Transport in interacting disordered nanotubes

Reinhold Egger
Institut für Theoretische Physik
Universität Düsseldorf
A. De Martino, C. Mora, A. Komnik
A. Altland, A. Bachtold, A.Gogolin, A.Tsvelik
DFG SFB Transregio 12

Overview

- Introduction
  - Bandstructure of single-walled nanotubes
  - Interaction effects: Luttinger liquid
  - Experimental evidence for Luttinger liquid
- Magnetotransport in chiral interacting SWNTs
  - Asymmetric contribution in magnetic field out of equilibrium
- Crossover from Luttinger liquid to Altshuler-Aronov behavior in MWNTs
- Crossed nanotube transport

Classification of carbon nanotubes

- Single-wall nanotubes (SWNTs):
  - One wrapped graphene sheet
  - Typical radius 1 nm, lengths up to several mm
- Ropes of SWNTs:
  - Triangular lattice of individual SWNTs (typically up to a few 100)
- Multi-wall nanotubes (MWNTs):
  - Russian doll structure, several inner shells
  - Outermost shell radius about 5 nm

Electronic transport in nanotubes

Basically all known mesoscopic effects appear in a single material system:

- UCF, weak and strong localization, Aharanov-Bohm physics
- Strong-interaction effects (Luttinger liquid)
- Kondo effect & quantum dot physics
- Superconductivity (both intrinsic and proximity induced)
- Spin transport, spin FET
- …

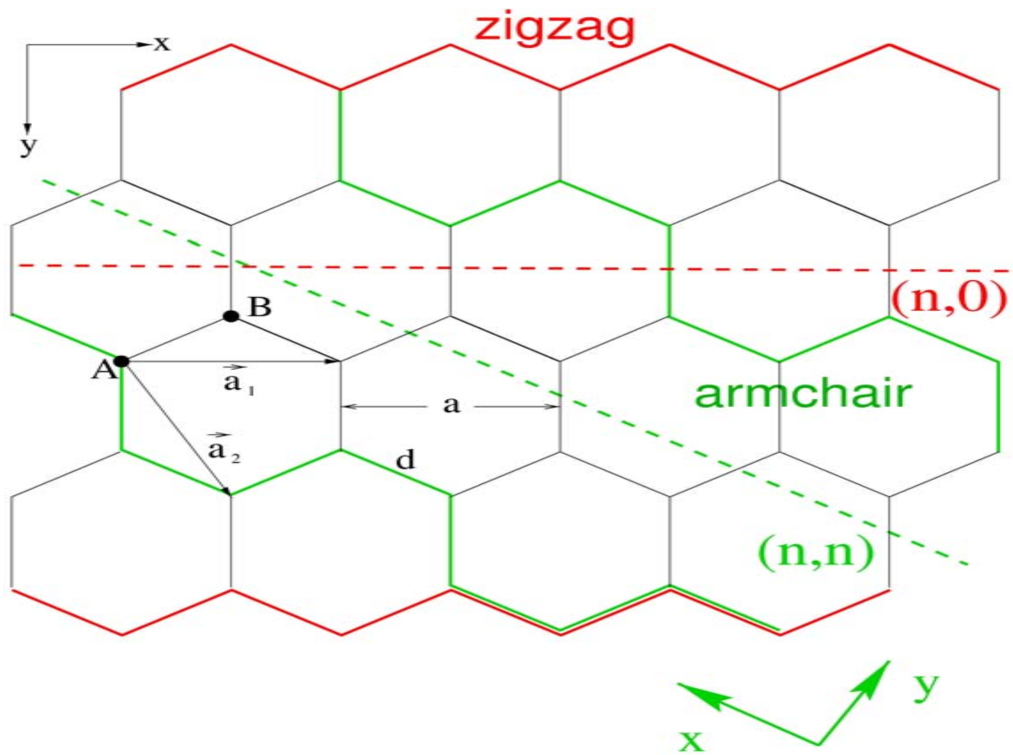
Wrapped 2D graphene sheet

Basis contains two atoms

$$a = \sqrt{3}d\,, d = 0.14nm$$

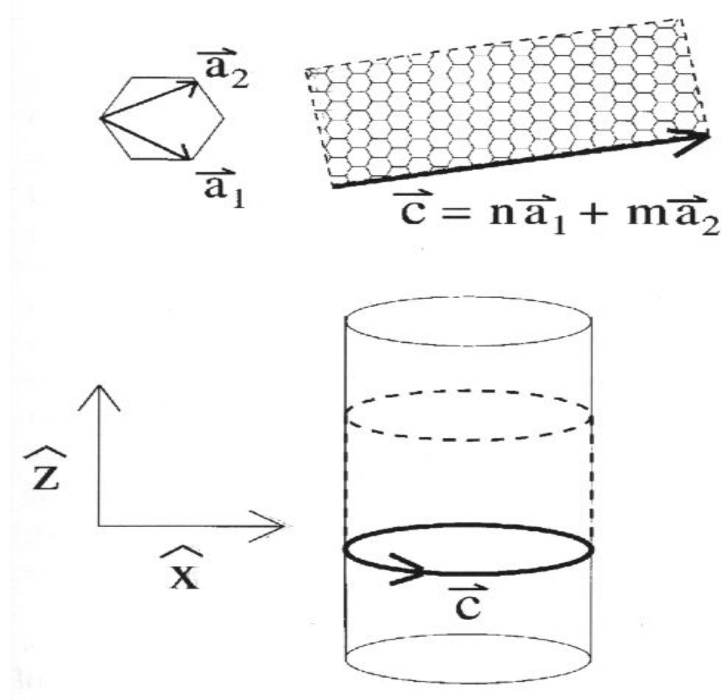
(n,m) indices: wrapping of sheet onto cylinder

Chiral angle θ: defined with respect to zigzag (n,0) tube



Nanotube as rolled graphene sheet

- *(n,m)* nanotube specified by superlattice vector
- imposes transverse momentum quantization
- Chiral angle determined by *(n,m)*
- Important effect on electronic structure

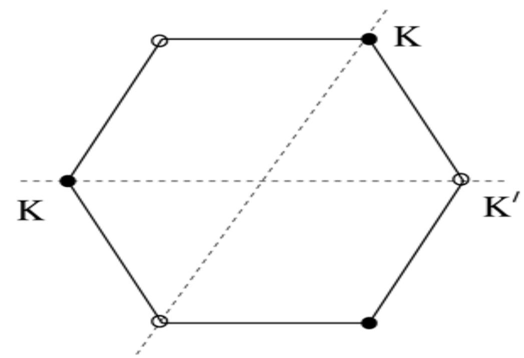


Band structure: Graphene

Exactly two independent corner points K, K´ in first Brillouin zone.

Band structure: valence and conduction bands touch at corner points *(E=0)*, these are the Fermi points in graphene

- Lowest-order k•p scheme: Dirac light cone dispersion
- Deviations at higher energies: trigonal warping



