Unconventional quantum Hall effect and Berry's phase of 2π in bilayer graphene

K. S. Novoselov[1], E. McCann[2], S. V. Morozov[1,3], V. I. Falko[2], M. I. Katsnelson[4],
A. K. Geim[1], F. Schedin[1] & D. Jiang[1]
[1]Manchester Centre for Mesoscience and Nanotechnology, University of Manchester, Manchester, M13 9PL, UK
[2]Department of Physics, Lancaster University, Lancaster, LA1 4YB, UK
[3]Institute for Microelectronics Technology, 142432, Chernogolovka, Russia
[4]Institute for Molecules and Materials, Radboud University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands

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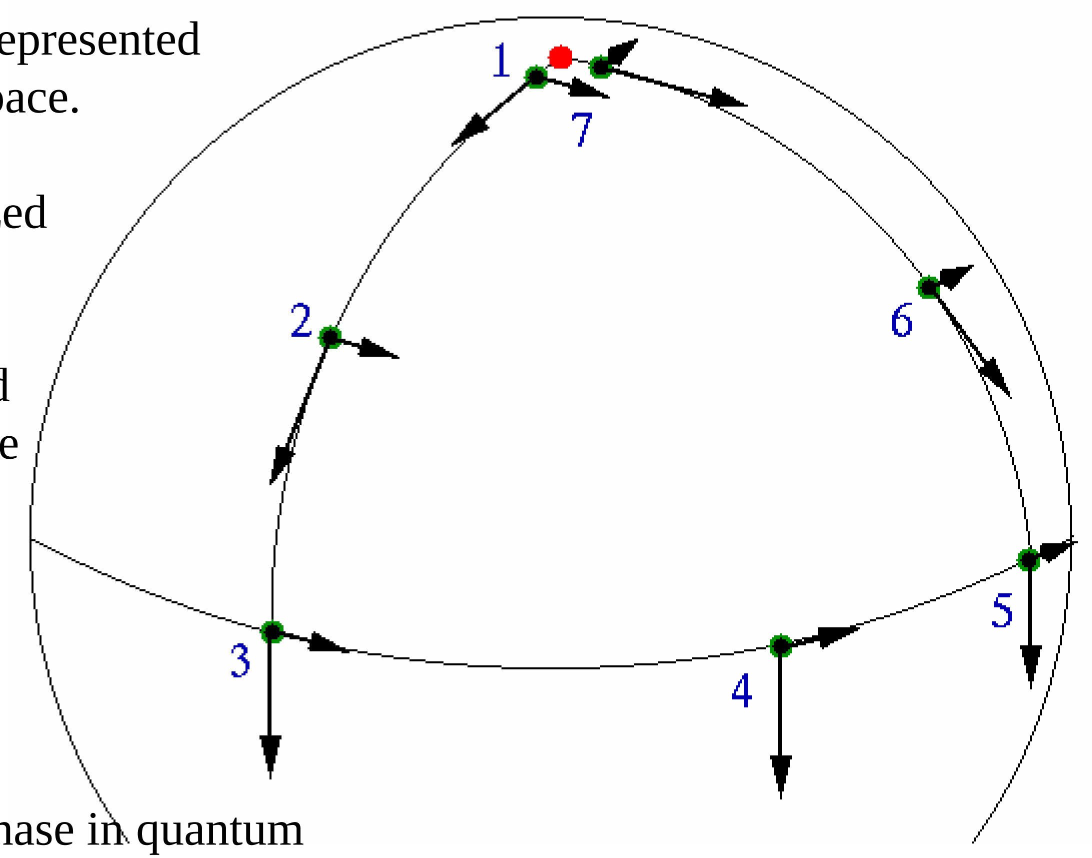
Example of Berry’s phase in elementary geometry

States in quantum mechanics are represented by vectors in a linear (complex) space.

In many cases they can be visualized as wave functions.

There is no reason that they should make exceptions to the general rule of acquiring phase angles after parallel transport along loops.



