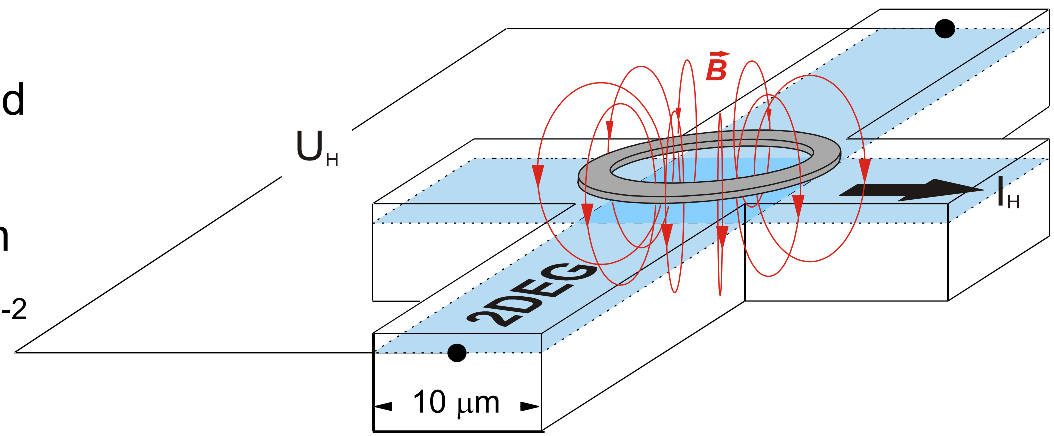


supercurrent response of the loop detectable via magnetic moment !

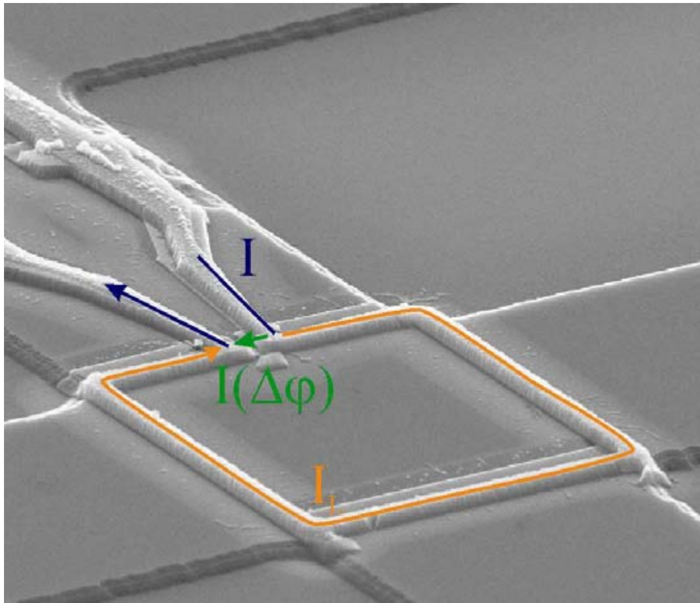
experimental realization: Micro-Hall-Magnetometer

deposition of the loop on top of a micron-sized Hall-cross

- high mobility 2DEG is realized in a GaAs heterostructure
- depth of 2DEG layer: 190 nm
- carrier density: 1,25 x 10^{15} m^{-2}
- mobility: 2 10^{6} cm^{2}/Vs



- sensitivity: 30 nT or 10^{-3} Φ_0
- temperature range: T = 300 mK to 6 K
- Hall current: I_H= 1 to 20 µA
- external magnetic field: B_{ext}= -4 to +4 mT



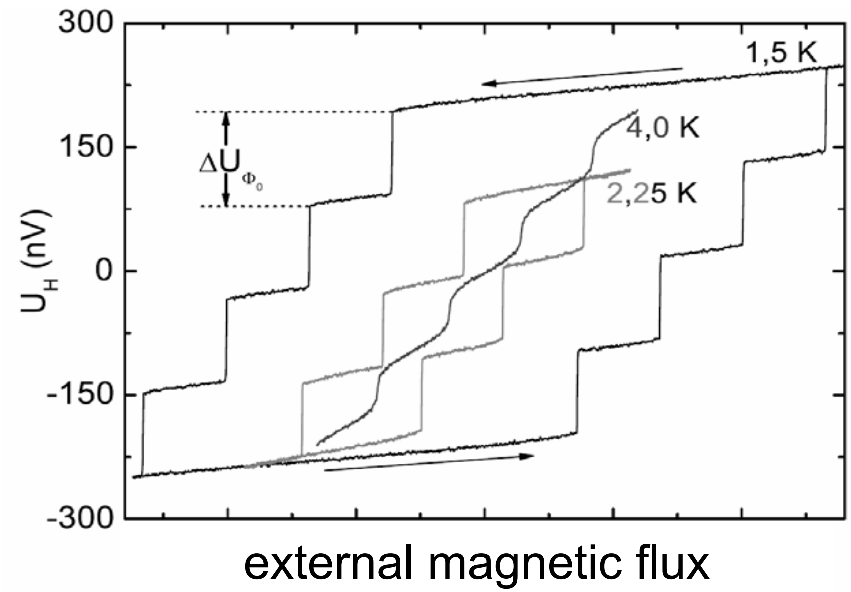
see also J. Waldram (1975)

preview: typical experimental data

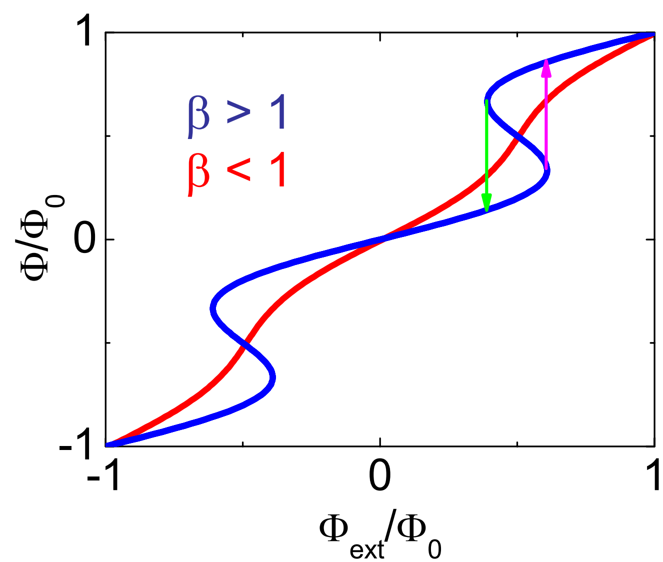
- the sum of external flux (linear back-ground) and the loop flux (steps) is detected by the Hall magnetometer