$$E(\vec{q}) = v|\vec{q}|$$

$$\vec{q} = \vec{k} - \vec{K}$$

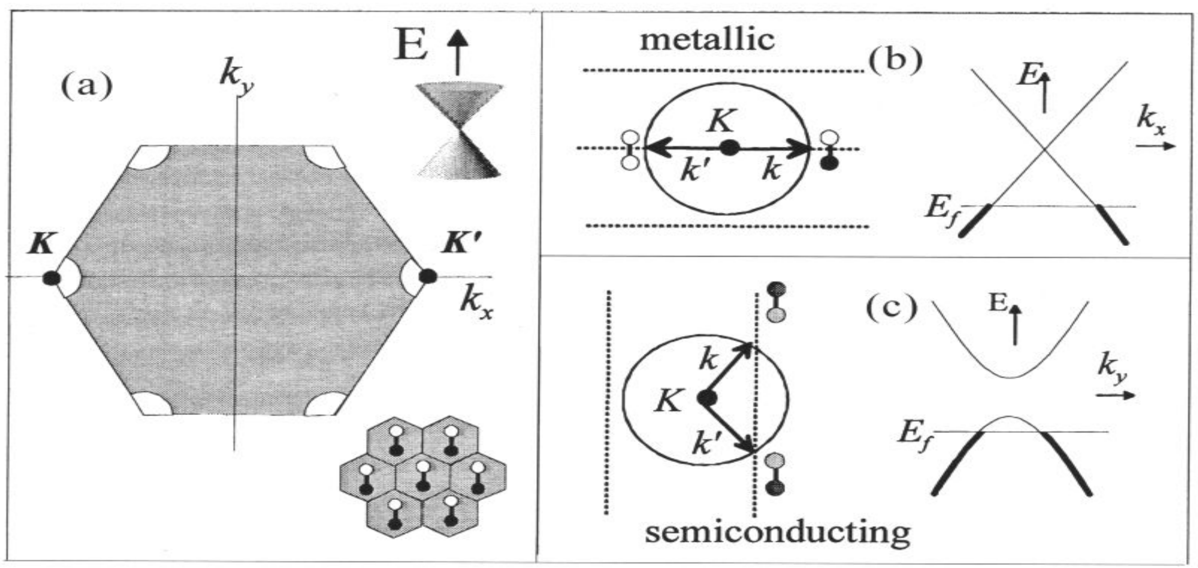
$$v = 8 \times 10^{5}\, m / \sec$$

Periodic boundary conditions: SWNTs

Transverse momentum must be quantized

Nanotube metallic only if K point has allowed transverse momentum

gives necessary condition: *2n+m = 3 x integer*



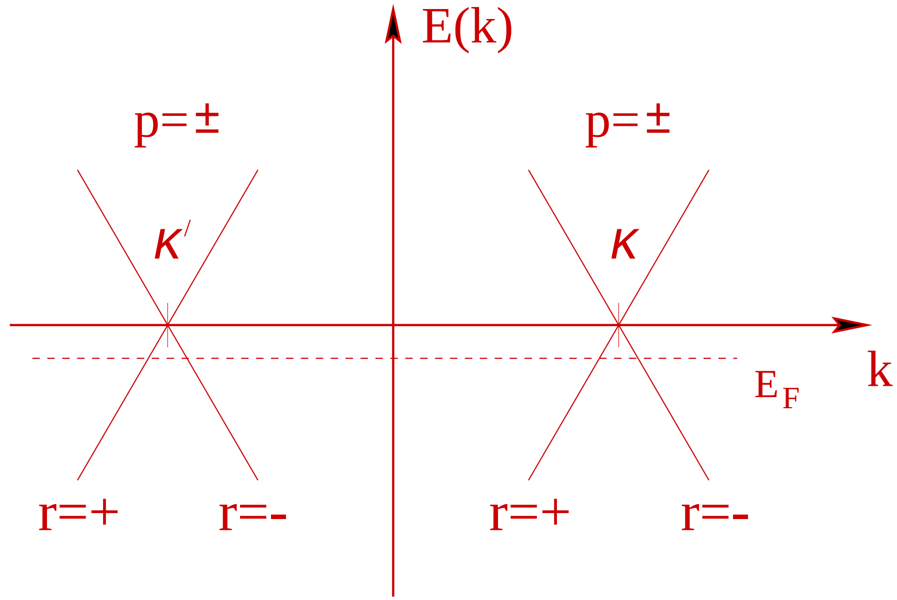
Electronic structure

- Band structure predicts three types:
  - Semiconductor if *(2n+m)/3* not integer. Band gap:

$$\Delta E = \frac{2v}{3R} \approx 1eV$$

  - Metal if *n=m*: Armchair nanotubes
  - Small-gap semiconductor otherwise (curvature-induced gap)
- Experimentally observed: STM map plus conductance measurement on same SWNT
- In practice intrinsic doping, Fermi energy typically 0.2 to 0.5 eV

Metallic SWNTs



Transverse momentum quantization: keep only $k_\perp = 0$

Individual SWNT represents ideal 1D quantum wire:

only *N*=2 spin-degenerate channels at Fermi level

Conductance of ballistic SWNT

- Two spin-degenerate transport bands
- Landauer formula: For good contact to voltage reservoirs, conductance is

$$G = N \frac{2e^2}{h} = 4e^2 / h$$

- Experimentally (almost) reached recently
- Ballistic transport is possible
- What about interactions?

Breakdown of Fermi liquid in 1D

- Landau quasiparticles unstable in 1D because of electron-electron interactions
- Reduced phase space
- Stable excitations: Plasmons (collective electron-hole pair modes)
- Often: Luttinger liquid

Luttinger, JMP 1963; Haldane, J. Phys. C 1981

- Physical realizations now emerging: Semiconductor wires, nanotubes, FQH edge states, cold atoms, long chain molecules,…

Field theory: metallic SWNTs

Egger & Gogolin, PRL 1997, EPJB 1998
Kane, Balents & Fisher, PRL 1997

Keep only two bands at Fermi energy:

Low-energy expansion of electron operator

- 1D fermion operators: Bosonization applies
  allows for nonperturbative handling of Coulomb interactions
- predicts Luttinger liquid behavior
- Simplest paradigm for non-Fermi liquid state with fractionalized quasiparticle excitations

Some Luttinger liquid basics

- Gaussian field theory, exactly solvable
- Plasmons: Bosonic displacement field
- Without interactions: Harmonic chain problem

$$H_0 = \frac{v}{2}\int dx\left[\Pi^2(x) + (\partial\varphi/\partial x)^2\right]$$

- Bosonization identities

$$\Psi_{R/L}(x) \propto \exp\left[\mp i\left(k_F x + \varphi(x)\right) - i\int^{x} dx' \Pi(x')\right]$$

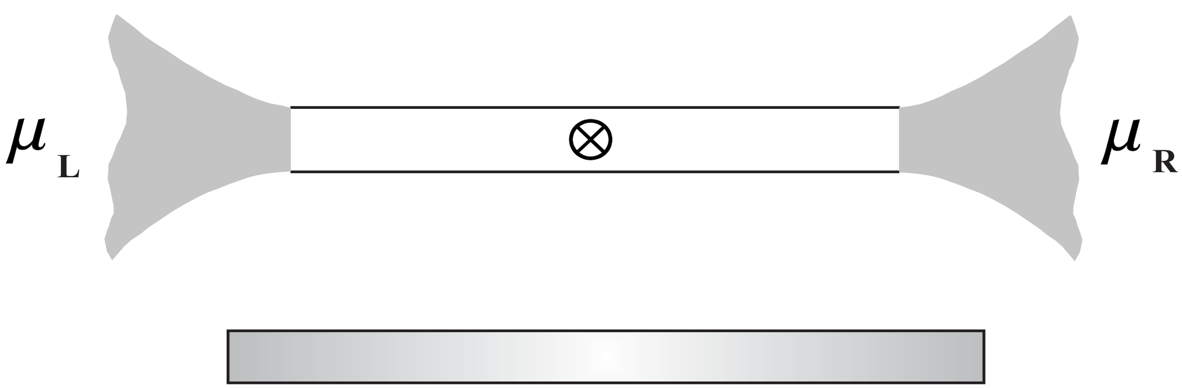
$$\rho(x) = \frac{k_F}{\pi} + \frac{\partial\varphi}{\partial x} + \frac{k_F}{\pi}\cos\left(2k_F x + \varphi(x)\right)$$

Coulomb interaction

- 1D interaction potential externally screened by gate

$$U(x-x') = \frac{e^2}{|x-x'|} - \frac{e^2}{\sqrt{(x-x')^2 + 4d^2}}$$

- Effectively short-ranged on large distance scales
- Retain only *q=0* Fourier component



Luttinger interaction parameter

- Dimensionless parameter $g = \frac{1}{\sqrt{1 + U_0 / \pi v}}$ describes interaction strength
- Density-density interaction from $\rho_{slow} = \partial\varphi / \partial x$ (forward scattering) gives Luttinger liquid

$$H = \frac{v}{2} \int dx \left[\Pi^2 + \frac{1}{g^2} \left(\partial\varphi / \partial x \right)^2 \right]$$

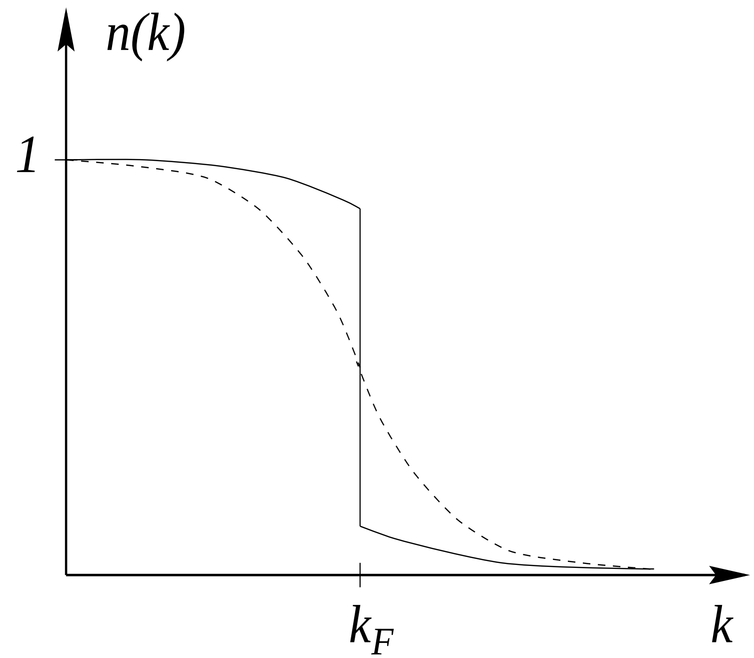
- Fast-density interactions (backscattering) open only very tiny gaps. Thermally smeared in practice…

