Spin relaxation in single and double quantum dots

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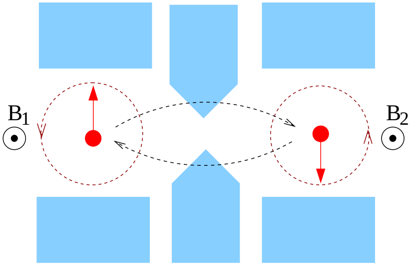
supported by US ONR

Outline

Quantum computing with spin qubits

single and double qubit operations

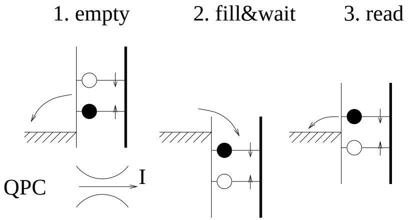


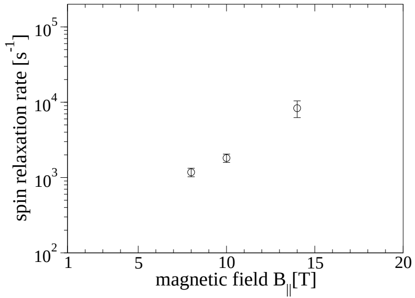
D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120, (1998).

$$H(t) = \underbrace{\mu_B g \sum_{i=1,2} \mathbf{B}_i(t).\mathbf{S}_i}_{\text{single qubit op.}} + \underbrace{J_{12}(t)\mathbf{S_1}.\mathbf{S_2}}_{\text{double qubit op.}}$$

Experiment
Elzermann et al, Nature 430, 431 (2004)

- single electron in single dot
- inplane magnetic field
- scheme (energy resolved tunneling):





Existing works

- mainly in single dots – analytical solution for electron in a parabolic potential
- spin relaxation in single dots

A. V. Khaetskii and Y. V. Nazarov, Phys. Rev. B **64**, 125316 (2001)

I. Aleiner and V. I. Falko, Phys. Rev. Lett. **87**, 256801 (2001)

V. I. Falko, B. L. Altshuler and O. Tsyplyatev, Phys. Rev. Lett. **95**, 76603 (2005)

D. V. Bulaev and D. Loss, Phys. Rev. B **71**, 205324 (2005)

M. Florescu and P. Hawrylak, Phys. Rev. B **73**, 45304 (2006)

- our work on double dots

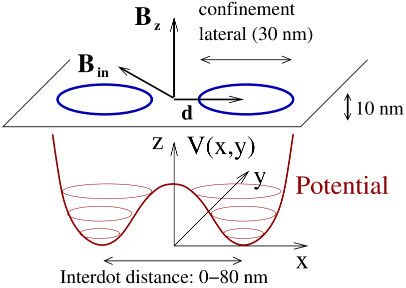
P. Stano and J. Fabian, Phys. Rev. B **72**, 155410 (2005)

P. Stano and J. Fabian, Phys. Rev. Lett. **96**, 186602 (2006)

P. Stano and J. Fabian, cond-mat/0604633, Phys. Rev. B in press (2006)

Electron Hamiltonian

effective mass and 2D approximation



$$H = \frac{\mathbf{p}^2}{2m} + V_C + \frac{g}{2}\mu_B \mathbf{B}.\boldsymbol{\sigma} + H_{so}$$

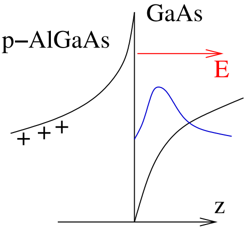
$$V_C = \frac{\hbar^2}{2ml_0^4}\min\{(\mathbf{r}-\mathbf{d})^2, (\mathbf{r}+\mathbf{d})^2\}$$

spin-orbit Hamiltonian

$$H_{so} = \underbrace{H_{br}}_{\text{Bychkov-Rashba}} + \underbrace{H_d}_{\text{Dresselhaus}}$$