$$\Phi = \Phi_{ext} - L I_s(\Phi)$$

L: loop inductance

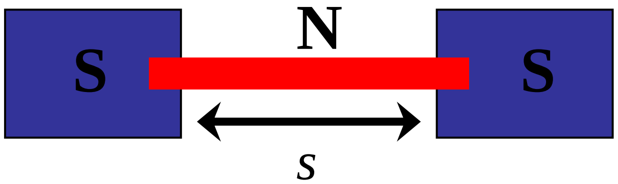


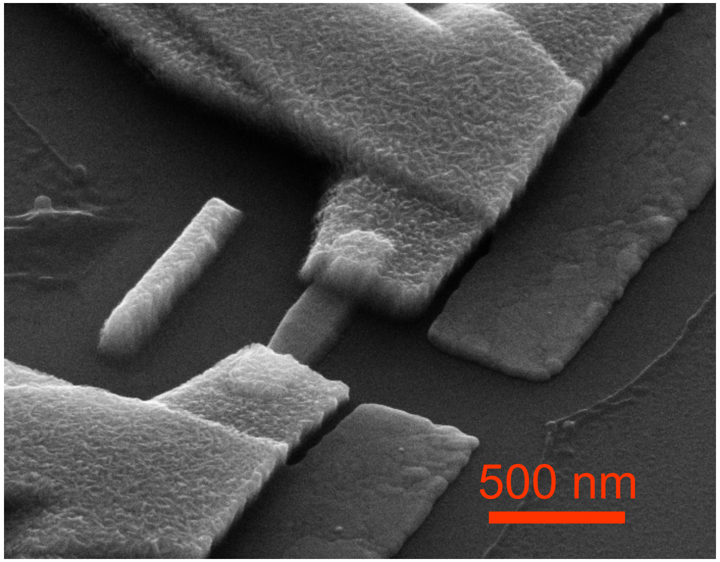
- depending on the parameter $\beta = 2\pi L I_C / \Phi_0$, one observes a hysteretic (β>1) and a non-hysteretic (β<1) regime



- Experimental determination of current-phase relation!

unconventional Josephson junctions:



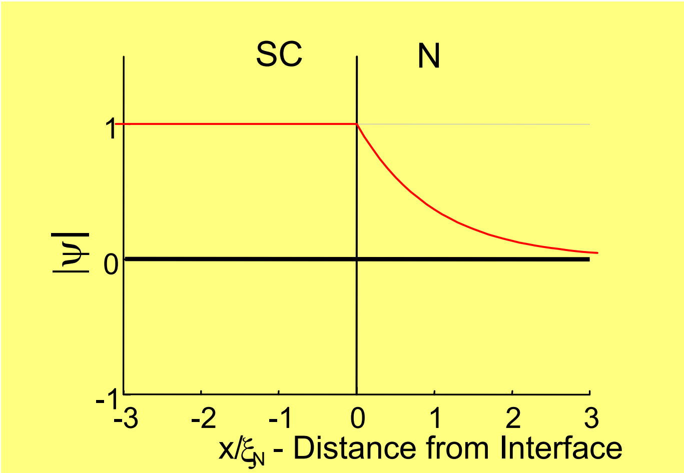


a piece of normal metal as weak link

naïve view:
penetration of macroscopic wave function ψ into normal conductor –

new length scale: ξ_N

⇨ *proximity effect*



proximity effect leads to Josephson coupling between two superconductors connected by a normal metal

BCS - theory

weak attractive interaction leads to Cooper-pairing at low T :

$$\mathcal{H}-\mu\mathcal{N} = \sum_{k,\sigma} \xi_k \, a^\dagger_{k,\sigma} a_{k,\sigma} + \frac{1}{2} \sum_{kk',\sigma\sigma'} V_{kk'} \, a^\dagger_{k,\sigma} a^\dagger_{-k,-\sigma} a_{k',\sigma} a_{-k',\sigma}$$

where $\xi_k = v_F \, \hbar (k - k_F)$ single particle energies for $V_{kk'} = 0$

mean field approximation with the ‚pairing-field' Δ

$$\Delta := -V \textstyle\sum_k \underbrace{\langle a_{k,\uparrow} a_{-k,\downarrow} \rangle} \propto \psi_{GL}$$ (self-consistency relation)

F_k : pair amplitude

$$\mathcal{H}_{\mathrm{MF}} - \mu\mathcal{N} = \sum_{k,\sigma} \xi_k \, a^\dagger_{k,\sigma} a_{k,\sigma} + \frac{1}{2} \sum_{k} \left\{ \Delta^* a_{k,\uparrow} a_{-k,\downarrow} + \text{h.c.} \right\}$$

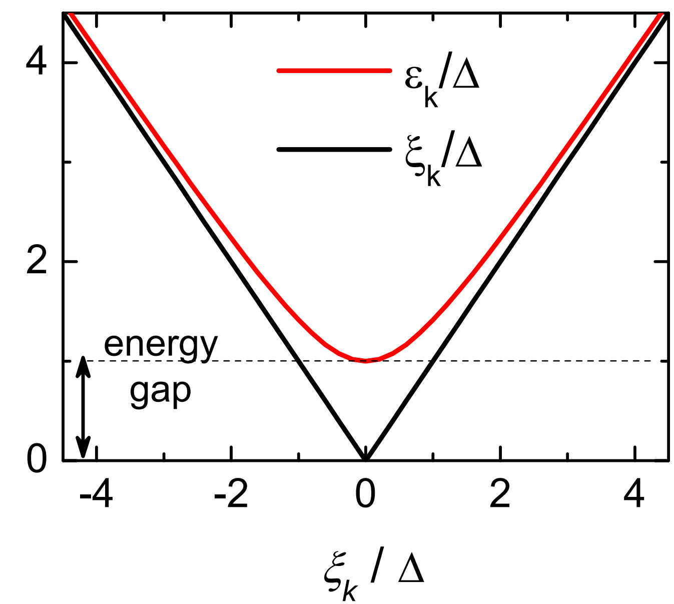
solution by Bogoliubov transform

Bogoliubov-transform

define new quasi-particles by:

$$b_{k0} = u_k\, a_{k\uparrow} \quad - v_k\, a^{\dagger}_{-k\downarrow}$$
$$\boldsymbol{b_{k1} = u_k\, a_{-k\downarrow} - v_k\, a^{\dagger}_{k\uparrow}}$$

linear combinations of particles and holes!



quasi-particle spectrum

diagonalized mean field hamiltonian :

$$\mathcal{H}_{\mathrm{MF}} - \mu\mathcal{N} = E_0 + \sum_{k,\sigma} \epsilon_k\, b^{\dagger}_{k,\alpha} b_{k,\alpha}$$

for
$$u_k = \sqrt{(1+\xi_k/\epsilon_k)/2}$$
$$v_k = \sqrt{(1-\xi_k/\epsilon_k)/2}$$

$E_0 = E_{\mathrm{normal}} - \frac{1}{2}N(\epsilon_F)\Delta^2(T=0)$
is the energy of the BCS-ground state:

$$|\psi_{BCS}\rangle = \prod_k \left(u_k + v_k\, a^{\dagger}_{k,\uparrow} a^{\dagger}_{-k,\downarrow}\right)\, |vac\rangle$$

and $\quad \epsilon_k = \sqrt{\xi_k^2 + \Delta^2}$

Δ defines new length scale: ξ_0 BCS – coherence length

Bogoliubov-de Gennes equations

generalization for inhomogeneous systems with NS-interfaces:

$$\begin{pmatrix} \mathcal{H}_0 & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & \mathcal{H}_0 \end{pmatrix} \begin{pmatrix} u_n(\vec{r}) \\ v_n(\vec{r}) \end{pmatrix} = \epsilon_n \begin{pmatrix} u_n(\vec{r}) \\ v_n(\vec{r}) \end{pmatrix}$$

where $\mathcal{H}_0 = \frac{1}{2m}\left(-i\hbar\vec{\nabla} - e\vec{A}\right)^2 + U_0(\vec{r}) - \mu$ (single-particle hamiltonian)

and $\Delta(\vec{r}) = -V \sum_n u_n(\vec{r}) v_n^*(\vec{r}) \left(1 - 2f(\epsilon_n, T)\right)$ (self-consistency)

⇨ coupled system of equations for the electron and hole wave functions:

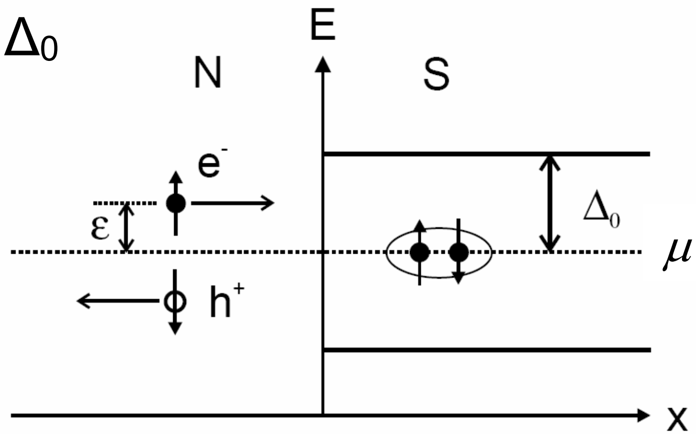
$u_n(\vec{r})$, $v_n(\vec{r})$ with the energy eigenvalues: ε_n

‚Andreev-states‘

Andreev-reflection

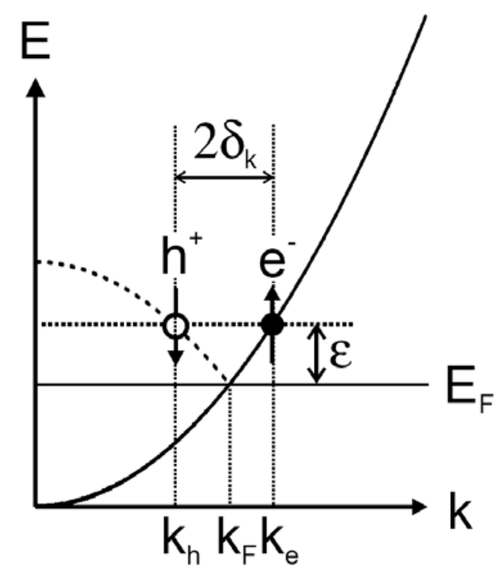
hybrid structure with S/N-interface

no single-particle states below energy the gap Δ_0



Andreev-reflection:
incoming electron is reflected as a hole, thus creating a Cooper-pair in SC

retro-reflection:
- the hole traces back the time-reversed path of the incoming electron;
- energy-dependent phase shift $\delta\varphi$ between electron and hole



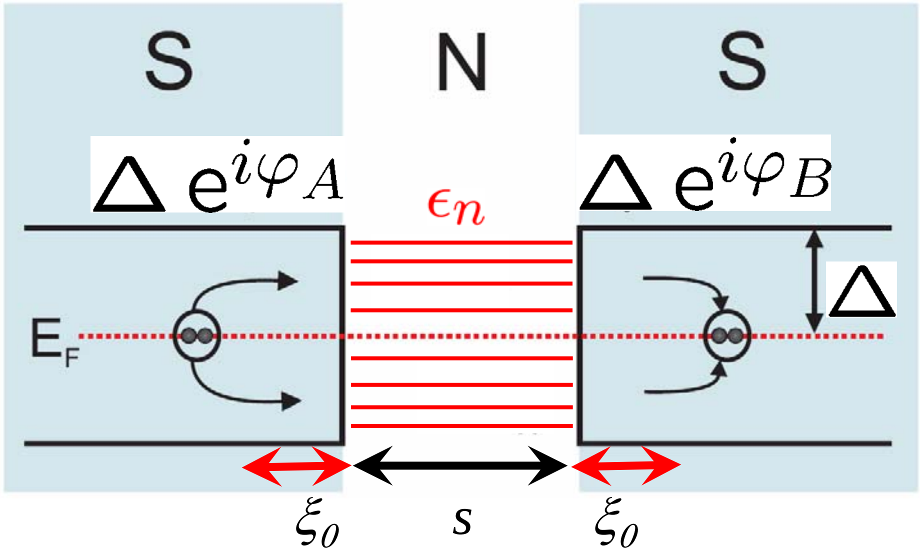
$$\delta\varphi = 2\,\delta k\;\ell = 2\frac{\epsilon}{\hbar v_F}\,\ell \lesssim \pi$$

constructive interference within a certain path length ℓ

multiple Andreev-reflection

gap confines quasiparticles close to the N part of the junction

formation of Andreev bound states (ABS) when wave functions u_n, v_n satisfy boundary condition:



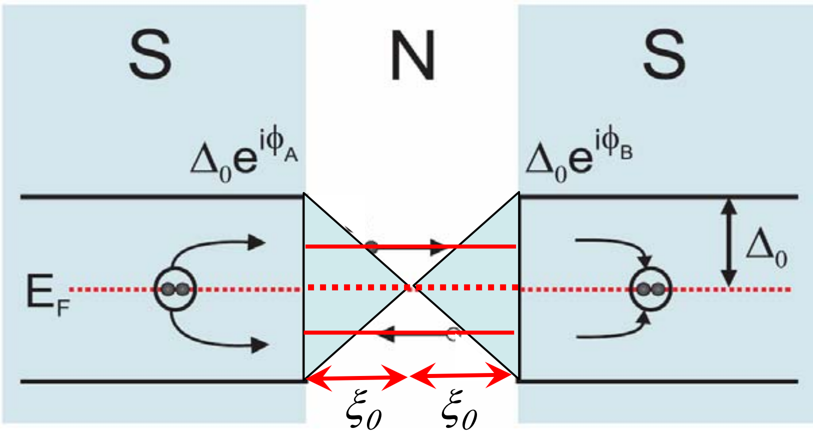
$$-2\arccos\frac{\epsilon_n^{\pm}}{\Delta} \pm (\varphi_A - \varphi_B) + \frac{2\epsilon_n^{\pm}}{\hbar v_F} s = 2\pi n,\ n \text{ integer},\quad E_{Th} = \frac{\hbar v_F}{s} \text{ Thouless energy}$$

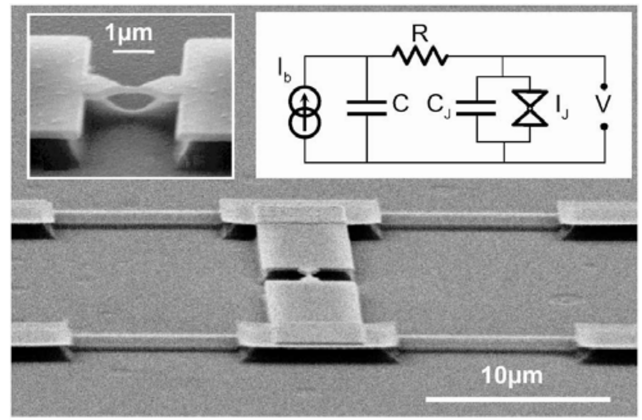
short junctions ($s << \xi_0$, $E_{Th} >> \Delta$) : $\epsilon_n^{\pm}(\Delta\varphi) = \pm\Delta \cos \Delta\varphi/2$

long junctions ($s >> \xi_0$, $E_{Th} << \Delta$) : $\epsilon_n^{\pm}(\Delta\varphi) = E_{Th}\,[\pi(2n+1) \pm \Delta\varphi]$

$$\Rightarrow\quad I_s(\Delta\varphi) = -\frac{2e}{\hbar}\sum_n \frac{\partial \epsilon_n^{\pm}(\Delta\varphi)}{\partial\,\Delta\varphi}\,(1 - 2f(\epsilon_n(\Delta\varphi), T))$$ supercurrent !