Luttinger liquid properties I

- Electron momentum distribution function: Smeared Fermi surface at zero temperature
- Power law scaling

$$n\left(k \to k_F\right) \propto \left|k - k_F\right|^{(g+1/g-2)/4}$$

- Similar power laws: Tunneling DoS, with geometry-dependent exponents



TDoS of multi-band Luttinger liquid

Power-law suppressed TDoS reflects orthogonality catastrophe: Electron splinters into true quasiparticles

Geometry dependence

$$\rho(x,\omega) = \mathrm{Re}\int_0^\infty dt e^{i\omega t} \left\langle \Psi(x,t)\Psi^+(x,0) \right\rangle \propto \omega^\eta$$

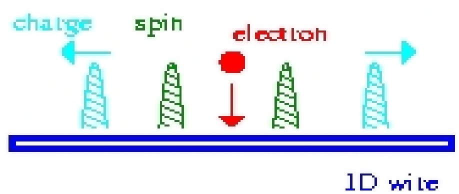
$$\eta_{bulk} \equiv \eta = \left(g + 1/g - 2\right)/2N$$

$$\eta_{end} = (1/g - 1)/N > 2\eta$$

Matveev & Glazman, PRL 1993
Egger, PRL 1999

Luttinger liquid properties II

- Electron fractionalizes into spinons and holons (solitons of the Gaussian field theory)
- Laughlin-type quasiparticles with fractional statistics and fractional charge



- Spin-charge separation: Additional electron decays into decoupled spin and charge wave packets
  - Different velocities for charge and spin
  - Spatial separation of electronic spin and charge!
  - Could be probed in nanotubes by magnetotunneling, electron spin resonance, or spin transport

Bosonized SWNT Hamiltonian

Four bosonic fields, index $a = c+, c-, s+, s-$

Low-energy theory: Luttinger liquid

$$H = \sum_a \frac{v_a}{2} \int dx \left[g_a \Pi_a^2 + g_a^{-1} \left(\partial_x \varphi_a \right)^2 \right]$$

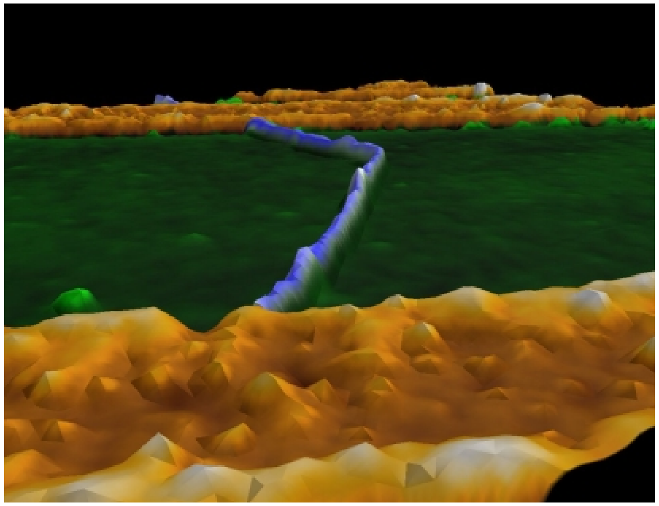
$$g_{a \neq c+} \cong 1 \qquad g \equiv g_{c+} \approx 0.2$$

$$v_{c+} = v / g, \; v_{a \neq c+} = v$$

Conductance probes TDoS

- Conductance across kink:

$$G \propto T^{2\eta_{end}}$$



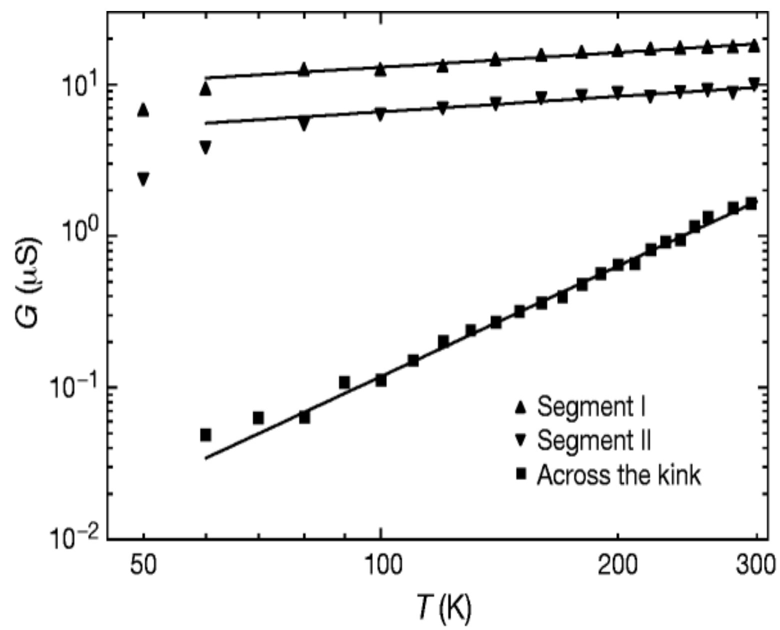
Delft group

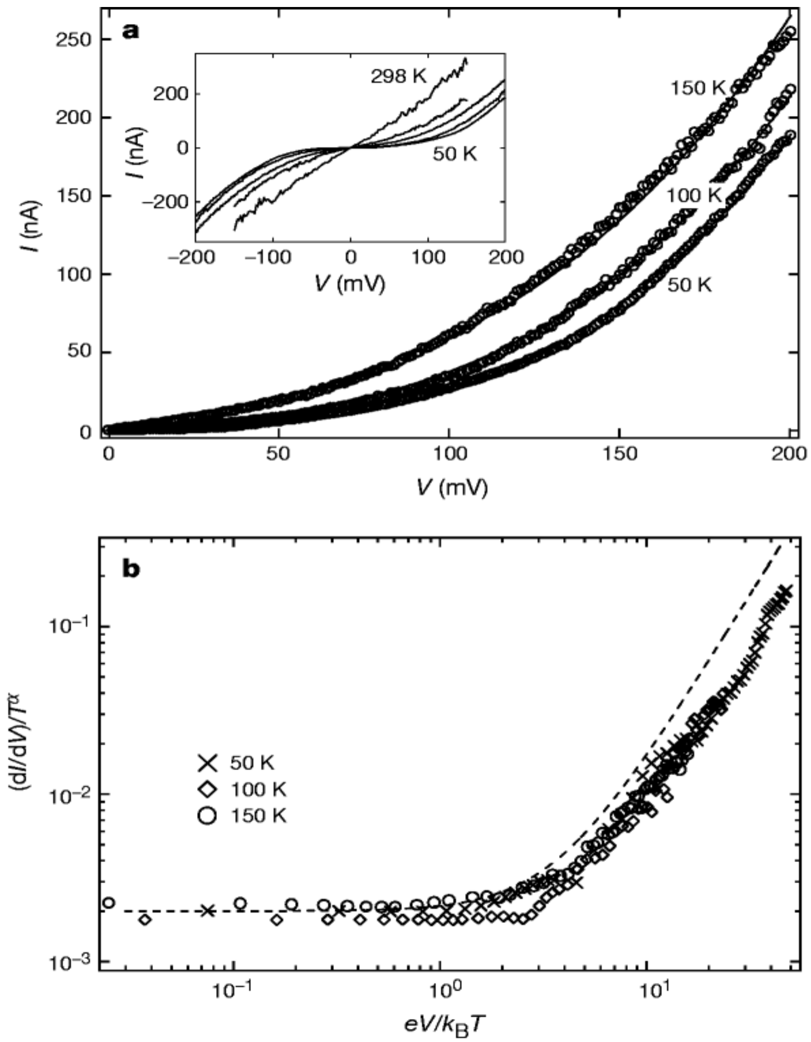
- Universal scaling of nonlinear conductance:

$$T^{-2\eta_{end}} dI / dV \propto \sinh\left[\frac{eV}{2k_B T}\right] \left|\Gamma\left(1+\eta_{end}+\frac{ieV}{2\pi k_B T}\right)\right|^2$$

$$\cdot \left[\coth\left(\frac{eV}{2k_B T}\right) - \frac{1}{2\pi} \mathrm{Im}\,\Psi\left(1+\eta_{end}+\frac{ieV}{2\pi k_B T}\right)\right]$$

Evidence for Luttinger liquid





gives *g* around 0.22

Yao et al., Nature 1999

Experimental evidence for Luttinger liquid in SWNTs

- Tunneling density of states (many groups)
- Resonant tunneling *Postma et al., Science 2001*
- Transport in crossed geometry (no tunneling)

  Gao, Komnik, Egger, Glattli & Bachtold, PRL 2004
- Photoemission spectra (spectral function)

  Ishii, Kataura et al., Nature 2003
- STM probes of density pattern *Lee et al. PRL 2004*
- Spin-charge separation & fractionalization so far not observed in nanotubes!