Spin-orbit interations

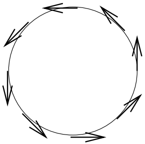
Bychkov-Rashba term



analogy of the relativistic correction – spin moving in electric field sees a magnetic field $\mathbf{B} = \frac{1}{c^2}\mathbf{v} \times \mathbf{E}$
energy of the spin in this field is

$$-\mu_B \boldsymbol{\sigma}.\mathbf{B} = -\frac{\mu_B}{c^2}\boldsymbol{\sigma}.\mathbf{v}\times\mathbf{E} = -\frac{\mu_B}{mc^2}\mathbf{E}.\boldsymbol{\sigma}\times\mathbf{p}$$

$$H_{BR} = \frac{\hbar}{2ml_{br}}(\sigma_x p_y - \sigma_y p_x)$$

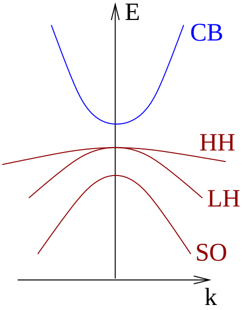


strength $\propto$ electric field **E** (asymetry of the QW)

Spin-orbit interations

Dresselhaus terms for [001] growing direction

correcton in **k**.**p** to a single conduction band description

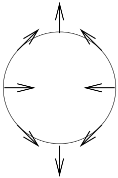


$$H_{D3}^{\text{bulk}} = \frac{\gamma_c}{2}[\sigma_x p_x(p_y^2 - p_z^2) + c.p. + h.c.]$$

by quantum averaging $p_z^2 \to \langle p_z^2 \rangle$ one gets

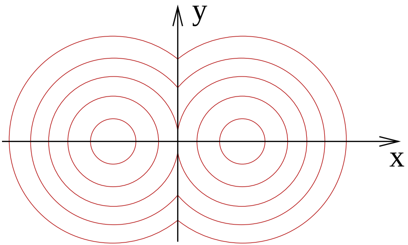
$$\frac{\hbar}{2ml_d}(-\sigma_x p_x + \sigma_y p_y) + \frac{\gamma_c}{2}(\sigma_x p_x p_y^2 - \sigma_y p_y p_x^2 + h.c.)$$

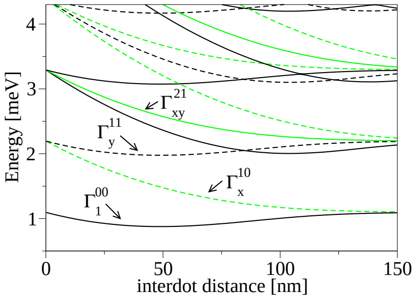
strength $\propto$ thickness of QW



Symmetry C_{2v}

spatial symmetry classes of the potential



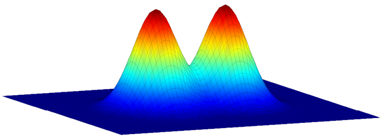


- symmetry operations: $I_x(x \to -x)$, $I_y(y \to -y)$, $I = I_x I_y$
- inplane (or no) field – $1, x, xy, y$
- perpendicular field – S $(1, xy)$ and A (x, y)
- spin-orbit terms – spatial symmetry x and y