extremely short junctions: atomic point contacts





Goffman et al.
PRL ***85***,
170 (2000)

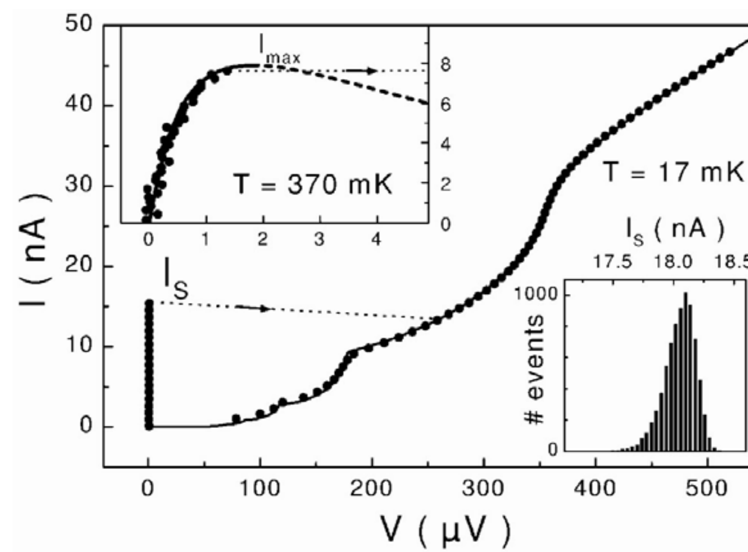
even for length s = 0, a pair of Andreev-bound states survives in the junction!

$\epsilon_0^{\pm}(\Delta\varphi) = \pm\Delta\,[1 - \mathcal{T}\sin(\Delta\varphi/2)]^{1/2}$ where $\mathcal{T}$ is the transmission probability of the channel.

supercurrent

$$I_s(\Delta\varphi) = -\frac{2e}{\hbar}\sum_n \frac{\partial\epsilon_n^{\pm}(\Delta\varphi)}{\partial\,\Delta\varphi}\,(1 - 2f(\epsilon_n, T))$$

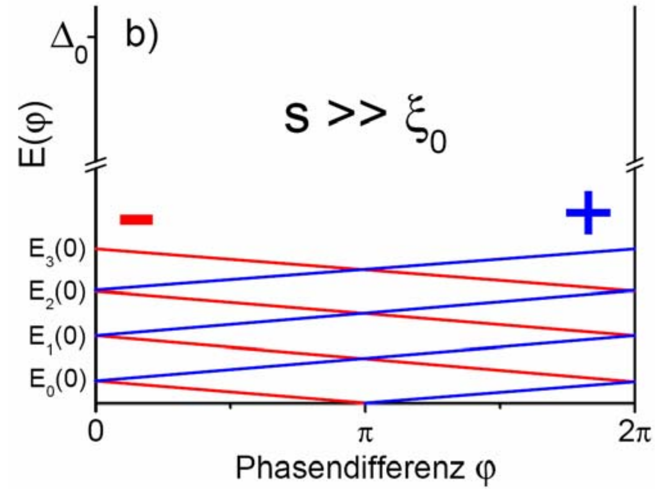
a single atomic orbital can carry a maximum supercurrent of ~ 40 nA !



long ballistic vs. diffusive SNS junctions

discrete spectrum: equidistant energy levels seperated by E_{Th}

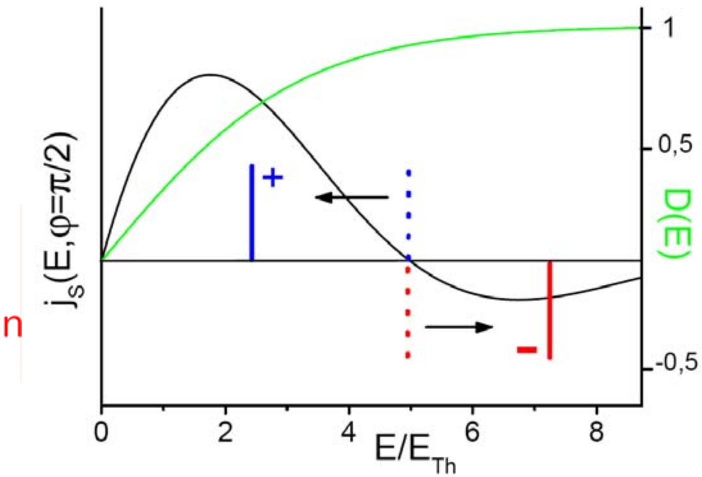
$$I_s(\Delta\varphi) = -\frac{2e}{\hbar}\sum_n \frac{\partial \epsilon_n^{\pm}(\Delta\varphi)}{\partial\,\Delta\varphi}\,(1-2f(\epsilon_n,T))$$



BUT: in a diffusive normal conductor, the effective trajectory lengths will vary due to scattering

smearing of the discrete spectrum:

$$I_s(\Delta\varphi) = -\frac{2e}{\hbar}\int d\epsilon\,\underbrace{\frac{\partial \epsilon(\Delta\varphi)}{\partial\,\Delta\varphi}}_{j_s(\epsilon,\Delta\varphi)}\cdot\underbrace{(1-2f(\epsilon,T))}_{\text{thermal occupation}}$$



$j_s(\varepsilon,\Delta\varphi)$: spectral supercurrent density

characteristic energy scale is still the Thouless-Energy:

$$\Rightarrow\quad E_{Th} = \frac{\hbar}{\tau_{\text{flight}}} = \frac{\hbar D}{s^2}$$

theoretical predictions for highly transparent junctions

In general, when transmission probability $\mathcal{T}$ of order 1:

$$I_S = \sum_{n=1}^{\infty} I_c^n \sin\left(n\Delta\varphi\right)$$

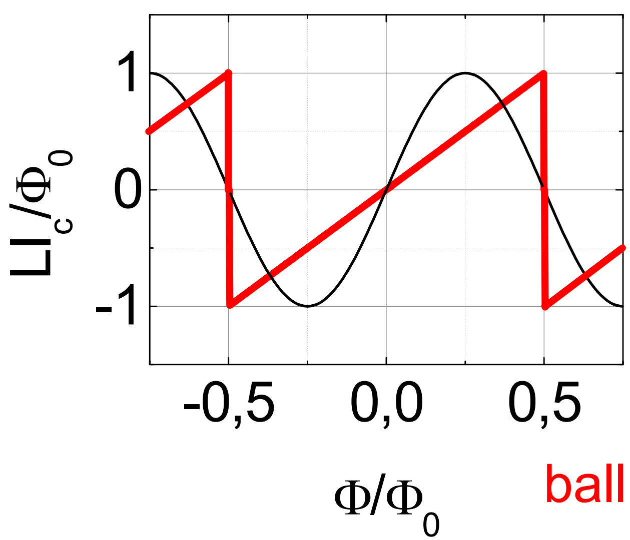
[Heikkilä et al.,Phys Rev B, **66** , 184513 (2002)]

Higher harmonics in the **c**urrent **p**hase **r**elation (CPR) can be interpreted as correlated transfer of multiple Cooper pairs.

