Electromechanical properties of a biphenyl transistor





- Weak coupling to the leads + low temperature -> Coulomb blockade
- Gate voltage
- Torsional degree of freedom of the molecule

The Hamiltonian

The hamiltonian of the device can be written as

$$H = H_{Mol} + H_{Leads} + V$$

Where

$$H_{Mol} = T_{\theta} + H_{PPP}(\hat{\theta})$$

$$H_{Leads} = \sum_{\alpha k\sigma} \epsilon_{k,\alpha} c^{\dagger}_{\alpha k\sigma} c_{\alpha k\sigma}$$

$$V = t \sum_{\alpha k \sigma} (c^{\dagger}_{\alpha k \sigma} c_{\alpha 3 \sigma} + c^{\dagger}_{\alpha 3 \sigma} c_{\alpha k \sigma})$$

Isolated biphenyl

H_{PPP} is the Pariser-Parr-Pople hamiltonian

$$\begin{aligned} H_{PPP}(\theta) &= \sum_{i\sigma} b_{i\,i+1}(\theta) (c_{i\sigma}^{\dagger} c_{i+1\sigma} + c_{i+1\sigma}^{\dagger} c_{i\sigma}) + \sum_{i} U_{i}(\theta) \left(\hat{n}_{i\uparrow} - \frac{z_{i}}{2} \right) \left(\hat{n}_{i\downarrow} - \frac{z_{i}}{2} \right) \\ &+ \sum_{\langle i < j \rangle} V_{ij}(\theta) (\hat{n}_{i} - z_{i}) (\hat{n}_{j} - z_{j}) \end{aligned}$$

+ H-H steric repulsion (Lehnard-Jones potential)



Potential surfaces (I)



•ground state electronic energy

•neutral biphenyl molecule

•Hartree-Fock approximation of the PPP Hamiltonian

Potential surfaces (II)

The Hartree-Fock approximation fails for the anionic state CORRELATIONS!!!

(work in progress to go beyond Hartree-Fock)



Model ... of the model



- Harmonic approximation for the anionic state
- Low lying states of a double (harmonic) well potential for the neutral state
- The model is identifyed by three parameters:

$$\alpha = \sqrt{\frac{\omega_1}{\omega_0}} \qquad \lambda = \frac{\Delta\theta}{\sqrt{\theta_{z0}\theta_{z1}}} \qquad \epsilon_a = \frac{EA}{\hbar\sqrt{\omega_0\omega_1}}$$

Definition of Unity

• We write the previous approximation in terms of unity operator:

$$\mathbf{1} = |N\rangle\langle N|(\mathcal{P}_{+} + \mathcal{P}_{-}) + |N+1\rangle\langle N+1|\mathcal{P}_{0}$$

• where $\mathcal{P}_{\pm} = \sum_{n=0}^{N_{\pm}} |n_{\pm}\rangle \langle n_{\pm}|$, $\mathcal{P}_{0} = \sum_{n=0}^{\infty} |n_{0}\rangle \langle n_{0}|$

that define the effective Hilbert space.

Effective Hamiltonian

$$\begin{split} H_{eff} &= \sum_{\alpha k\sigma} \epsilon_{k,\alpha} c^{\dagger}_{\alpha k\sigma} c_{\alpha k\sigma} + \\ &+ \sum_{\alpha k\sigma,n,m} \left[t^{(+)}_{n,m} c^{\dagger}_{\alpha k\sigma} c_{d\sigma} |n_{+}\rangle \langle m_{0}| + t^{(-)}_{n,m} c^{\dagger}_{\alpha k\sigma} c_{d\sigma} |n_{-}\rangle \langle m_{0}| + t^{(+)}_{n,m} c^{\dagger}_{d\sigma} c_{\alpha k\sigma} |m_{0}\rangle \langle n_{+}| + t^{(-)}_{n,m} c^{\dagger}_{d\sigma} c_{\alpha k\sigma} |m_{0}\rangle \langle n_{-}| \right] + \\ &+ \left(1 - \sum_{\sigma} c^{\dagger}_{d\sigma} c_{d\sigma} \right) \left\{ \epsilon_{0} + \hbar \omega_{0} \left[\mathcal{P}_{+} \left(d^{\dagger}_{+} d_{+} + \frac{1}{2} \right) \mathcal{P}_{+} + \mathcal{P}_{-} \left(d^{\dagger}_{-} d_{-} + \frac{1}{2} \right) \mathcal{P}_{-} \right] \right\} + \\ &+ \sum_{\sigma} c^{\dagger}_{d\sigma} c_{d\sigma} \left[\epsilon_{1} + \mathcal{P}_{0} \left(d^{\dagger} d + \frac{1}{2} \right) \mathcal{P}_{0} \right] \end{split}$$