Symmetric ground state Γ_1^{00}

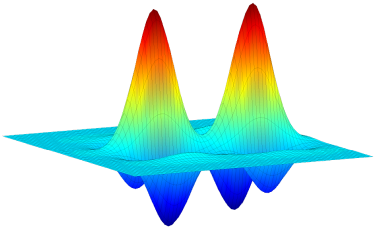
in zero and finite magnetic field

zero magnetic field



symmetric w.r.t. I_x, and I_y

finite magnetic field

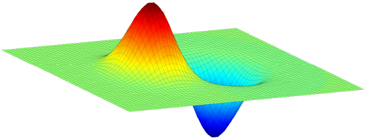


symmetric w.r.t. $I = I_x I_y$

Antisymmetric excited state Γ_x^{10}

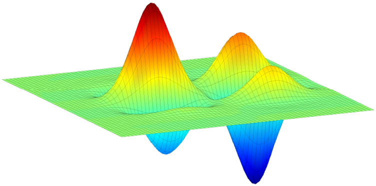
in zero and finite magnetic field

zero magnetic field



antisymmetric w.r.t. I_x, and I_y

finite magnetic field



antisymmetric w.r.t. $I = I_x I_y$

Phonons

piezoelectric and deformation electron-phonon potentials

$$H^{\mathrm{df}} = \sigma_e \sum_{\mathbf{Q}} \sqrt{\frac{\hbar Q}{2\rho V c_\lambda}} (b_{\mathbf{Q}l} + b^\dagger_{-\mathbf{Q}l}) e^{i\mathbf{Q}\cdot\mathbf{R}}$$

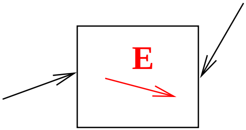
$$H^{\mathrm{pz}} = -ieh_{14} \sum_{\mathbf{Q}\lambda} \sqrt{\frac{\hbar}{2\rho V c_\lambda}} M_\lambda \underbrace{(b_{\mathbf{Q}\lambda} + b^\dagger_{-\mathbf{Q}\lambda})}_{\text{phonon operators}} \underbrace{e^{i\mathbf{Q}\cdot\mathbf{R}}}_{\text{relevant for overlaps}}$$

- phonons are plain waves
- only acoustic phonons play role

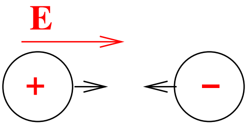
Electron-phonon interactions

- deformation potential
  $\delta E_c = \sigma_e \frac{\delta V}{V}$

- piezoelectric interaction



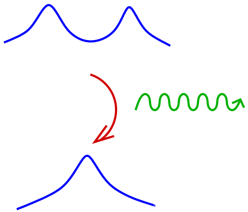
- Fröhlich coupling



Relaxation rate

Fermi's golden rule

$$\text{rate}_{i\to f} = \frac{2\pi}{\hbar}\sum_{\mathbf{Q}\lambda} |\langle \Gamma_i | H^{\text{e-ph}}(\mathbf{Q},\lambda)|\Gamma_f\rangle|^2 \delta(E_i - E_f - \hbar\omega_Q)$$



relaxation rate is proportional to

- overlap of initial and final wavefunctions
- density of states of phonons

Strategy & tactics

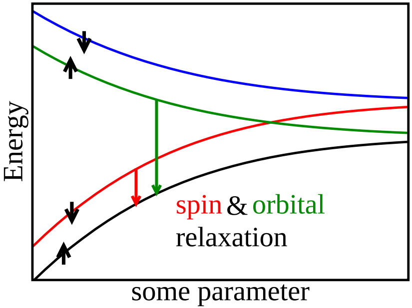
numerics

- diagonalize electron Hamiltonian (50x50)
- overlap due to phonons – Fast Fourier Transform
- numerical integration over phonons

analytics

- unitary transformation of the Hamiltonian
- lowest order perturbation to get eigenstates
- integration possible only in limiting cases

Orbital and spin relaxation



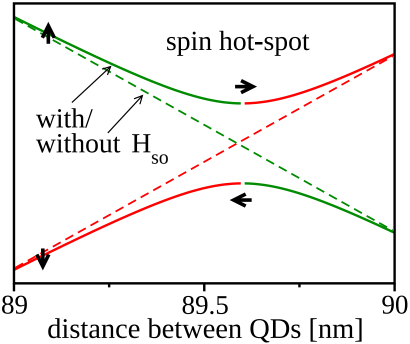
$$\begin{pmatrix} E_x^{\downarrow} & \triangle_{so} \\ \triangle_{so} & E_1^{\uparrow} \end{pmatrix}$$

$$\widetilde{\Gamma}_1^{\downarrow} = \Gamma_1^{\downarrow} + \frac{\triangle_{so}}{E_1^{\downarrow} - E_x^{\uparrow}} \Gamma_x^{\uparrow}$$

$$\underbrace{\langle \widetilde{\Gamma}_1^{\uparrow} | H^{\mathrm{e-ph}} | \widetilde{\Gamma}_x^{\uparrow} \rangle}_{\text{orbital}} \gg \underbrace{\langle \widetilde{\Gamma}_1^{\uparrow} | H^{\mathrm{e-ph}} | \widetilde{\Gamma}_1^{\downarrow} \rangle}_{\text{spin}}$$