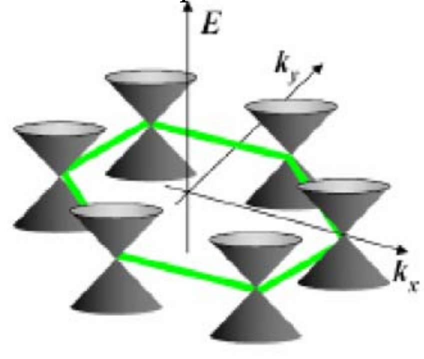
Two classical examples of Berry phase in quantum mechanics (known long before Berry' s systematization) are:

1) The rotation of a spinor.

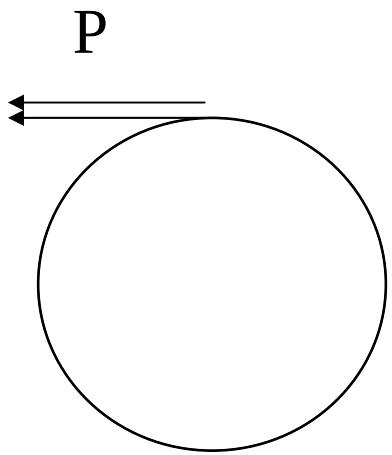
2) The Aharonov-Bohm experiment: a geometric phase acquired by electrons due to differences in local values of the magnetic *vector* potential.

Berry's phase in graphene

Cones of dispersion relation



Cyclotron orbit →



l - circuit

$(2\pi/\lambda)l = 2\pi n$ GaAs etc.

$(2\pi/\lambda)l = 2\pi(n+1/2)$ 1L graphene

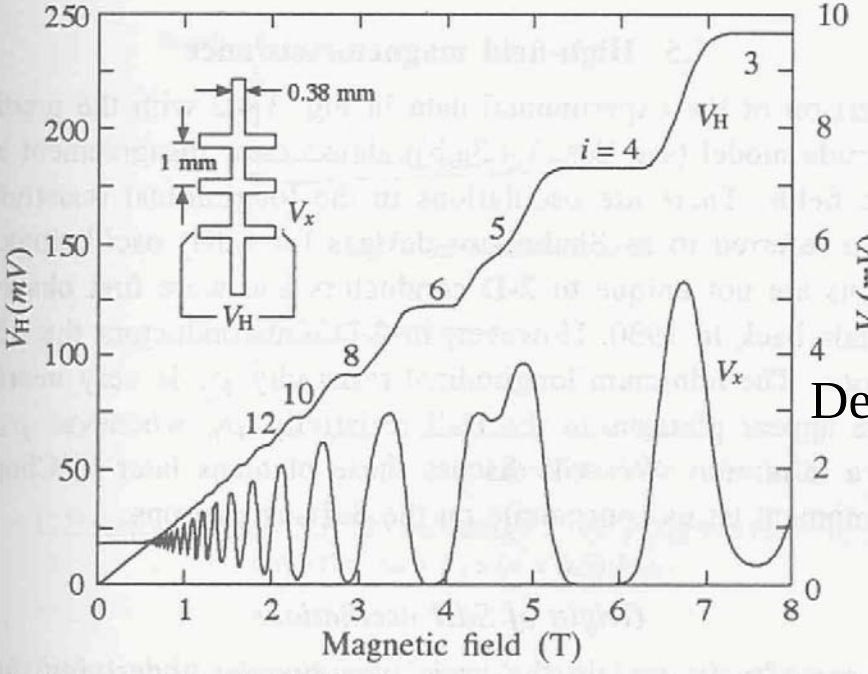
$(2\pi/\lambda)l = 2\pi(n+1)$ 2L graphene

If a quasiparticle encircles a closed contour in the momentum space, a phase shift known as Berry's phase is gained by the quasiparticle's wavefunction.

Berry's phase can be viewed as arising due to rotation of pseudospin, when a quasiparticle repetitively moves between different carbon sublattices (*A* and *B* for 1L graphene, and *A1* and *B2* for 2L graphene)