where

$$\begin{split} c_{d\sigma}^{\dagger}|N\rangle &\equiv |N+1,\sigma\rangle \\ t_{n,m}^{(\pm)} &= t \langle N|c_{3\alpha}|N+1 \rangle \langle n_{\pm}|m_{0}\rangle \\ \hline \end{split} \qquad \begin{array}{l} \text{Franck-Condon} \\ \text{coefficient} \end{array} \\ \\ \text{We assume it independent of } \theta \end{split}$$

Generalized Master Equation (I)

The density matrix evolves according to the Liouville-von Neumann equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho]$$

We define the reduced density matrix:

$$\sigma = \mathrm{Tr}_{Leads}\{\rho\}$$

To second order in the coupling to the leads and in the Markov approximation:

$$\dot{\sigma} = -\frac{i}{\hbar} [H_{Mol}, \sigma] - \frac{1}{\hbar^2} \int_0^\infty \mathrm{d}\tau \operatorname{Tr}_{Leads} \{ [V, [\tilde{V}_I(-\tau), \sigma]] \otimes \rho_{Leads} \}$$

GME (II)

• coherencies between different charge states vanish

•due to the mechanical degeneracies of the neutral state we MUST keep coherencies between displaced mechanical states

• we write the GME in the basis that diagonalize the molecule hamiltonian (Bloch-Redfield form)

$$\dot{\sigma}_{ij} = \sum_{mn} R_{ijmn} \sigma_{mn}$$

GME (III)

The elements of the Redfield tensor R_{ijmn} depend on

- Fermi factors the Pauli esclusion principle prevents some transitions to occur
- Bare tunneling rates given by the density of states in the leads times the electrical coupling leads-molecule
- Franck-Condon coefficients each state transition in the system is electromechanical

The Franck-Condon parabola



GME (IV)

$$\begin{split} \dot{\sigma}_{00}^{r\tau \ s\tau'} &= -\frac{i}{\hbar} \sum_{l\rho} \left[H_{r\tau \ l\rho}^{(0)} \sigma_{00}^{l\rho \ s\tau'} - \sigma_{00}^{r\tau \ l\rho} H_{l\rho \ s\tau'}^{(0)} \right] \qquad \dot{\sigma}_{11}^{k,k'} = \\ &- \sum_{l\rho \ \alpha} \left[W_{r\tau \ l\rho}^{\alpha} \sigma_{00}^{l\rho \ s\tau'} - \sigma_{00}^{r\tau \ l\rho} W_{l\rho \ s\tau'}^{\alpha} \right] \\ &+ \sum_{m \ m' \ \alpha} \left[R_{r\tau \ s\tau' \ m' \ m}^{\alpha} + R_{s\tau' \ r\tau \ m \ m'}^{\alpha} \right] \sigma_{11}^{m' \ m} \end{split}$$

$$\begin{split} {}^{k,k'}_{11} &= -\frac{i}{\hbar} \sum_{l} \left[H^{(1)}_{k\,l} \sigma^{l\,k'}_{11} - \sigma^{k\,l}_{11} H^{(1)}_{l\,k'} \right] \\ &- \sum_{l\,\alpha} \left[W^{\alpha}_{k\,l} \sigma^{l\,k'}_{11} - \sigma^{k\,l\rho}_{11} W^{\alpha}_{l\,k'} \right] \\ &+ \sum_{n'\tau'\,n\tau\,\alpha} \left[R^{\alpha}_{k\,k'\,n'\tau'\,n\tau} + R^{\alpha}_{k'\,k\,n\tau\,n'\tau'} \right] \sigma^{n'\tau'\,n\tau}_{00} \end{split}$$