Anticrossing (spin hot spot)

drastic effect on spin relaxation

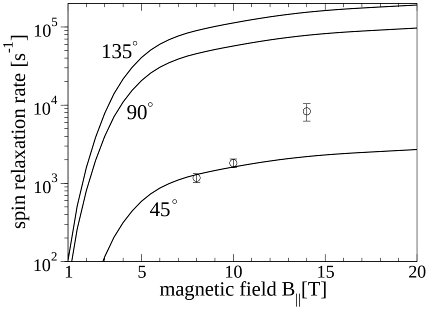


$$\begin{pmatrix} E & \triangle_{so} \\ \triangle_{so} & E \end{pmatrix}$$

$$\tilde{\Gamma}_1^{\downarrow} = \frac{1}{\sqrt{2}}(\Gamma_1^{\downarrow} + \Gamma_x^{\uparrow})$$

$$\underbrace{\langle \tilde{\Gamma}_1^{\uparrow} | H^{\mathrm{e-ph}} | \tilde{\Gamma}_x^{\uparrow} \rangle}_{\text{orbital}} \approx \underbrace{\langle \tilde{\Gamma}_1^{\uparrow} | H^{\mathrm{e-ph}} | \tilde{\Gamma}_1^{\downarrow} \rangle}_{\text{spin}}$$

Adjusting spin-orbit parameters for GaAs

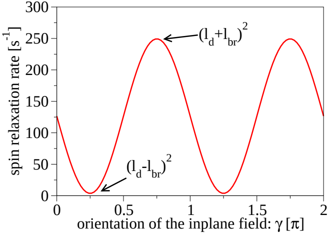


P. Stano and J. Fabian, Phys. Rev. Lett. **96**, 186602, (2006)

- 14 T beyound our theory (3D effects of the field)
- (not really a) fit: $l_{br} = 1.8\,\mu$m, $l_d = 1.3\,\mu$m
- anisotropy?

Anisotropy

Spin-orbit colinear with the field does not flip the spin



$$\text{rate} \propto \frac{1}{l_d^2} + \frac{1}{l_{br}^2} - 2\frac{1}{l_d l_{br}} \sin(2\gamma)$$

"explanation":

$$H_{so}^{\text{lin}} \propto \frac{1}{l_d}(\sigma_y p_y - \sigma_x p_x) + \frac{1}{l_{br}}(\sigma_x p_y - \sigma_y p_x)$$

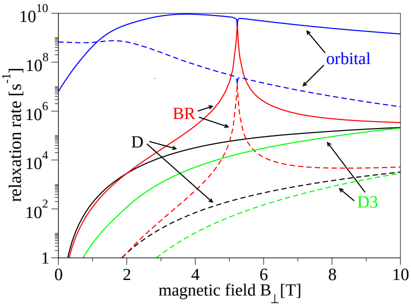
$$H_{so}^{\text{lin}}(l_d = l_{br}) \propto (\sigma_x + \sigma_y)(p_x + p_y)$$

spin relaxation is zero if

$$H_Z \propto (\sigma_x + \sigma_y)$$

Piezoelectric vs deformation

given by the energy difference



solid: piezoelectric
dashed: deformation

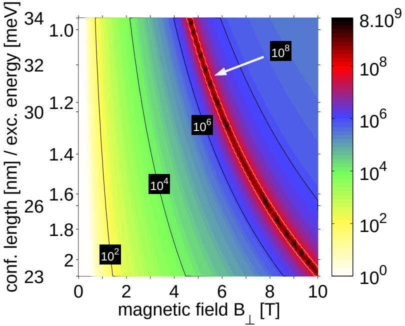
D. V. Bulaev and D. Loss, Phys. Rev. B **71**, 205324 (2005)