Quantum Hall effect in 2DEG

1.4 Low-field magnetoresistance 25



Drude model

$$\rho_{xx} = \sigma^{-1}$$

$$\rho_{yx} = -\rho_{xy} = (\mu B)/\sigma = B/|e|n_s$$

Density of sates for 2D conductor, [cm $^{-2}$eV^{-1}]

$$N_s(E) = \frac{m}{\pi \hbar^2} \vartheta(E - E_s)$$

Degeneracy of Landau level

$N_S \hbar\omega_c$, $\omega_c = eB/m$

1.5 High-field magnetoresistance 27

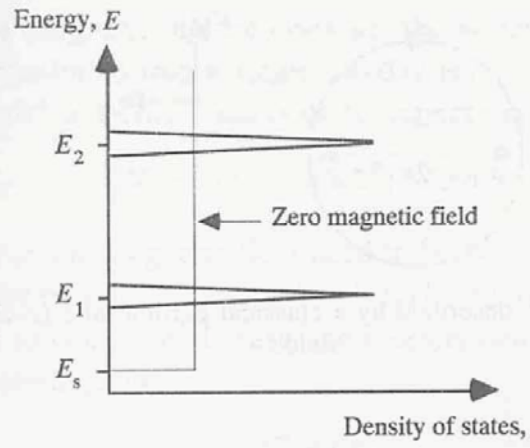


Fig. 1.5.1. Density of states $N_s(E,B)$ vs. energy E for a 2-DEG in a magnetic field. Note that $E_n = E_s + (n + 0.5)\,\hbar\omega_c$.

Filling factor ν and carrier concentration

Filling factor, ν

$$\frac{n_s}{2eB_1/h} - \frac{n_s}{2eB_2/h} = 1$$

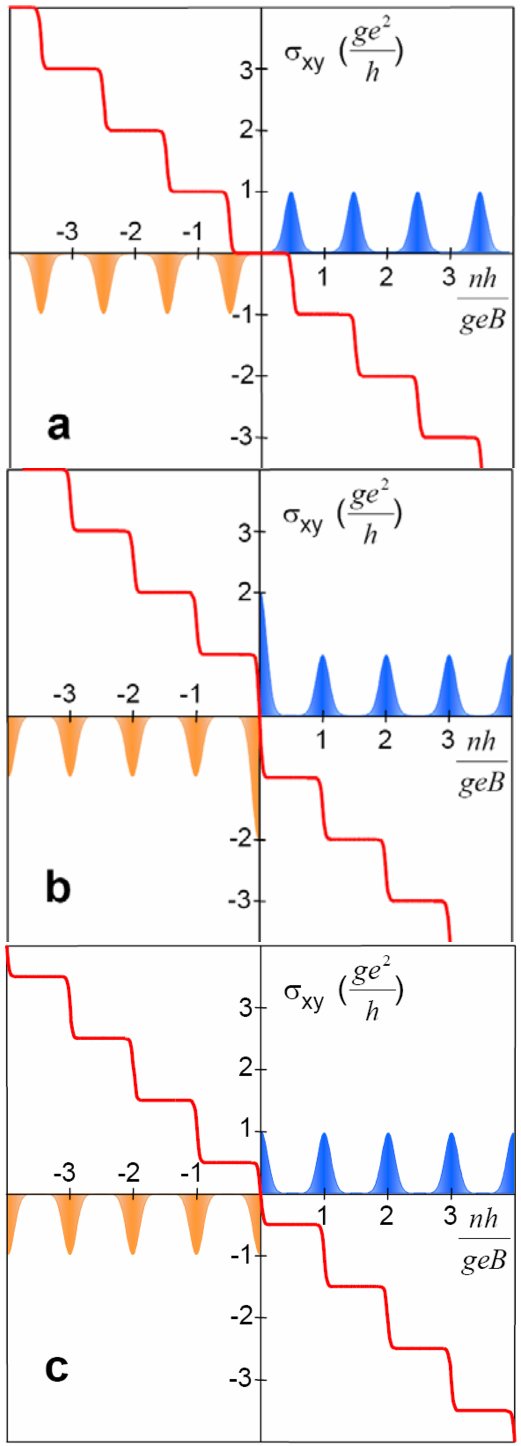
$$n_s = \frac{2e}{h} \frac{1}{(1/B_1) - (1/B_2)}$$