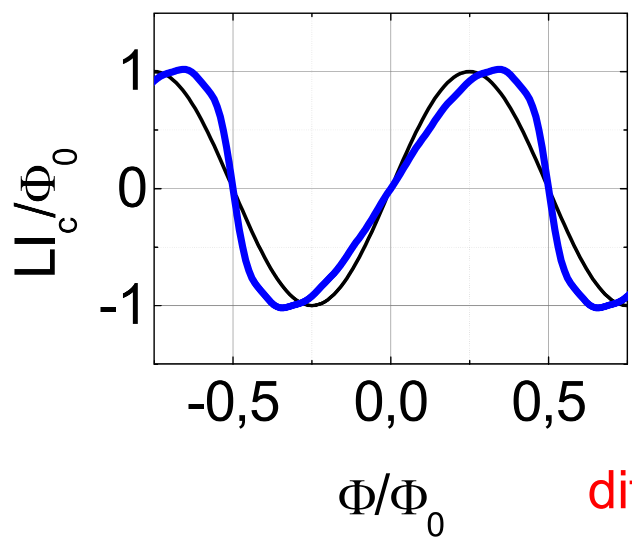
⇨ rapidly suppressed for $k_B T > E_{Th}$

$$\mathrm{T}=1:\ \mathrm{I}_c^n \propto -(-1)^n \, 1/n$$

$$0<\mathrm{T}<1:\ \mathrm{I}_c^n \propto -(-1)^n \, 1/n^2$$



ballistic



diffusive

meaning of higher harmonics in the CPR

allow for transfer of more than one Cooper pair at a time:

$$\mathcal{H}_{JJ} = -\left\{\frac{E_J}{2}\sum_n \left(|n_c + 1 >< n| + \text{h.c.}\right)\right\} - \left\{\frac{E_J}{2}\sum_n \left(|n_c + 2 >< n| + \text{h.c.}\right)\right\} - \cdots$$

with: $E_J = \frac{\hbar I_C}{2e}, \qquad E_C = \frac{(2e)^2}{2C_\Sigma}$

analog tight binding Hamiltonian for Bloch electrons in crystal lattice !

representation in the basis of eigenstates the number operator $\widehat{n}_C = \frac{\hbar}{i}\frac{\partial}{\partial \Delta\varphi}$

Josephson current: $I_s(\Delta\varphi) = -\langle \widehat{n}_C \rangle = -\langle [\mathcal{H}_{JJ}, \widehat{n}_C] \rangle = \frac{2e}{\hbar}\frac{\partial \langle \mathcal{H}_{JJ} \rangle}{\partial \Delta\varphi}$

change to the basis of eigenstates the phase operator $\Delta\widehat{\varphi}$

$$\mathcal{H}_{JJ} = -E_J \cos\Delta\widehat{\varphi} \left[+ E_C \left(\frac{\hbar}{i}\frac{\partial}{\partial \Delta\widehat{\varphi}} - n_G\right)^2 \right]$$

$\Delta\widehat{\varphi}$ analog position of a particle in a washboard potential

„quantum version of RCSJ-model“

meaning of higher harmonics in the CPR

allow for transfer of more than one Cooper pair at a time:

$$\mathcal{H}_{JJ} = -\left\{\frac{E_J}{2}\sum_n \left(|n_c + 1 >< n| + \text{h.c.}\right)\right\} - \left\{\frac{E_J}{2}\sum_n \left(|n_c + 2 >< n| + \text{h.c.}\right)\right\} - \cdots$$

change to phase representation:

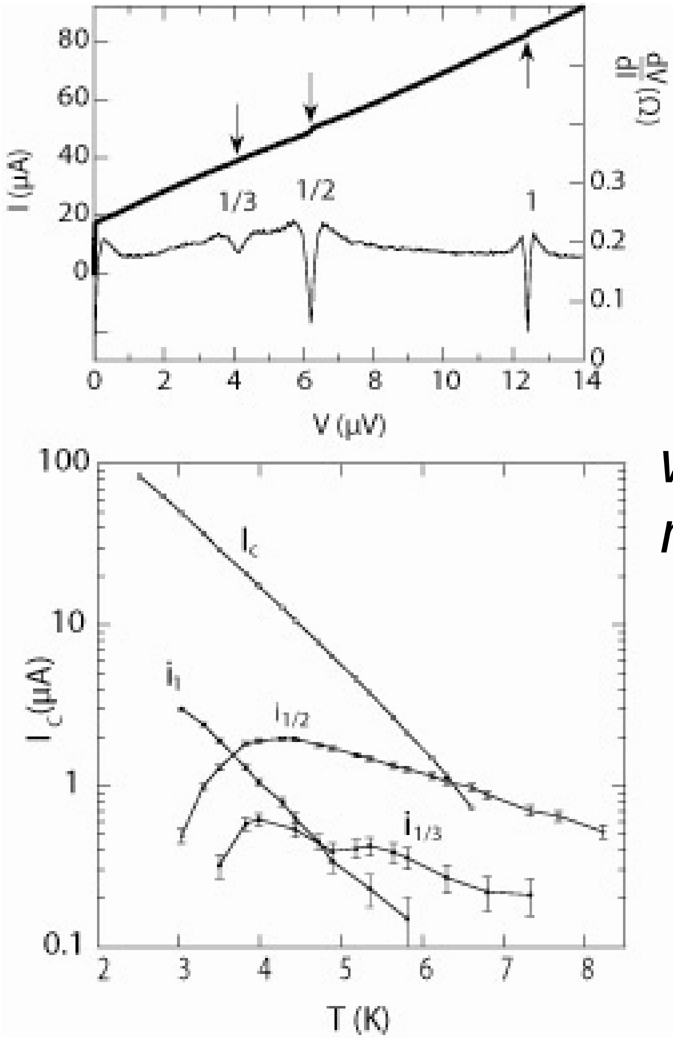
$$\mathcal{H}_{JJ} = -E_J^{(1)} \cos \Delta\hat{\varphi} \;-\; E_J^{(2)} \cos 2\,\Delta\hat{\varphi} - \cdots$$

corresponding CPR:

$$I_s(\Delta\varphi) = -\langle[\mathcal{H}_{JJ}, \hat{n}_C]\rangle = I_C^{(1)} \sin \Delta\varphi + I_C^{(2)} \sin 2\Delta\varphi + \cdots$$

where $$I_C^{(j)} = \frac{2e}{\hbar}\, j\, E_J^{(j)}$$

Previous attempts to measure the CPR in SNS-junctions:



look at current-voltage characteristics under rf-irradiation ω_{ext}

Shapiro steps

when Josephson frequencies $\omega_J^{(i)} = i\,\frac{2e}{\hbar}\,V$ *match integer multiples of* ω_{ext}

- higher harmonics in CPR induce subharmonic shapiro steps
- amplitude of higher harmonics can increase with temperature
  - so far not understood!