- piezoelectric dominates
- with one exception
- only Bychkov-Rashba induces anticrossing

Flavour of analytics

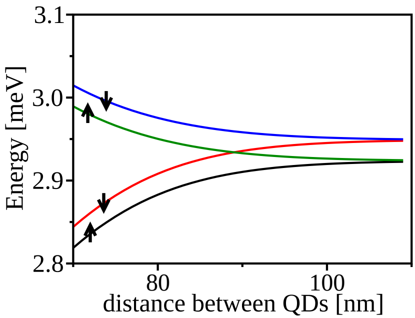
low mag. field	orbital	Γ^{df}	$(\pi\gamma_{\mathrm{df}}\hbar^3 c_l/m) l_0^{-4}(1 - B_\perp e l_0^2/2\hbar)$	$\lesssim 0.6$ T
		Γ_l^{pz}	$(459\pi\gamma_{\mathrm{pz}} c_l^5 m^5/4\hbar^5) l_0^4(1 + 5B_\perp e l_0^2/2\hbar)$	$\lesssim 0.5$ T
		Γ_t^{pz}	$(61\pi\gamma_{\mathrm{pz}} c_l^2 c_t m^3/4\hbar^3) l_0^2(1 + 3B_\perp e l_0^2/2\hbar)$	$\lesssim 0.8$ T
	spin	Γ^{df}	$(128\pi\gamma_{\mathrm{df}} m^2/3\hbar^7 c_l^3) l_0^8 \lvert\mu B_\perp\rvert^7 l_D^{-2}$	$\lesssim 4$ T
		Γ_l^{pz}	$(128\pi\gamma_{\mathrm{pz}} m^2/35\hbar^7 c_l^3) l_0^8 \lvert\mu B_\perp\rvert^5 l_D^{-2}$	$\lesssim 4$ T
		Γ_t^{pz}	$\Gamma_l^{\mathrm{pz}} \times 4c_l^5/3c_t^5$	$\lesssim 4$ T
high mag. field	orbital	Γ^{df}	$(2\pi\gamma_{\mathrm{df}}\hbar^{13}/3e^6 m^5 c_l^3) l_0^{-20} B_\perp^{-6}$	$\gtrsim 4$ T
		Γ_l^{pz}	$(8\pi\gamma_{\mathrm{pz}}\hbar^7/35e^4 m^3 c_l^3) l_0^{-12} B_\perp^{-4}$	$\gtrsim 4$ T
		Γ_t^{pz}	$\Gamma_l^{\mathrm{pz}} \times 4c_l^5/3c_t^5$	$\gtrsim 6$ T
	spin	Γ^{df}	$(32\pi\gamma_{\mathrm{df}}\lvert\mu\rvert^5/3\hbar e^2 c_l^3) B_\perp^3 l_D^{-2}$	$\gtrsim 8$ T
		Γ_l^{pz}	$(32\pi\gamma_{\mathrm{pz}}\lvert\mu\rvert^3/35\hbar e^2 c_l^3) B_\perp l_D^{-2}$	$\gtrsim 7$ T
		Γ_t^{pz}	$\Gamma_l^{\mathrm{pz}} \times 4c_l^5/3c_t^5$	$\gtrsim 7$ T

Single dot in perpendicular field



- hot spot dominates
- fixing confinement – previous case

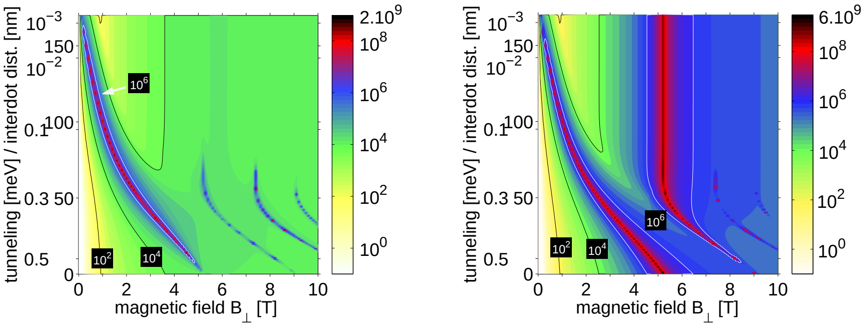
Difference to the single dot case



- ground state is degenerate at large interdot distance – anticrossing is always present
- Dresselhaus terms also induce anticrossings