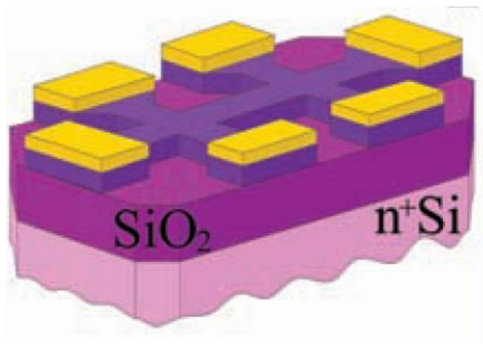
Three types of the integer quantum Hall effect

a – conventional 2D free fermion;
semiconductor 2DEG

b – chiral quasiparticle with Berry phase 2π;
$\varepsilon(p) = p^2/2m$
bilayer graphene

b – chiral quasiparticle with Berry phase π;
massless Dirac fermions
$\varepsilon(p) = pc^*$; $c^*=10^6$ m/s
monolayer graphene

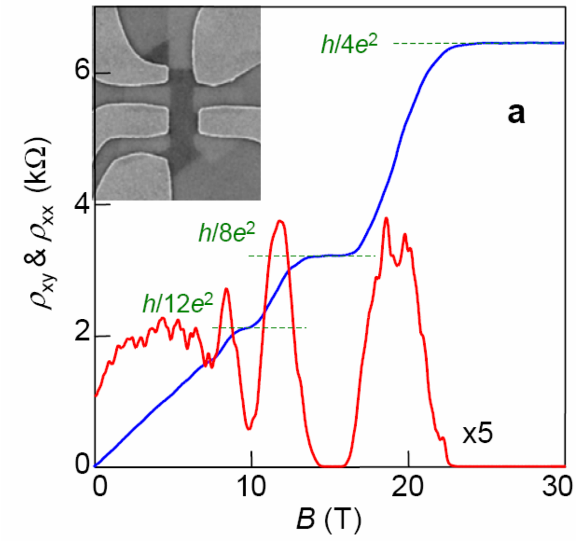




$$\sigma_{xy} = \rho_{xy}/(\rho_{xy}^2+\rho_{xx}^2)$$

