Electrons in one dimension: an introduction to the Tomonaga-Luttinger liquid

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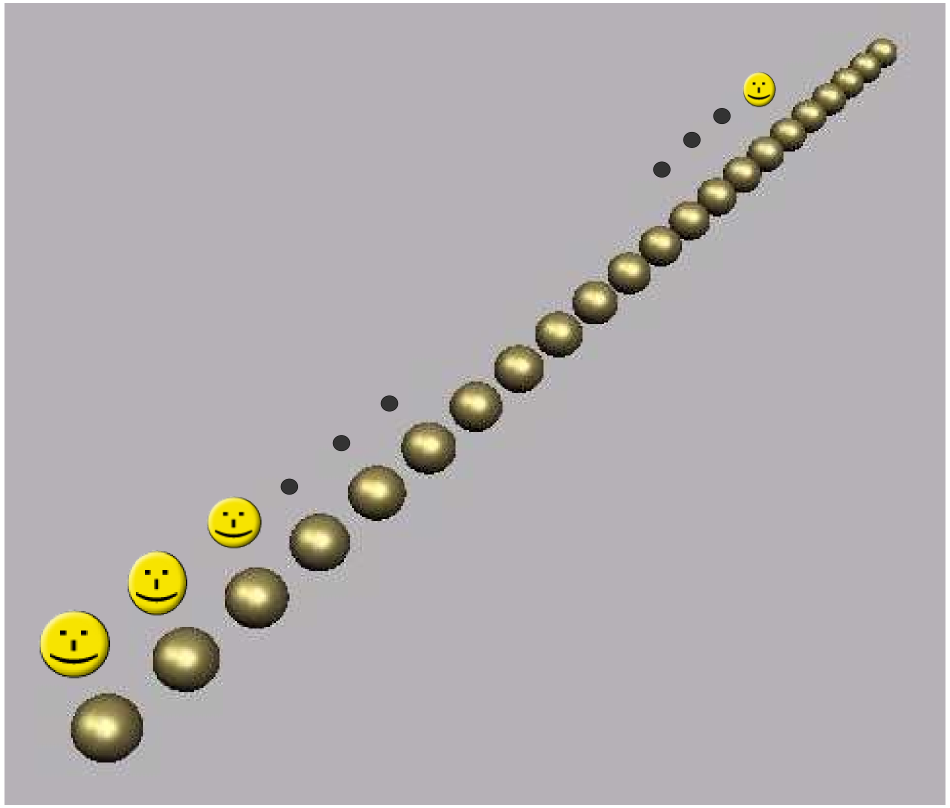


Outline

- The Fermi liquid concept
- Elementary excitations in 3D
- Breakdown of the Fermi liquid in 1D
- The Tomonaga-Luttinger liquid
- Carbon nanotubes: a Luttinger liquid?

Some references:

K. Schoenhammer, "Luttinger liquids: the basic concepts", cond-mat/0305035

A. J. Schofield, "Non-Fermi liquids", Contemporary Physics **40**, 95 (1999)

The Fermi liquid concept I

- $C_V^{el} \sim k_B T/E_F \Leftrightarrow$ classically $3k_B/2$ per electron ? $\rightsquigarrow$ Pauli exclusion principle $\rightsquigarrow$ Fermi see +Fermi surface. Only electrons with $k_B T/E_F \ll 1$ contribute (reduced phase space). Nice, but $\cdots$ why should free electron picture work for an interacting system?

- Landau (1956)$\rightsquigarrow$ concept of adiabatic continuity: turning on the interactions slowly (adiabatically) allows for a one-to-one map of low-energy excitations of the interacting system onto those of the non-interacting system.

- Excitations near E_F (quasi-particles) are labeled with the same quantum numbers as for non-interacting particles $\rightsquigarrow$ QP dynamical properties are renormalized

- Luttinger theorem: the Fermi surface is preserved after switching on the interactions (important to define the ground state of the Fermi liquid !)