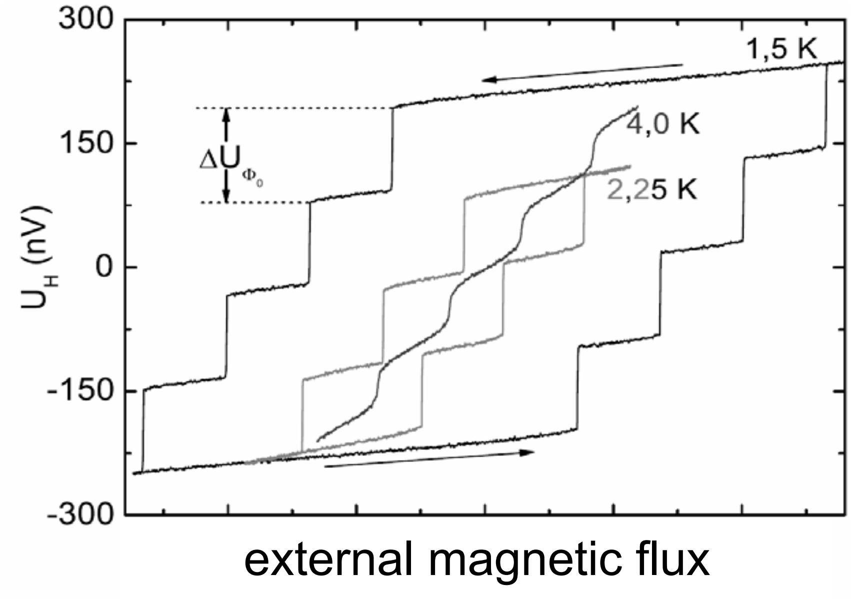
Dubos *et al.*, PRL **87**, 206801 (2001)

from Hall voltage to CPR

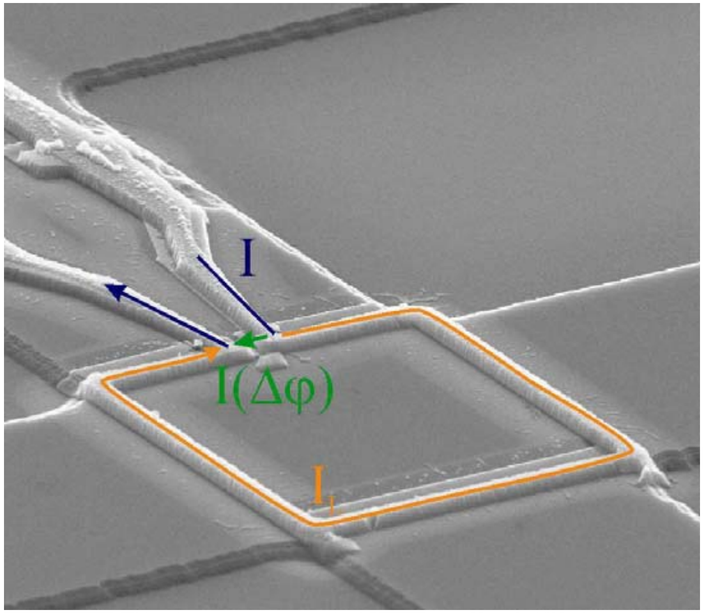
- the sum of external flux (linear back-ground) and the loop flux (steps) is detected by the Hall magnetometer

$$\Phi = \Phi_{ext} - LI_s(\Phi)$$

L: loop inductance



- depending on the parameter $\beta = 2\pi L I_C / \Phi_0$, one observes a hysteretic (β>1) and a non-hysteretic (β<1) regime

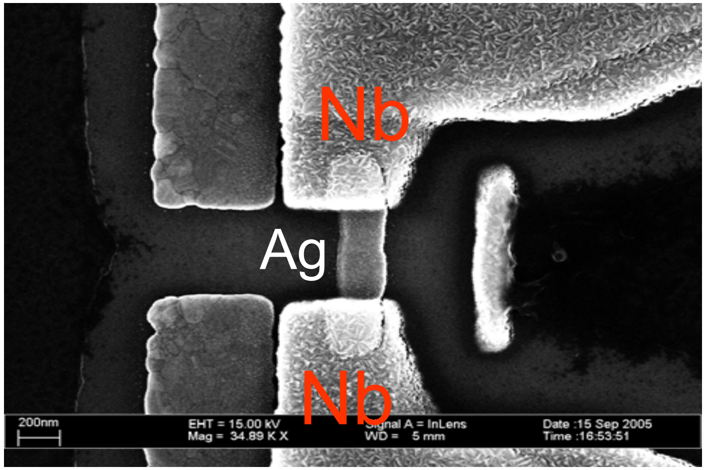


- Experimental determination of current-phase relation!

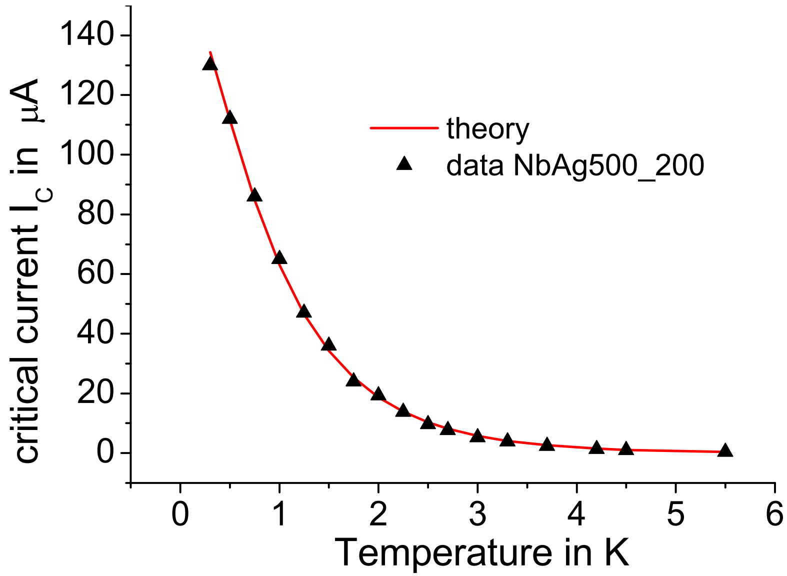
critical current of Nb/Ag/Nb-junctions

- layer thickness: 40nm Ag, 150nm Nb
- length of Ag bridge s: 495 nm
- width of Ag bridge: 205 nm
- diffusion constant D = ?

$$E_{Th} = \frac{\hbar D}{s^2}$$



determine D from the temperature dependence of $I_c(T)$:



fit theoretical $I_c(T)$ curve to measured data:

only fitting parameter: D

D = 0,014 m²/s

E_{Th} = 24 µeV

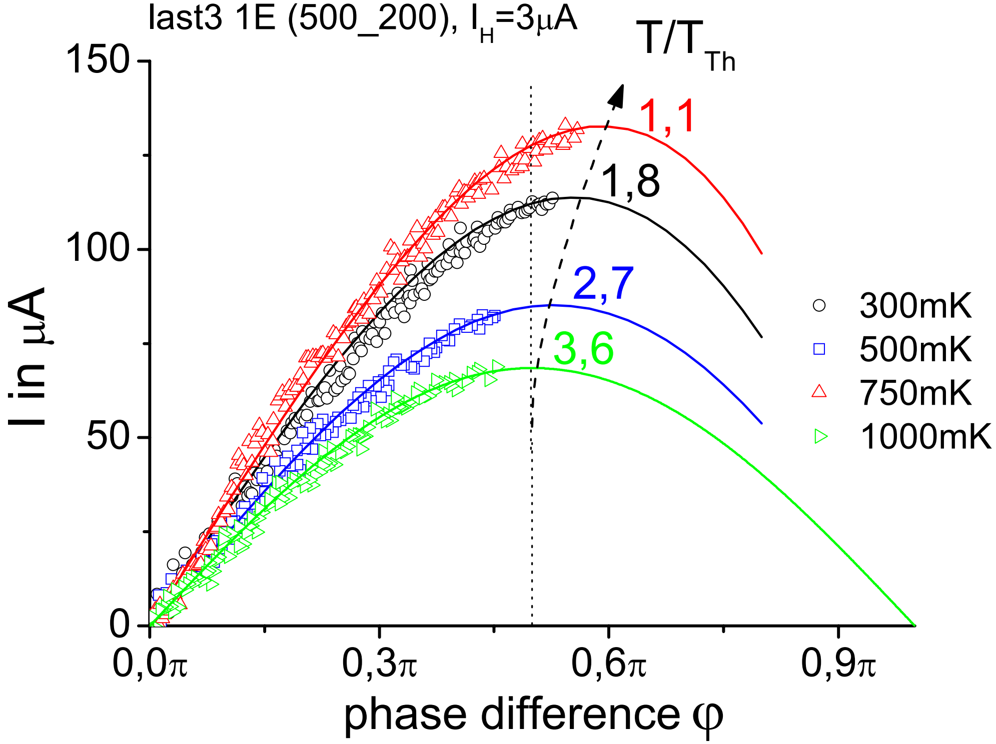
A.D. Zaikin and G.F. Zharkov, Sov. J. Low Temp. Phys. **7**, 184 (1981)

current-phase-relation of long diffusive SNS junctions

experimental data:

$E_{Th} = 24\ \mu eV$ ⟹ $T_{Th} = E_{Th}/k_B = 280\ mK$

$R_N = 1{,}46\ \Omega$

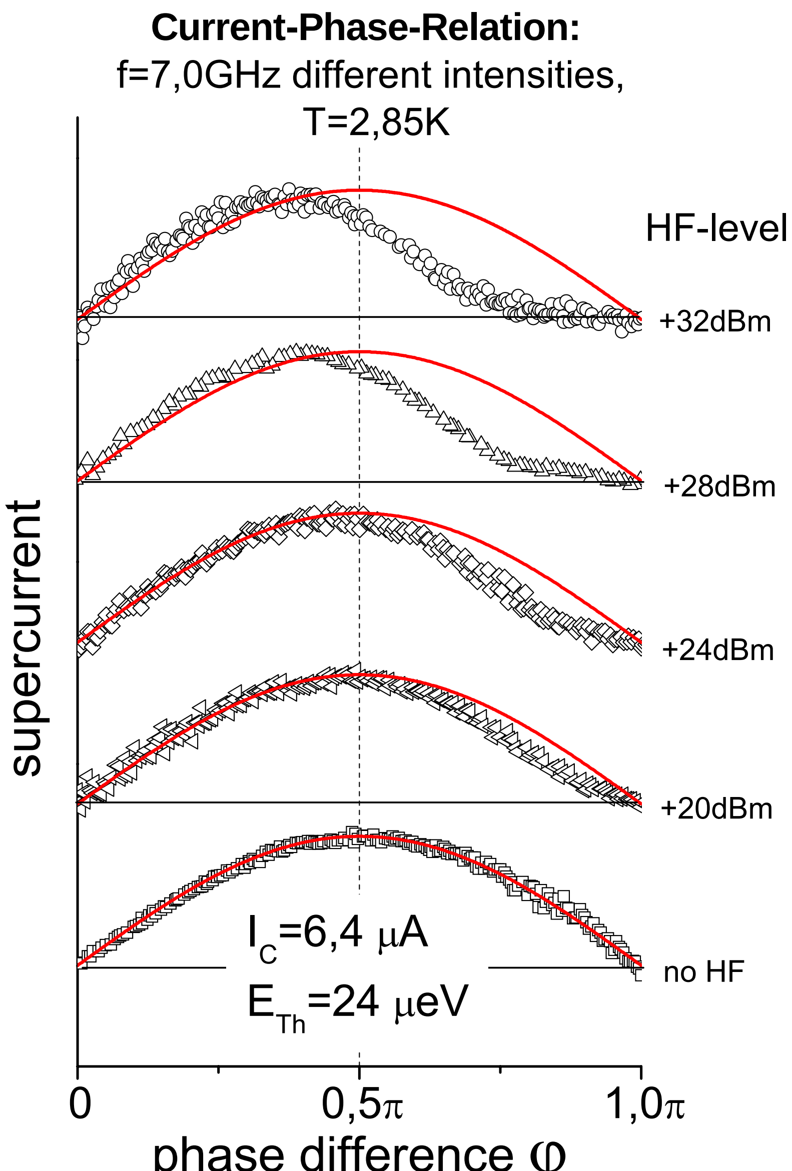


- the theoretical CPR are calculated numerically
- only variable parameter: $\mathcal{E}_{Th}$
- no fitting parameters between data and theoretical CPR
- the curves are in good agreement with theory
- for T > 750 mK, a sinusoidal CPR is recovered

Theory: Heikkilä et al., PRB 2002

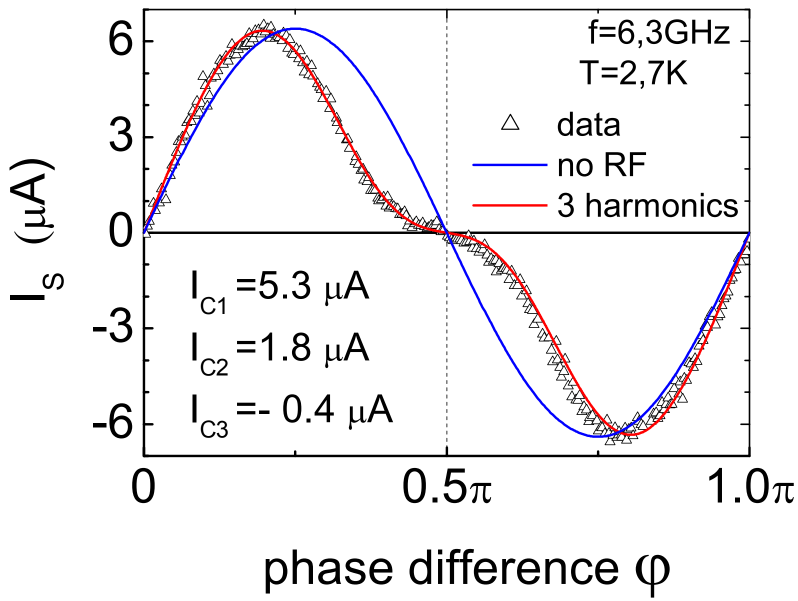
measurements are in good agreement with theory!

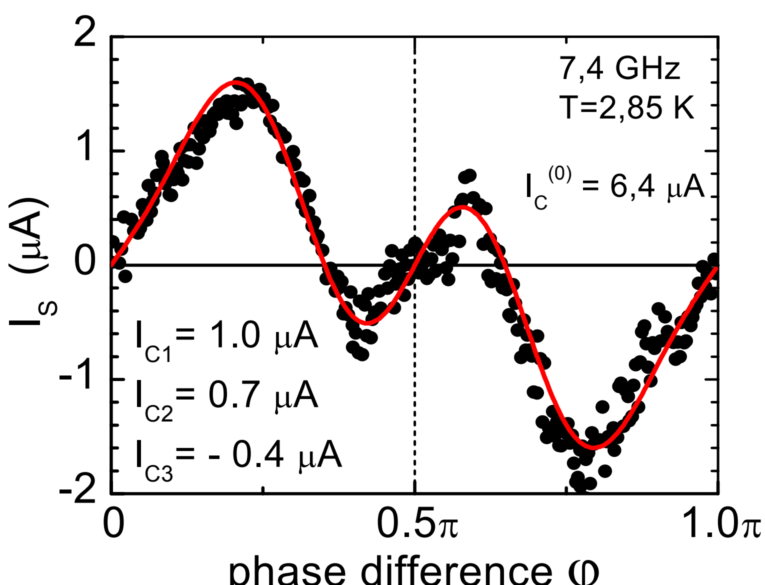
CPR under microwave irradiation



- the CPR is substantially altered, depending on the frequency and the intensity
- deviations from sinusoidal CPR are dominant in the falling branch → effects were measured in the non-hysteretic regime
- I_{max} at phase $\varphi < \pi/2$ possible
- a strong supression of I_c occurs for $\varphi > \pi/2$
- for signal input powers < +16dBm, no response for any frequency

radio frequency induced higher harmonics in the CPR





- At very high rf-amplitude the higher harmonics can become very strong
- maximal effect in the φ-ranges, where the induced minigap in the Ag-bridge becomes small

→ excitation of quasiparticles above the minigap!