Beyond lowest-order k•p scheme?

Dirac cone approximation: chirality drops out

To go beyond, one must include

- Trigonal warping: anisotropic & nonlinear dispersion relation
- Transverse momentum quantization: in parallel magnetic field *B*, including tube curvature

$$k_{\perp} = \frac{eBR^{2}}{4\pi\hbar} \pm \frac{a}{R}\cos 3\theta$$

- Net effect: R/L movers have different velocity

$$\delta = \frac{v_R - v_L}{v_R + v_L} \propto B \sin 6\theta$$

Nonlinear current-voltage relation

- Linear transport: Onsager-Casimir relation

$$G(B) = G(-B)$$

- Out of equilibrium: odd-in-B part allowed

$$I_e(V,B) = -I_e(V,-B)$$

this contribution is even in voltage!

- Fundamentally interesting because nonzero effect requires combined presence of
  - Electron-electron interactions
  - Chirality (handedness): broken inversion symmetry
  - Magnetic field: broken time reversal symmetry

Sanchez & Büttiker, PRL 2004
Spivak & Zyuzin, PRL 2004

How to include in low energy theory?

- Luttinger liquid theory now comes with left/right plasmon velocities, but still exactly solvable Gaussian theory

$$v_{c+,R/L}/v = g^{-1} \pm \delta$$

$$v_{a\neq c+,R/L} = v_{R/L} = v(1 \pm \delta)$$

- Consider long SWNT & good contacts
  - Effect requires (at least two) impurities
  - Here: 2 impurities separated by distance *d*
  - Nonequilibrium Keldysh approach

Odd-in-B current in a chiral SWNT

De Martino, Egger & Tsvelik, cond-mat/0605645

Analytical result

$$I_e \propto \sin(2k_F d)\Theta^{2g-1}e^{-g\Theta}\sin\left(\frac{(1-g^2)B}{gB_0}\sin(6\theta)U\right)$$

$$\times \mathrm{Im}\left[e^{iU}\frac{\Gamma(1+g-iU/\Theta)}{\Gamma(g)\Gamma(2-iU/\Theta)}F\left(g,1+g-iU/\Theta;2-iU/\Theta;e^{-2\Theta}\right)\right]$$

with dimensionless temperature/voltage $\Theta = \frac{2\pi k_B T}{\hbar v/gd},\ U = \frac{|eV|}{\hbar v/gd}$

Requires interactions (g<1) and chirality (sin6θ≠0)

odd in magnetic field *B*, even in bias voltage *V*

changes sign with handedness (enantioselective)

Available experimental results

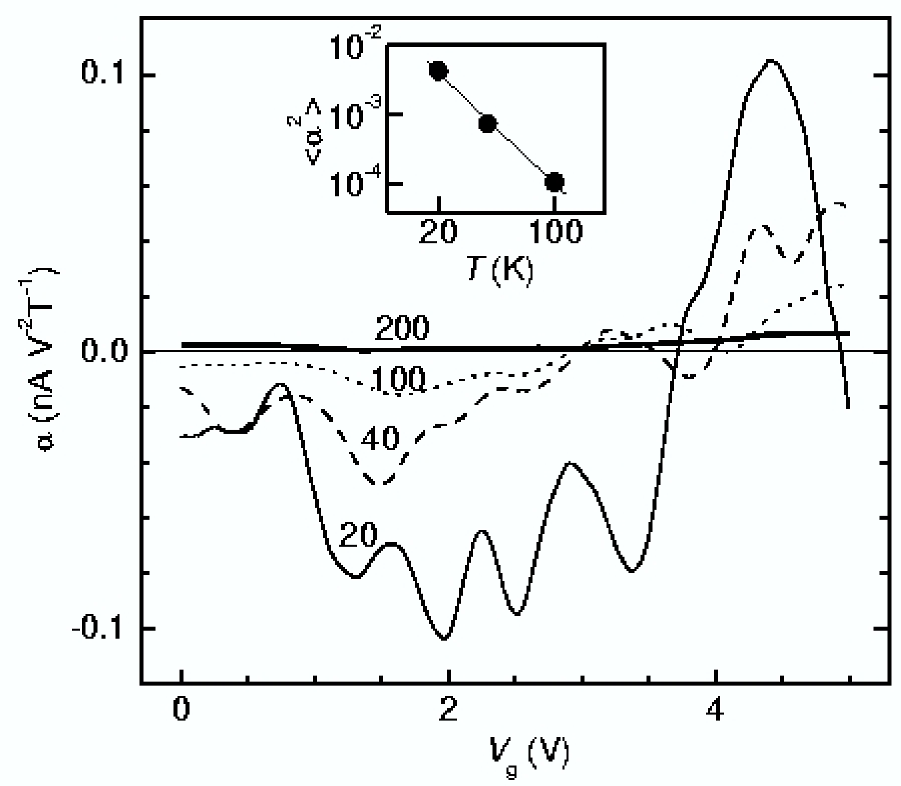
Wei, Cobden et al., PRL 2005

Measured:

$$\alpha(T) = \left[\frac{I_e(V,T,B)}{V^2 B}\right]_{V,B\to 0}$$

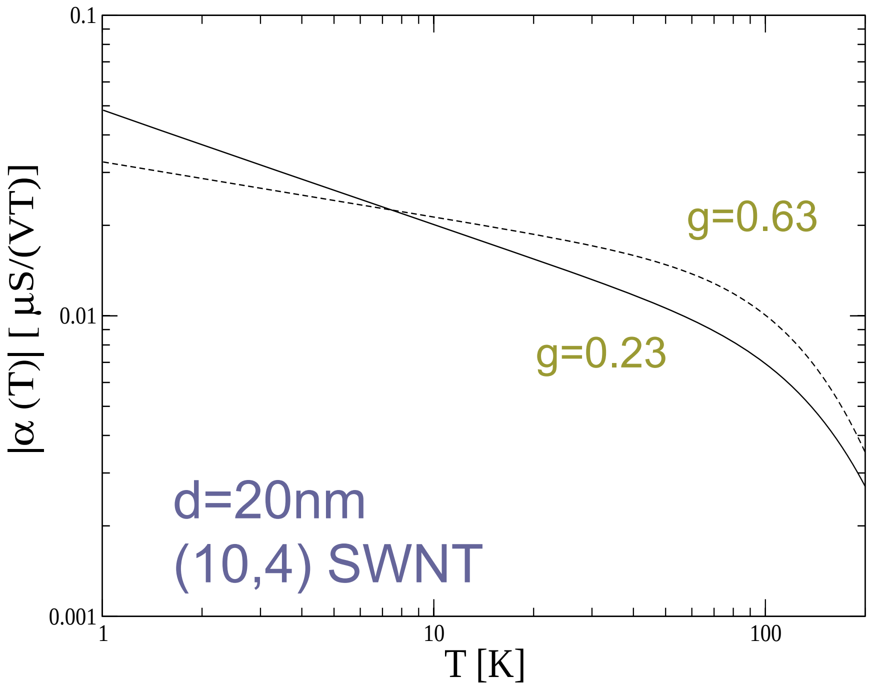
for individual SWNT (with several impurities)

- Oscillatory dependence on gate voltage; corresponds to $\sin(2k_F d)$ factor
- increases when lowering temperature



Theoretical result for *α(T)*

- Power-law scaling at low temperature
- Exponentially small at high temperature
- Order of magnitude as in experimental data
- Does not change sign as function of temperature



Prediction: Oscillations in $I_e(V)$

Zero temperature limit:

$$I_e \propto \sin\left[\frac{(1-g^2)B}{gB_0}\sin(6\theta)U\right] U^{g-1/2} J_{g-1/2}(U)$$