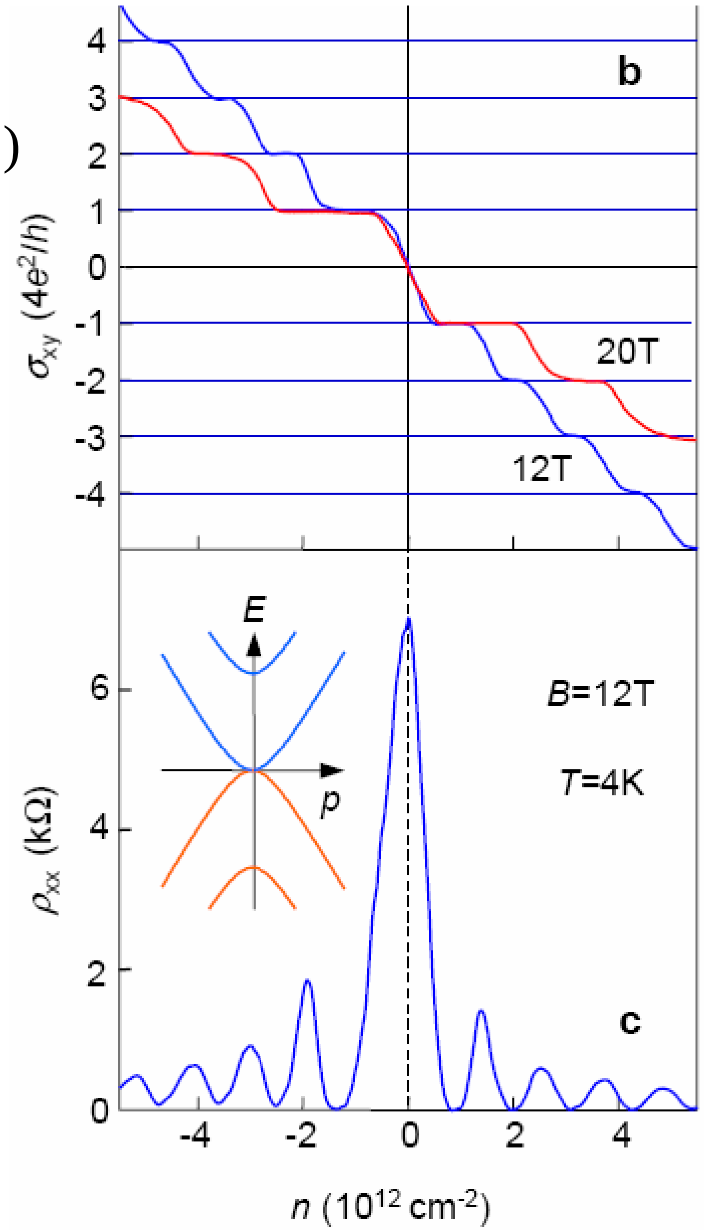
$$n=\alpha V_g$$

Quantum Hall effect in bilayer graphene

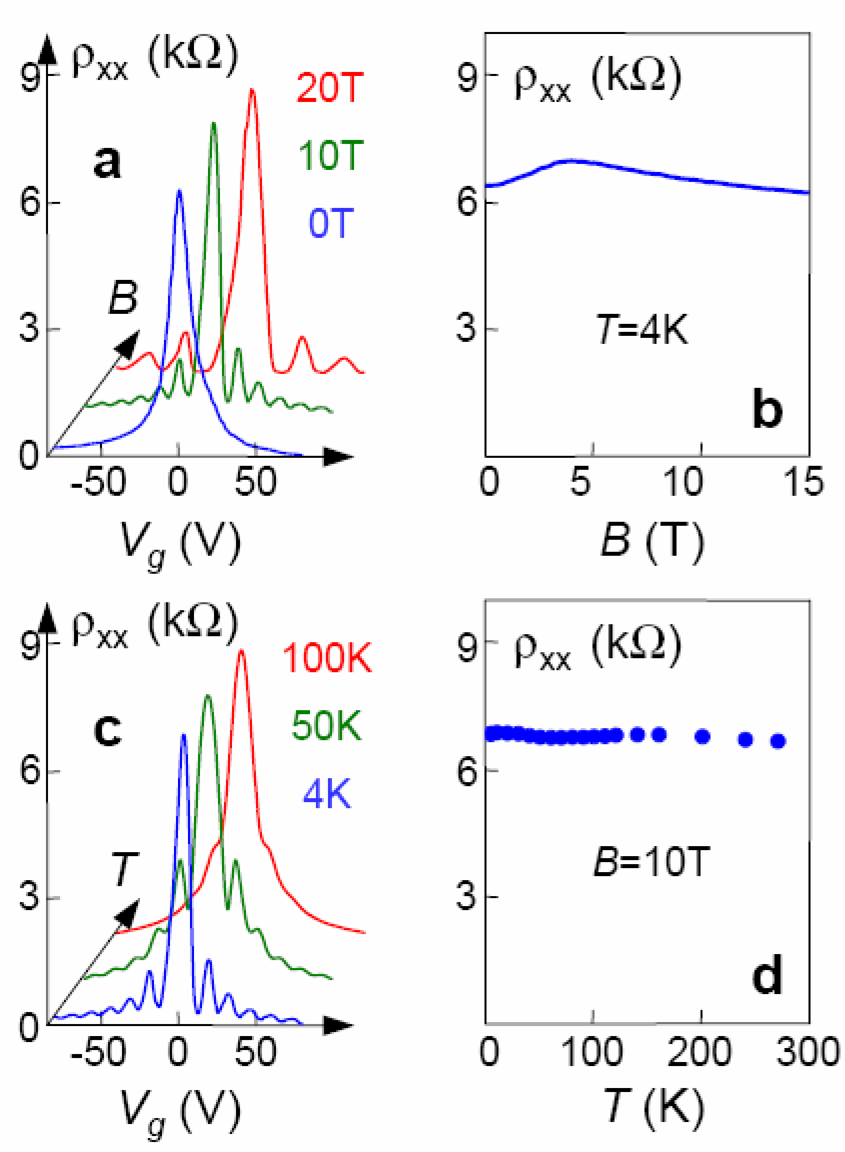


$\sigma_{xy} = \rho_{xy}/(\rho_{xy}^2+\rho_{xx}^2)$

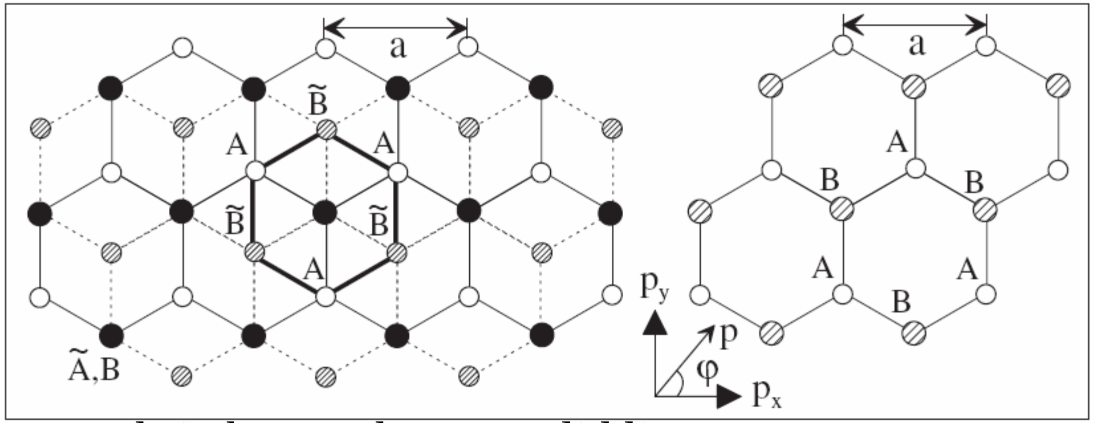
Degeneracy of central peak $2\times 4B/\Phi_0$



Resistivity of 2L graphene near zero concentrations

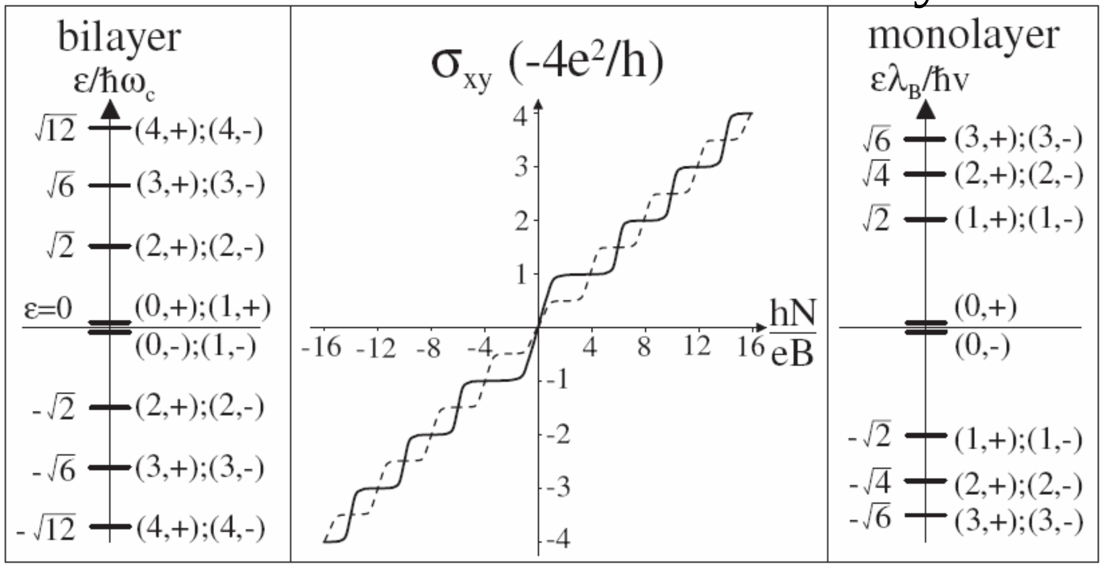


Lattice of bilayer   monolayer



Bonds in bottom layer – solid lines
Bonds in upper layer – dashed lines
Solid circles – dimers

Landau levels and Hall conductivity



$\varepsilon_N = \pm\hbar\omega_c[N(N-1)]^{1/2}$ $\varepsilon_N = \pm\, v_F(2e\hbar\, B\, N)^{1/2}$

Conclusions:

2L graphene adds a new member to the family of QHE systems

Its QHE behaviour reveals the existence of massive chiral Fermions with Berry's phase 2π

Observation of a finite metallic conductivity ~e^2/h at filling factor n=0 poses a challenge for the theory