where

$$\begin{split} W^{\alpha}_{r\tau \ l\rho} &= \sum_{m} \Gamma^{0}_{\alpha} f_{\alpha} (\epsilon_{c} + \hbar \omega_{1} (m + 1/2) - \hbar \omega_{0} (l + 1/2)) C_{r\tau \ m} C_{l\rho m} \\ R^{\alpha}_{r\tau \ s\tau' \ m' \ m} &= \frac{\Gamma^{0}_{\alpha}}{2} [1 - f_{\alpha} (\epsilon_{c} + \hbar \omega_{1} (m + 1/2) - \hbar \omega_{0} (s + 1/2))] C_{r\tau \ m'} C_{s\tau' \ m} \\ W^{\alpha}_{k \ l} &= \sum_{n\tau} \frac{\Gamma^{0}_{\alpha}}{2} [1 - f_{\alpha} (\epsilon_{c} + \hbar \omega_{1} (l + 1/2) - \hbar \omega_{0} (n + 1/2))] C_{n\tau \ k} C_{n\tau \ l} \\ R^{\alpha}_{k \ k' \ n'\tau' \ n\tau} &= \Gamma^{0}_{\alpha} f_{\alpha} (\epsilon_{c} + \hbar \omega_{1} (k + 1/2) - \hbar \omega_{0} (n + 1/2)) C_{n'\tau' \ k} C_{n\tau \ k'} \end{split}$$

The current calculation

- 1. Computation of the stationary reduced density matrix σ^{stat}
- 2. Identification of the current operators:

$$\begin{split} I = & \frac{dQ}{dt} = \sum_{m} \dot{\sigma}_{11}^{m\,m} \\ = & -\sum_{k\,l\,\alpha} \left(W_{k\,l}^{\alpha} \sigma_{11}^{l\,k} + \sigma_{11}^{k\,l} W_{l\,k}^{\alpha} \right) + \sum_{k\,n'\tau'\,n\tau\,\alpha} \left(R_{k\,k\,n'\tau'\,n\tau}^{\alpha} + R_{k\,k\,n\tau\,n'\tau'}^{\alpha} \right) \sigma_{00}^{n'\tau'\,n\tau} \\ = & -2 \mathrm{Tr}_{Mech} \{ W_{1}^{L} \sigma_{11} \} + 2 \mathrm{Tr}_{Mech} \{ W_{0}^{L} \sigma_{00} \} - 2 \mathrm{Tr}_{Mech} \{ W_{1}^{R} \sigma_{11} \} + 2 \mathrm{Tr}_{Mech} \{ W_{0}^{R} \sigma_{00} \} \\ = & \langle \hat{I}_{L} \rangle - \langle \hat{I}_{R} \rangle \end{split}$$

3. Calculation of the stationary current:

$$I_L^{stat} = \operatorname{Tr}_{Mech} \{ \sigma^{stat} \hat{I}_L \}$$
$$I_R^{stat} = \operatorname{Tr}_{Mech} \{ \sigma^{stat} \hat{I}_R \}$$

Stability diagrams

 Plot of differential conductance as a function of bias and gate voltage:

$$G = \frac{dI}{dV_b}(V_b, V_g)$$

• Helpful representation of tunneling spectroscopy:



Nanomechanical oscillations in a single-C₆₀ transistor

Hongkun Park*‡\$, Jiwoong Park†, Andrew K. L. Lim*, Erik H. Anderson‡, A. Paul Alivisatos*‡ & Paul L. McEuen†‡



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Single level



Franck-Condon parabola (II)





Preliminary results $\lambda = 1$





Preliminary results $\lambda = 2$





Preliminary results $\lambda = 3$



Franck Condon Blockade

Coherencies...examples



 $\lambda = 1 \Delta V = 5, \mu = -1$

15



Summary

- We derived an effective Hamiltonian for the biphenyl transistor
- We obtained the description of the electromechanical dynamics in terms of a GME
- We observed the Franck-Condon blockade and gave an interpretation in terms of the Franck-Condon rates
- We observed the relevance of the mechanical coherencies in the transport mechanism

Still (a lot) to do!

- Understand the anionic potential surface in terms of the PPP Hamiltonian;
- Understand the role of mechanical coherencies in transport through biphenyl;
- Use realistic values for the electron-vibron coupling in biphenyl (λ~50);
- Calculate the current noise