Perpendicular magnetic field

hot-spot dominated – Bychkov-Rashba vs Dresselhaus

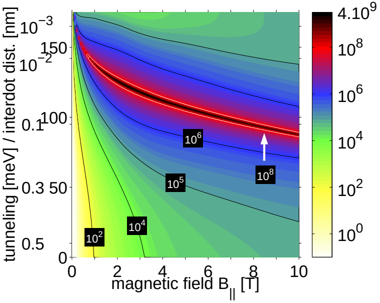


Dresselhaus – anticrossings only in true double dot regime

Bychkov-Rashba – anticrossings always there

P. Stano and J. Fabian, cond-mat/0604633, Phys. Rev. B in press (2006)

Inplane magnetic field



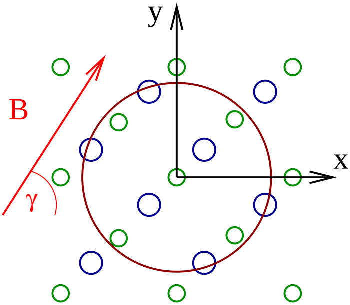
- turning the coupling on/off is going from the upper to the lower part
- but how to travel with spin?

Exploiting the anisotropy

idea – Dresselhaus refers to the crystallographic axes

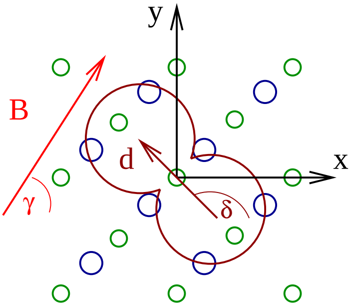
freedom in the single dot:

- orientation of the magnetic field



freedom in the double dot:

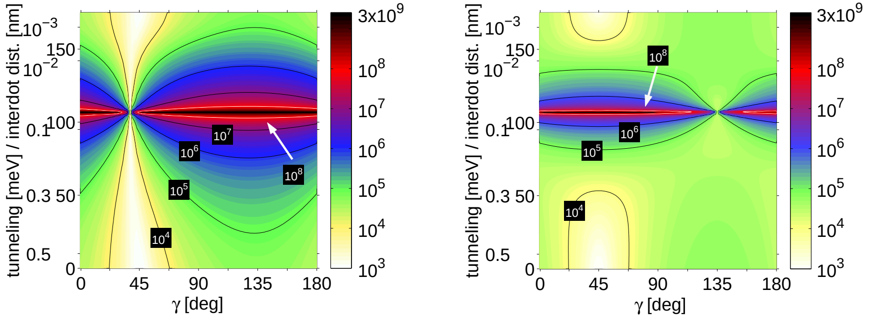
- orientation of the magnetic field and the double dot



Double dot – inplane magnetic field

anticrossing is sometimes not there!

dots oriented along [100]   dots oriented along [110]



P. Stano and J. Fabian, Phys. Rev. Lett. **96**, 186602 (2006)

Easy passage – derivation of effective Hamiltonian

1. unitary transformation to remove linear spin-orbit terms – find operator A such that $[A, H_0] = -H_{so}$

$$H = H_0 + H_{so} \to e^A H e^{-A} = H_0 + H_{so}^{(2)}$$

2. rotate spatial axes to be along the “main” axes of the potential

$$\mathbf{r}_{\rm new} = \mathbf{R}(\delta)\mathbf{r}_{\rm old}$$

3. rotate spin “axes” to be along the field

$$\boldsymbol{\sigma}_{\rm new} = \mathbf{R}(\gamma)\boldsymbol{\sigma}_{\rm old}$$