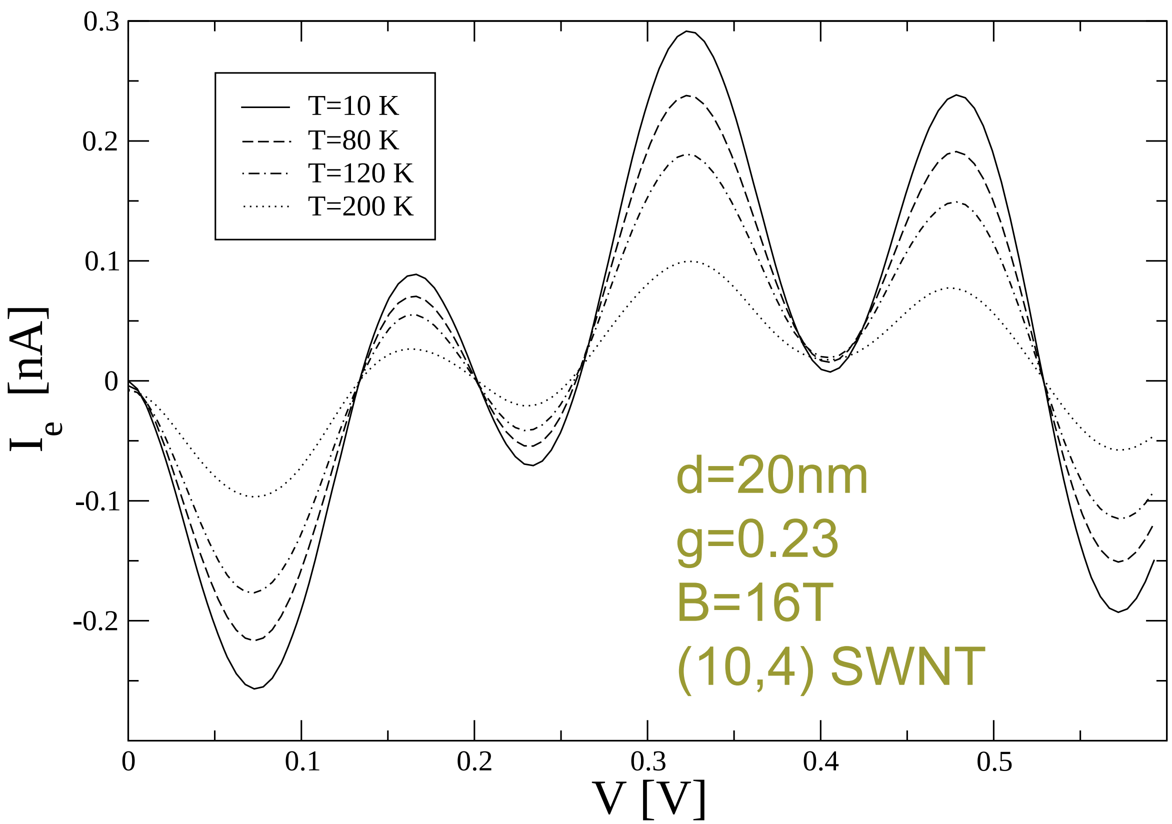
predicts oscillations as function of V with periods:

$$\Delta V_1 = \frac{hv}{egd}$$ yields Luttinger parameter

$$\Delta V_2 = \frac{B_0 g \Delta V_1}{B(1-g^2)\sin(6\theta)}$$ yields chirality

Low-voltage limit: Power-law scaling $I_e(V \to 0) \propto |V|^{2g}$

To be



direct observation of interaction physics possible

What about MWNTs?

- Electronic transport in MWNTs usually in outermost shell only
- Energy scales one order smaller
- Typically $N \approx 10$ bands due to doping
- Inner shells can also create disorder

White & Todorov, Nature 1998
Wang & Grifoni, PRL 2005

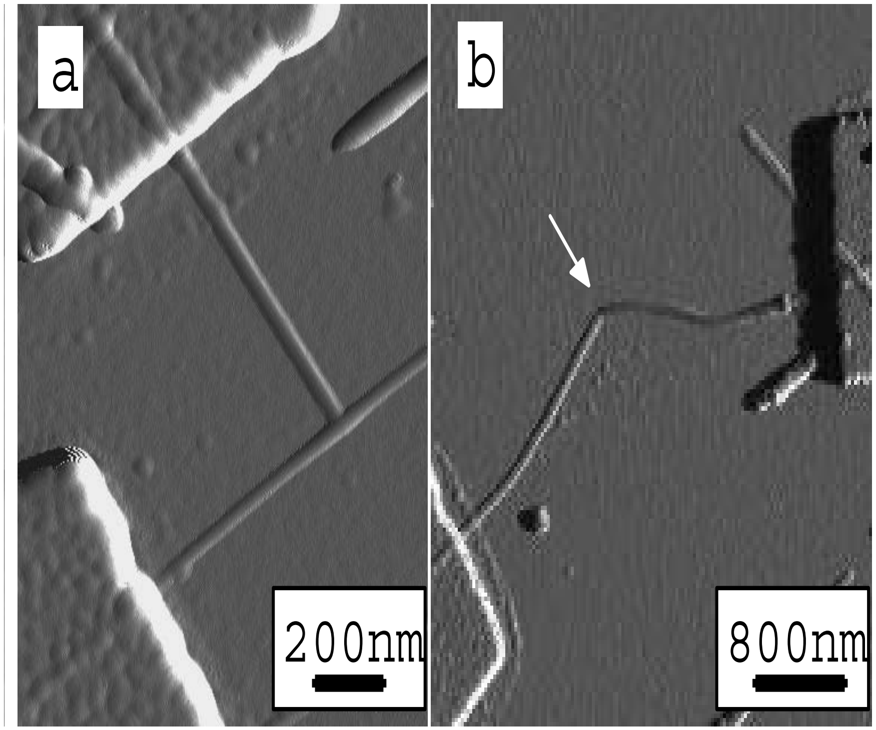
Experiments indicate mean free path $\ell > R$

- Also relevant for long SWNTs

Experiment: TDoS of MWNT

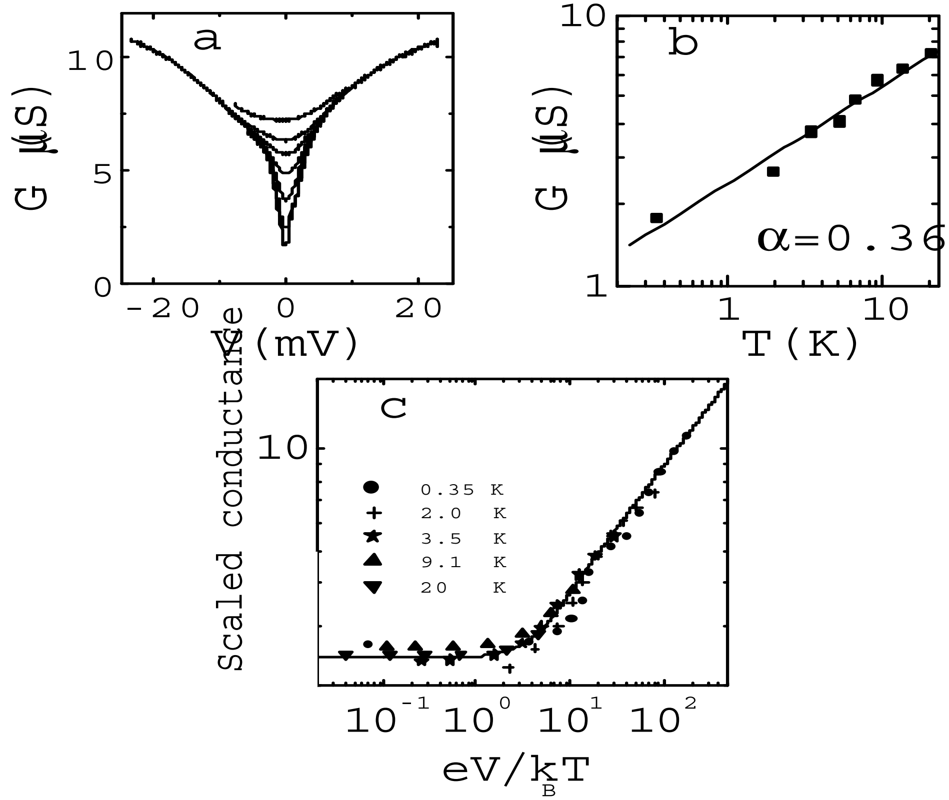
Bachtold et al., PRL 2001

- TDoS observed from conductance through tunnel contact
- Power law zero-bias anomalies
- Scaling properties similar to a Luttinger liquid, but: exponent larger than expected from Luttinger theory

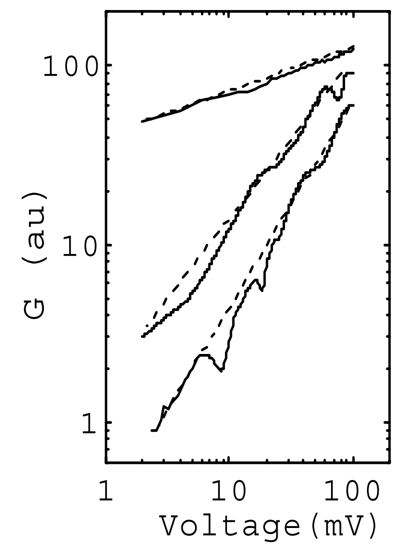


Tunneling DoS of MWNTs: experiment

Bachtold et al., PRL 2001



Geometry dependence



Luttinger exponent much smaller!