- Consequence of a nonthermal occupation of Andreev-bound states induced rf?

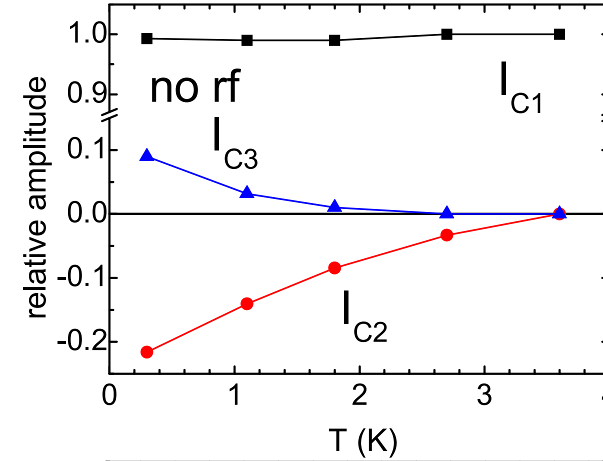
T-dependence of higher harmonics

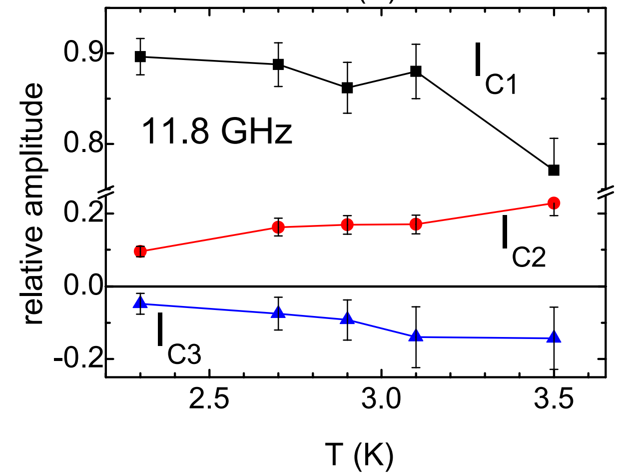
<u>intrinsic higher harmonics at low *T*:</u>

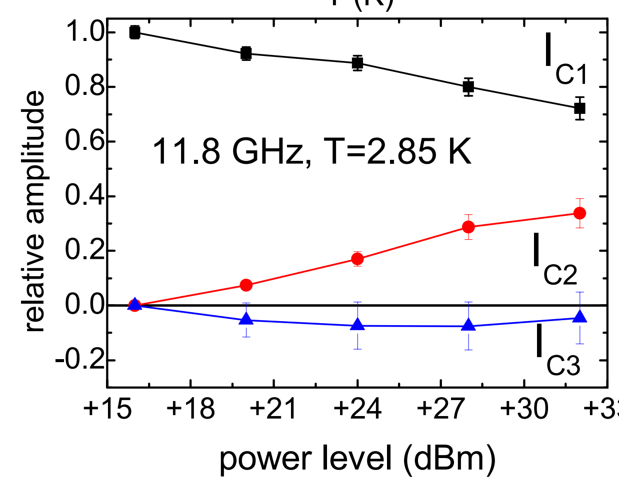
- alternating sign of I_{Cj}

<u>rf-induced higher harmonics at higher *T*:</u>

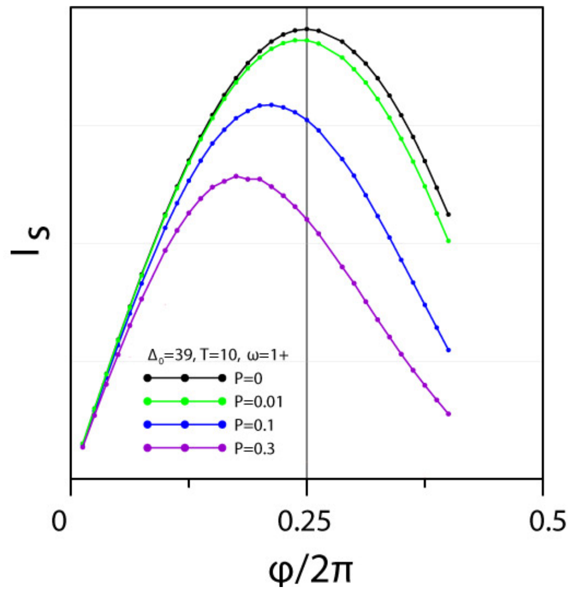
- I_{C2} and I_{C3} switch sign with respect to unperturbed case at lower *T* !
- At very high rf-amplitude the higher harmonics win importance with respect to the basic sinusoidal term
- basic term is reduced compared to unperturbed case
- higher harmonics *<u>increase</u>* with *T* !

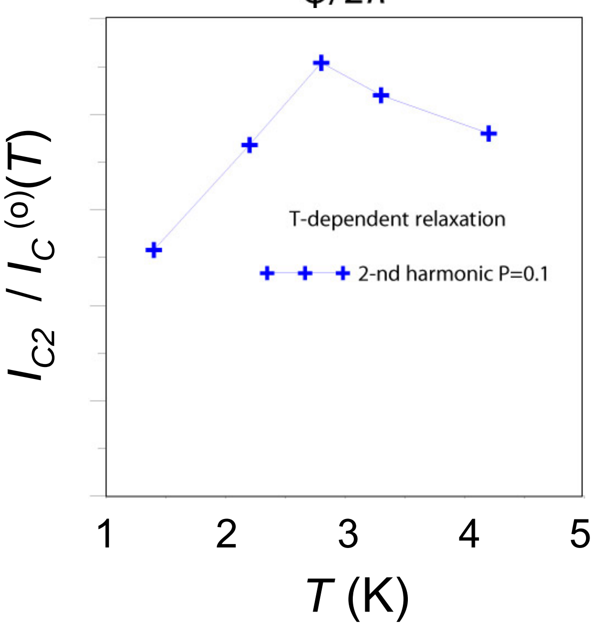






preliminary model calculations (D. Ryndyk, Regensburg)





- numerical calculation of supercurrent spectral density (T. Heikkilä, F. Wilhelm)
- calculation of non-equilibrium distribution function under rf- irradiation using Keldysh-technique

two competing contributions:

→ rf-induced stimulation of superconductivity (Klapwijk,Mooij '77)

→ pair breaking across the closing minigap near $\varphi = \pi$

→ nonmonotonic T-dependence of I_{C2}

- conclusions:

  - Microscopic understanding of the proximity effect
  - Detection and control of higher harmonics in the CPR of SNS-rings

- perspectives:

- study ballistics SNS junctions (Franziska, Tom)
- Use CNTs-quantum dots to create SNS-junctions with a few molecular orbitals (Markus)