Easy passage – explanation

- effective s-o Hamiltonian:

$$H_1 = -\mu B \sigma_z \{x[l_{br}^{-1}\cos(\gamma-\delta) - l_d^{-1}\sin(\gamma+\delta)] + y[l_{br}^{-1}\sin(\gamma-\delta) - l_d^{-1}\cos(\gamma+\delta)]\}$$

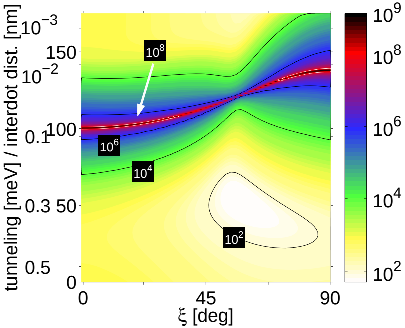
- crossing states are $\Gamma_1^{\downarrow}$ and $\Gamma_x^{\uparrow}$
  therefore only terms of x symmetry induce the anticrossing

$$\langle \Gamma_1 | H_1 | \Gamma_x \rangle \propto \int_{-\infty}^{\infty} 1 H_1(x) x \,\mathrm{d}x$$

- zero x term = no anticrossing
  100 dots orientation: $\tan\gamma = l_{BR}/l_D$
  110 dots orientation: $\gamma = 135^\circ$ (robust)
- perpendicular field?

Double dot – perpendicular magnetic field

try to remove the anticrossing – weak passage



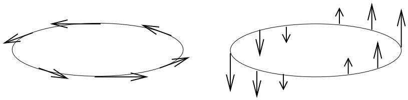
- similar effective Hamiltonian
- lower symmetry – crossing states are $\Gamma_S^\downarrow$, $\Gamma_A^\uparrow$
- therefore both x and y terms contribute
- the smaller the field the closer to previous case

Other growing directions

predictions for anisotropy

1. find out how the spin-orbit terms look like – Dresselhaus terms change

   [110] example: Dresselhaus out of plane



2. rotate spin and spatial axes
3. identify term responsible for the anticrossing
4. get conditions for this term to be zero

   [110] example: potential along crystalographic y axis, field perpendicular

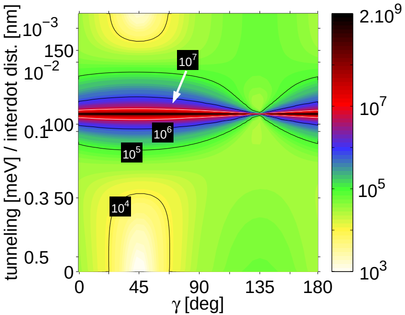
Other growing directions – results

predictions for anisotropy

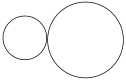
growing dir.	in-plane	general
[001]	$l_{BR}\cos(\gamma+\delta)=$ $=l_D\sin(\gamma-\delta)$	$l_D=l_{BR},\ \delta=\pi/4$
[111]	$\cos(\gamma-\delta)=0$	$2\sqrt{3}l_{BR}+l_D=0$
[110]	$\gamma=0,\ \delta=\pi/2$	$l_{BR}\cos\delta=\pm 2l_D\cot\xi,$ $\sin(\delta-\gamma)=\pm 1$
$[\cos\alpha\ \sin\alpha\ 0]$	$\delta=\pi/2,$ $l_D\tan\gamma=-l_{BR}\cos 2\alpha$	$l_D=-l_{BR}\cos 2\alpha,$ $\delta=\pi/4,\ \xi=0$

What if the potential is not precisely symmetric?

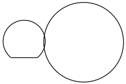
Symmetry downgrade = easy passage downgrade



- asymmetry along one axis does not harm



- *xy* asymetry does harm



- 10^3 V/m in one dot – see the figure

Summary

- Realistic calculations of spin and orbital relaxation rates in GaAs quantum dots.
- Spin relaxation in double dot dominated by spin-hot spots.
- Exploitation of the spin-hot spot influence is possible due to the potential symmetry.
- proposed geometry for spin based quantum computation: growth [001], double dot along [110], inplane magnetic field [1$\bar{1}$0]
- Outlook
    - more electron case
    - coupling to the leads