Role of disorder?

$$\eta_{end} = 2\eta_{bulk}$$

Interplay of disorder and interaction

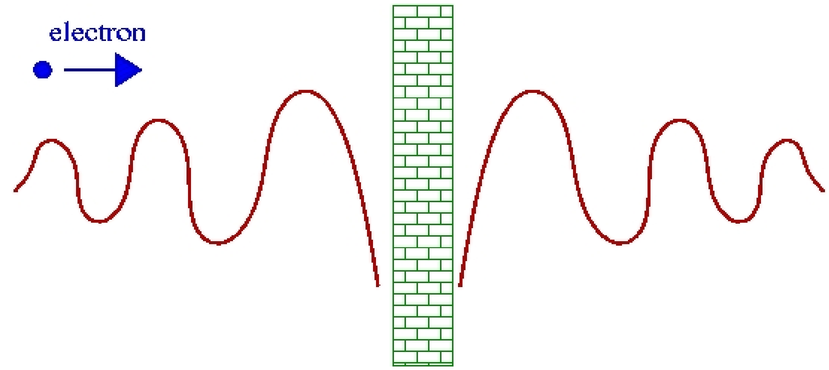
Mora, Egger & Altland, cond-mat/0602411

- Coulomb interaction enhanced by disorder
- Expected: crossover from quasiballistic Luttinger liquid at $\omega\tau > 1$ to diffusive/localized phase (e.g. Altshuler-Aronov diffusive anomalies) at $\omega\tau < 1$
- Field theory for multi-channel case and arbitrary disorder strength:

Interacting Nonlinear σ Model

Earlier versions:
Finkel'stein, Z. Phys. B 1983
Kamenev & Andreev, PRB 1999

Friedel oscillation



Mechanism unifying Luttinger liquid and Altshuler-Aronov corrections: *Matveev, Yue & Glazman, PRL 1993*

- Barrier (impurity) generates Friedel oscillation
- Incoming electron is also backscattered by Hartree-Fock potential of Friedel oscillation
- Energy dependence linked to Friedel oscillation asymptotics: very slow decay, $\delta\rho(x) \propto x^{-g}$

Egger & Grabert, PRL 1995

- Quantitative treatment of disorder difficult using this picture. Better suited: Nonlinear sigma model

Bulk TDoS

- Analytical result for $\rho(\omega, T)$ available
- Can be recast in terms of standard *P(E)* Coulomb blockade theory (microscopic derivation)

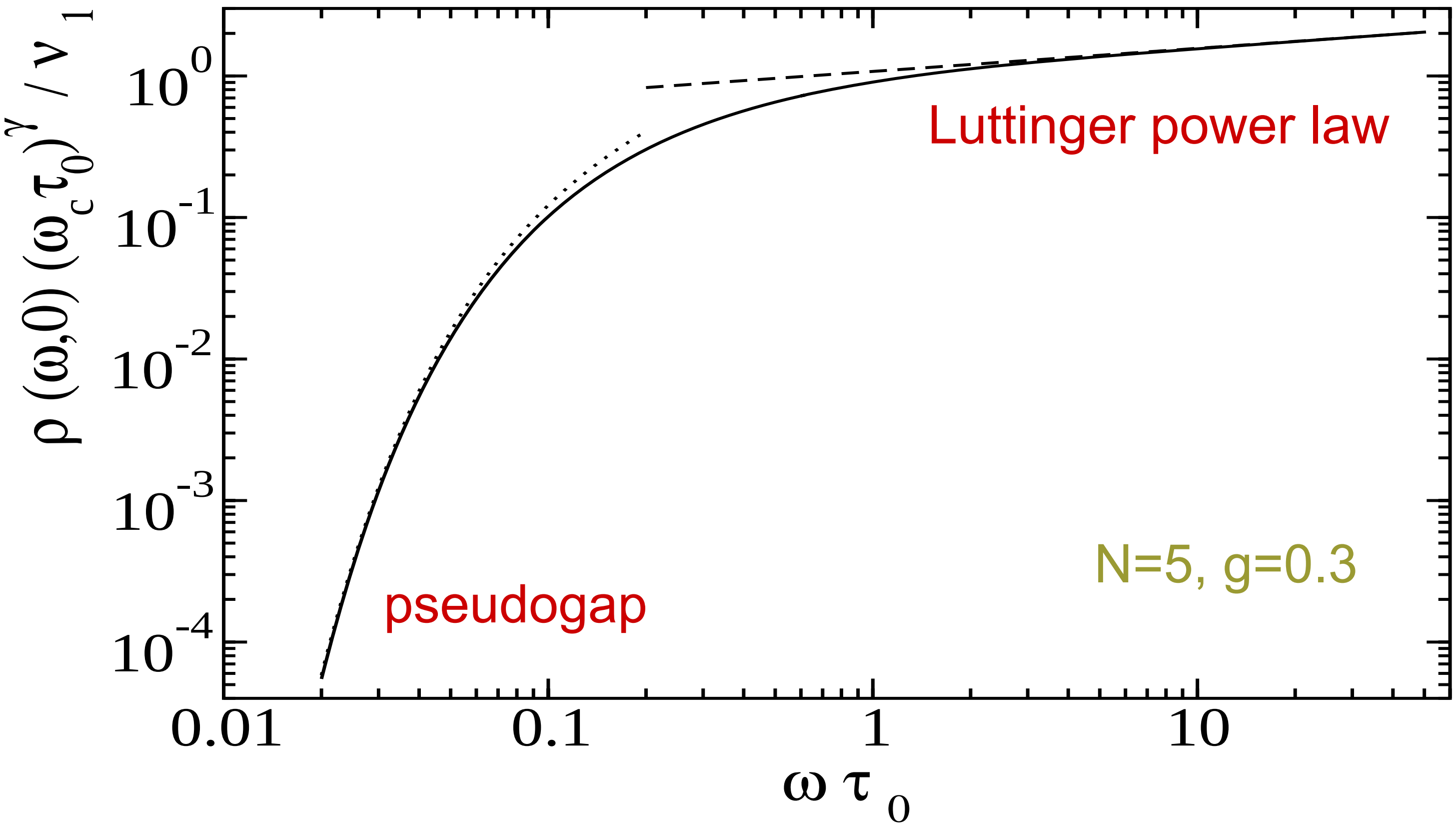
Egger & Gogolin, PRL 2001
Rollbühler & Grabert, PRL 2001

- Zero temperature: describes crossover from
  - Luttinger power law $\rho(\omega\tau > 1) \propto \omega^{\eta}$
  - to pseudogap at low energy:

$$\rho(\omega\tau < 1) \propto \frac{\sqrt{\omega\tau}}{\eta_{end}} \exp\left(-\frac{2\pi\eta_{end}^2}{\omega\tau} \right)$$

Nazarov, JETP 1989
Mishchenko et al., 2001

Bulk TDoS at T=0



Stronger suppression of TDoS due to disorder.
But does not really explain experimental results…

Interaction correction to conductivity

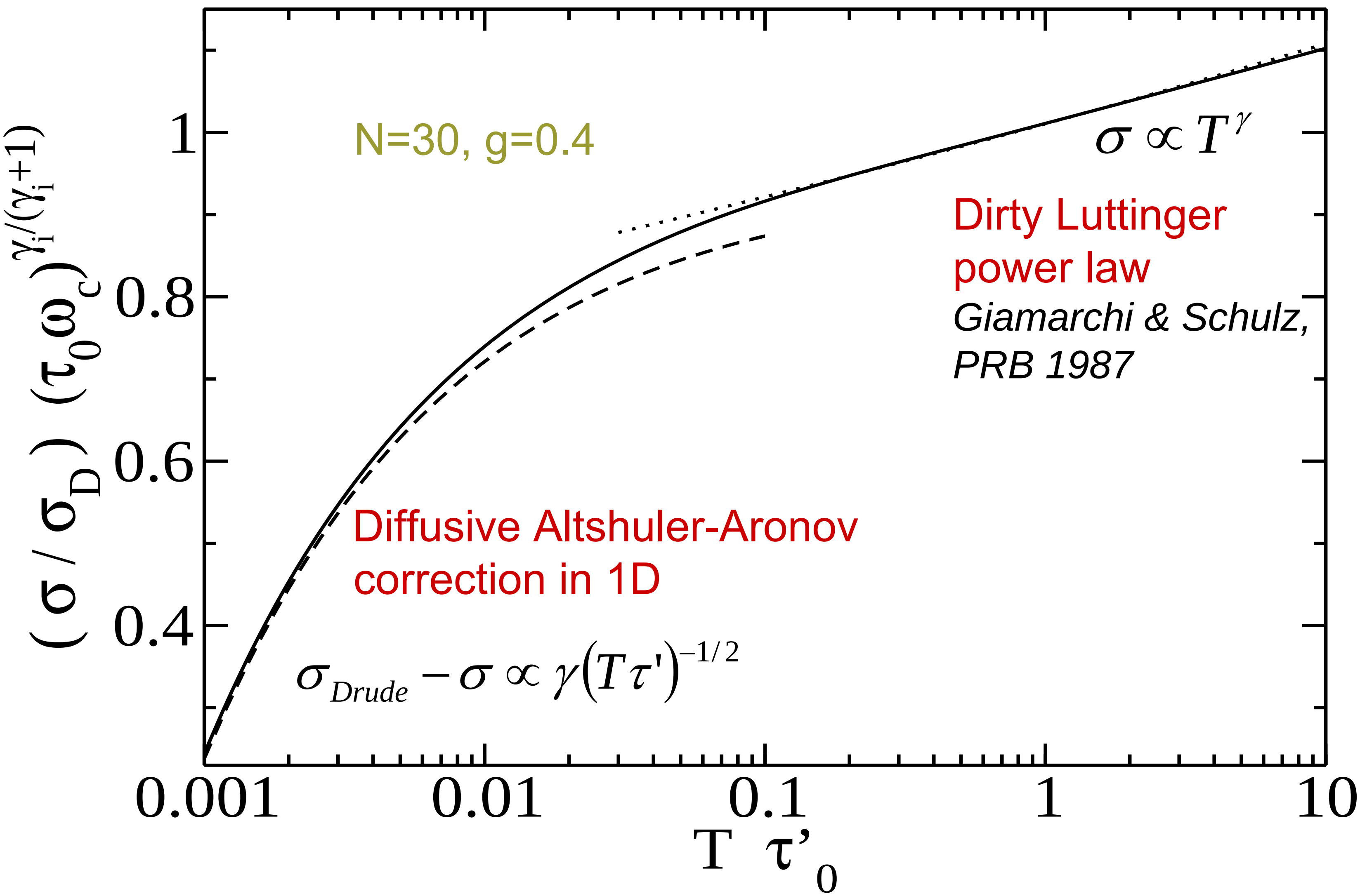
Complete crossover solution from ballistic to diffusive case, to lowest order in interaction:

$$\frac{\sigma(T)}{\sigma_{Drude}} = 1 + \gamma \ln(T\tau') - \gamma \int_0^\infty d\Omega \, \frac{\partial_\Omega \left[\Omega \coth \dfrac{\hbar\Omega}{2k_B T} \right]}{\Omega}$$

$$\times \left((1 + i/\Omega\tau')^{-1/2} - 1 + \frac{i}{2\tau'(1+g)\sqrt{\Omega^2 + i\Omega/\tau'}} \right)$$

Exponent for weak backscattering by single impurity in a Luttinger liquid $\quad \gamma = 2(1-g)/N$

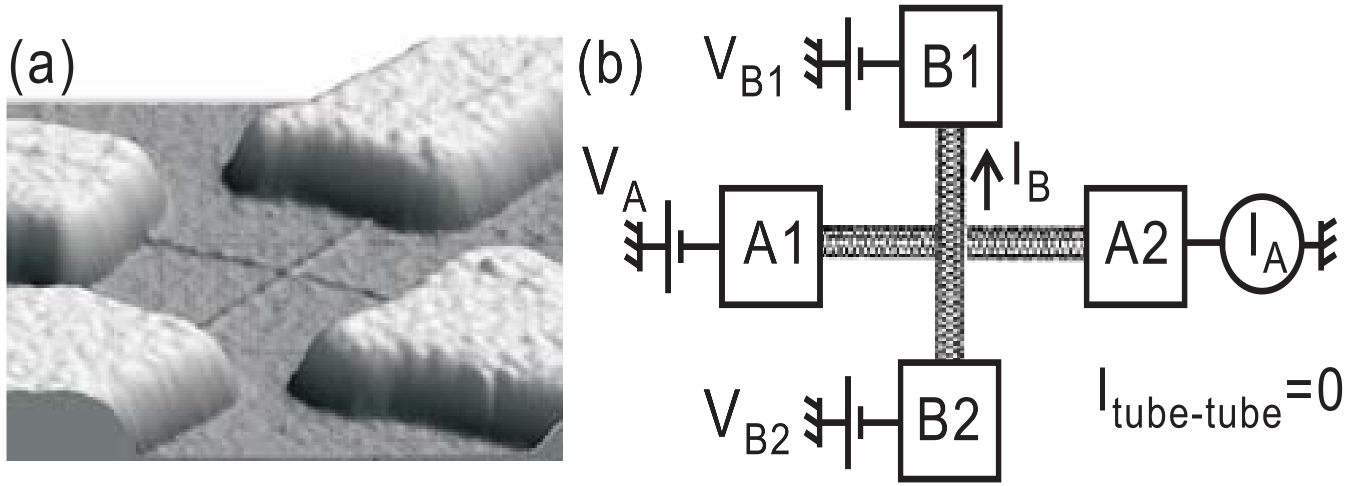
Renormalized mean free time $\quad \tau' \propto \tau^{1/(\gamma+1)}$



N=30, g=0.4
$\sigma \propto T^{\gamma}$
Dirty Luttinger
power law
Giamarchi & Schulz,
PRB 1987
Diffusive Altshuler-Aronov
correction in 1D
$\sigma_{Drude} - \sigma \propto \gamma (T\tau')^{-1/2}$
$(\sigma/\sigma_D)(\tau_0\omega_c)^{\gamma_i/(\gamma_i+1)}$
T τ'₀
1
0.8
0.6
0.4
0.001
0.01
0.1
1
10

Crossed tubes: Theory vs. experiment

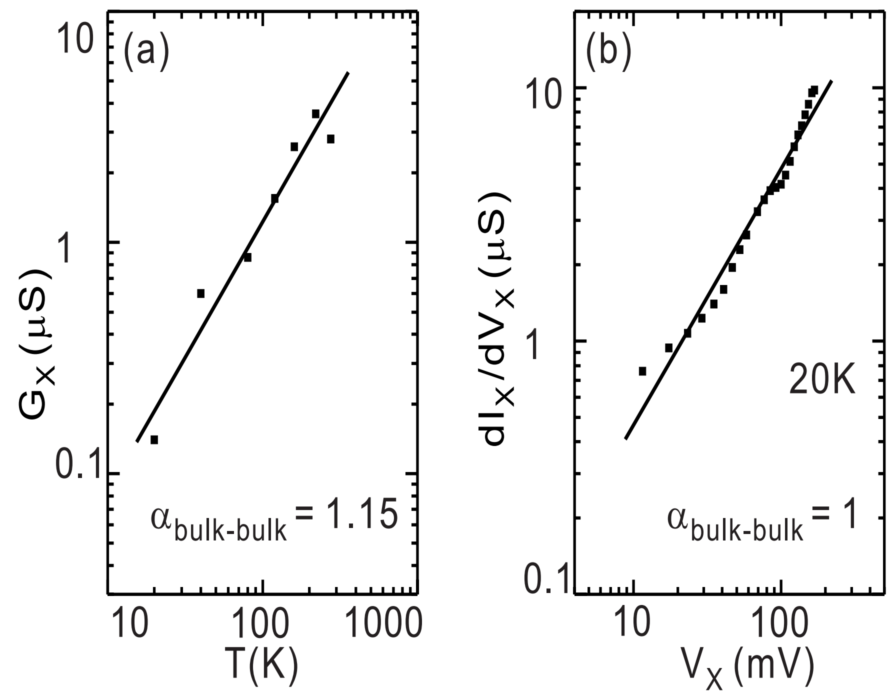
Komnik & Egger, PRL 1998, EPJB 2001
Gao, Komnik, Egger, Glattli, Bachtold, PRL 2004



- Weakly coupled crossed nanotubes
  - Single-electron tunneling between tubes irrelevant
  - Electrostatic coupling relevant for strong interactions
- Without tunneling: Local Coulomb drag

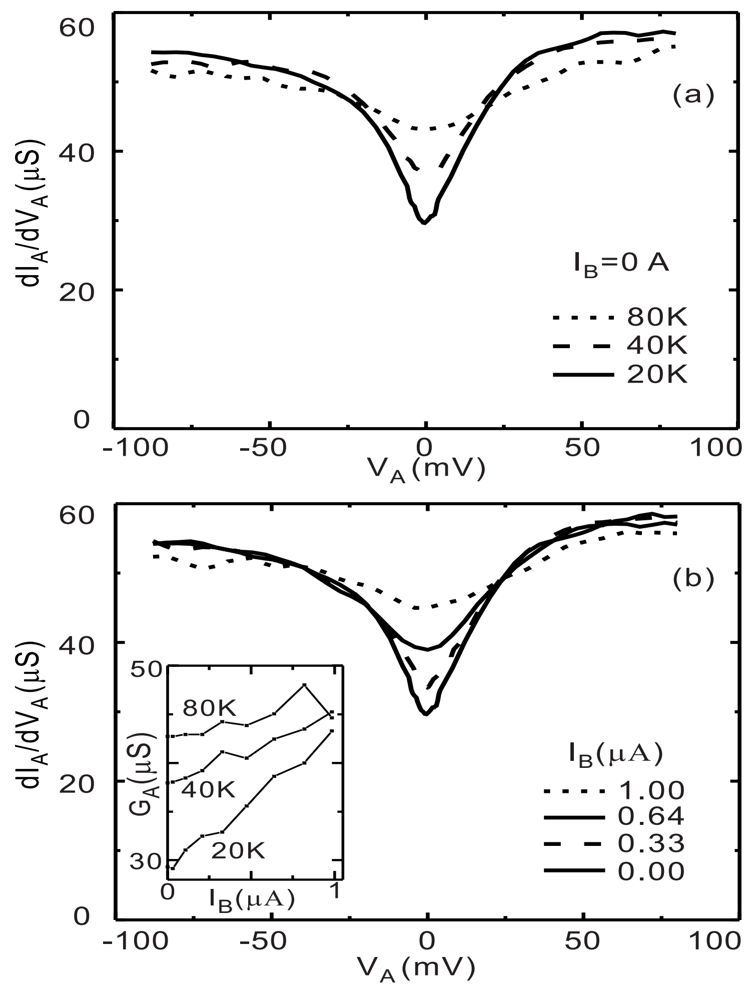
Characterization: Tunneling DoS

- Tunneling conductance through crossing: Power law, consistent with Luttinger liquid
- Quantitative fit gives *g=0.16*
- Evidence for Luttinger liquid beyond TDoS?



Dependence on transverse current

- Experimental data show suppression of zero-bias anomaly when current flows through transverse tube
- Coulomb blockade or heating mechanisms can be ruled out
- Prediction of Luttinger liquid theory?



Hamiltonian for crossed tubes

- Without tunneling: Electrostatic coupling and crossing-induced backscattering

$$H = H_0^A + H_0^B + \lambda_0 \rho_A(0) \rho_B(0) + \sum_{i=A/B} \lambda_i \rho_i(0)$$

$$H_0^i = \frac{1}{2} \int dx \left[\Pi_i^2 + (\partial_x \varphi_i)^2 \right]$$

- Density operator:

$$\rho_{A/B}(x) \propto \cos\left[\sqrt{16\pi g}\, \varphi_{A/B}(x) \right]$$

Renormalization group equations

- Lowest-order RG equations:

$$\frac{d\lambda_0}{dl} = (1-8g)\lambda_0 + 2\lambda_A\lambda_B$$

$$\frac{d\lambda_{A/B}}{dl} = (1-4g)\lambda_{A/B}$$

- Solution:

$$\lambda_{A/B}(l) = e^{(1-4g)l}\lambda_{A/B}(0)$$

$$\lambda_0(l) = e^{(1-8g)l}\left[\lambda_0(0) - 2\lambda_A(0)\lambda_B(0)\right] + 2e^{(2-8g)l}\lambda_A(0)\lambda_B(0)$$

- Here: inter-tube coupling most relevant!

Low-energy solution

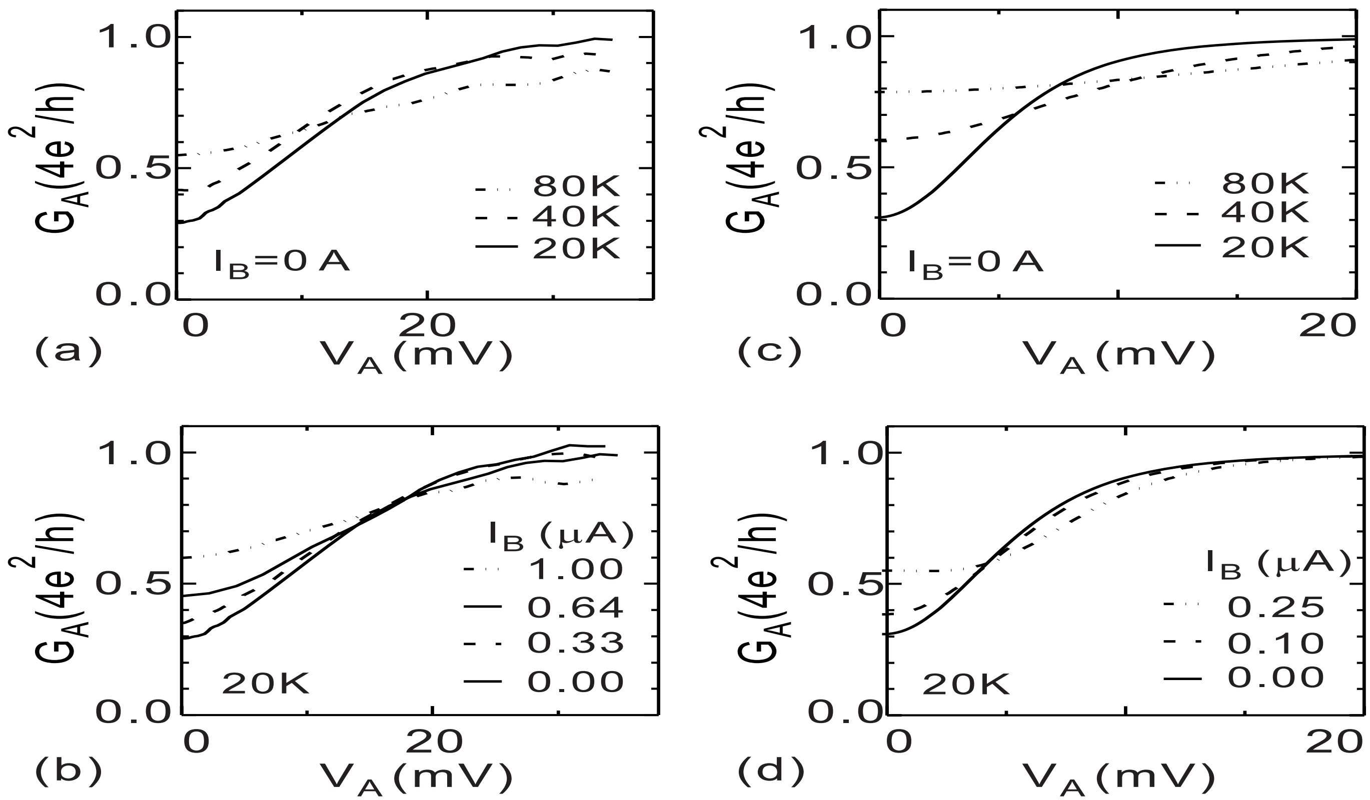
- Keeping only inter-tube coupling, problem is exactly solvable by switching to symmetric/antisymmetric (±) boson fields in *c*+ sector
- For *g=3/16=0.1875,* particularly simple:

$$I_{A/B} = \frac{4e^2}{h}\left[V_{A/B} - \frac{U_+ \pm U_-}{\sqrt{2}}\right]$$

$$eU_\pm = 2k_B T_B \,\mathrm{Im}\,\Psi\left(\frac{1}{2} + \frac{k_B T_B + ie\left(V_\pm - U_\pm\right)}{2\pi k_B T}\right)$$

$$V_\pm = \frac{V_A \pm V_B}{\sqrt{2}}$$

Comparison to experimental data



Experimental data

Theory

Only one fit parameter: $T_B = 11.6K$

Coulomb drag shot noise

Trauzettel, Egger & Grabert, PRL 2002

- Shot noise at T=0 gives important information beyond conductance

$$P(\omega) = \int dt e^{i\omega t} \langle \delta I(t) \delta I(0) \rangle$$

- For two-terminal setup & one weak impurity: DC shot noise carries no information about fractional charge $P = 2eI_{BS}$

Ponomarenko & Nagaosa, PRB 1999

- Crossed nanotubes: For $V_A = 0, V_B \neq 0 \Rightarrow P_A \neq 0$

  must be due to cross voltage (drag noise)

Shot noise transmitted to other tube

- Mapping to decoupled two-terminal problems in ± channels implies $\langle \delta I_+(t)\,\delta I_-(0) \rangle = 0$
- Consequence: Perfect shot noise locking

$$P_A = P_B = (P_+ + P_-)/2$$

  - Noise in tube A due to cross voltage is exactly equal to noise in tube B
  - Requires strong interactions, *g<1/2*
  - Effect survives thermal fluctuations

Conclusions & Outlook

- Nanotubes provide tailor-made laboratory for mesoscopic transport with strong interactions, e.g.
  - Luttinger liquid physics
  - Magnetochiral transport: Asymmetric in B terms
  - Disorder-interaction interplay
  - Local Coulomb drag in crossed SWNTs
- Some currently pursued topics
  - Interplay nanotube-graphene
  - Spin transport (no spin orbit, no hyperfine contributions)
  - Quantum dot physics (with superconducting and/or ferromagnetic contacts): Spin transistor, Josephon current
  - Intrinsic superconductivity in suspended ropes or MWNTs