The Fermi liquid concept II

- Microscopic theory : Single-particle Green functions

$$G(k,\omega) = \frac{1}{\omega - \epsilon_k - \Sigma(k,\omega)} = \frac{1}{\omega - \underbrace{(\epsilon_k + Re\,\Sigma(k,\omega))}_{E_k} - i\,Im\,\Sigma(k,\omega)}$$

- expand $\Sigma(k,\omega)$ near E_k

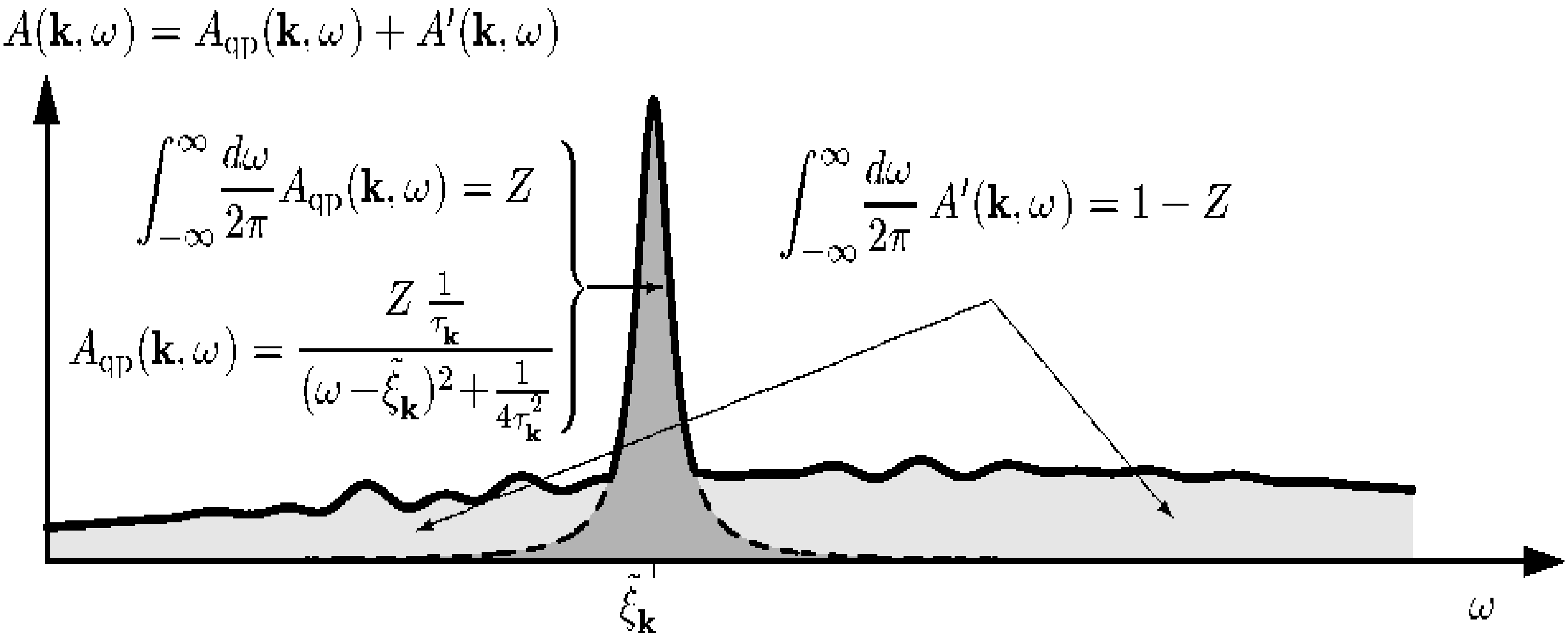
$$G(k,\omega) = Z_k \frac{1}{\omega - E_k + \dfrac{i}{2\tau_k(\omega)}}$$

$$Z_k^{-1} = 1 - \partial_\omega Re\,\Sigma(k,\omega)|_{\omega=E_k} \rightarrow \text{quasi} - \text{particle weight}$$

- Spectral density $A(k,\omega)$

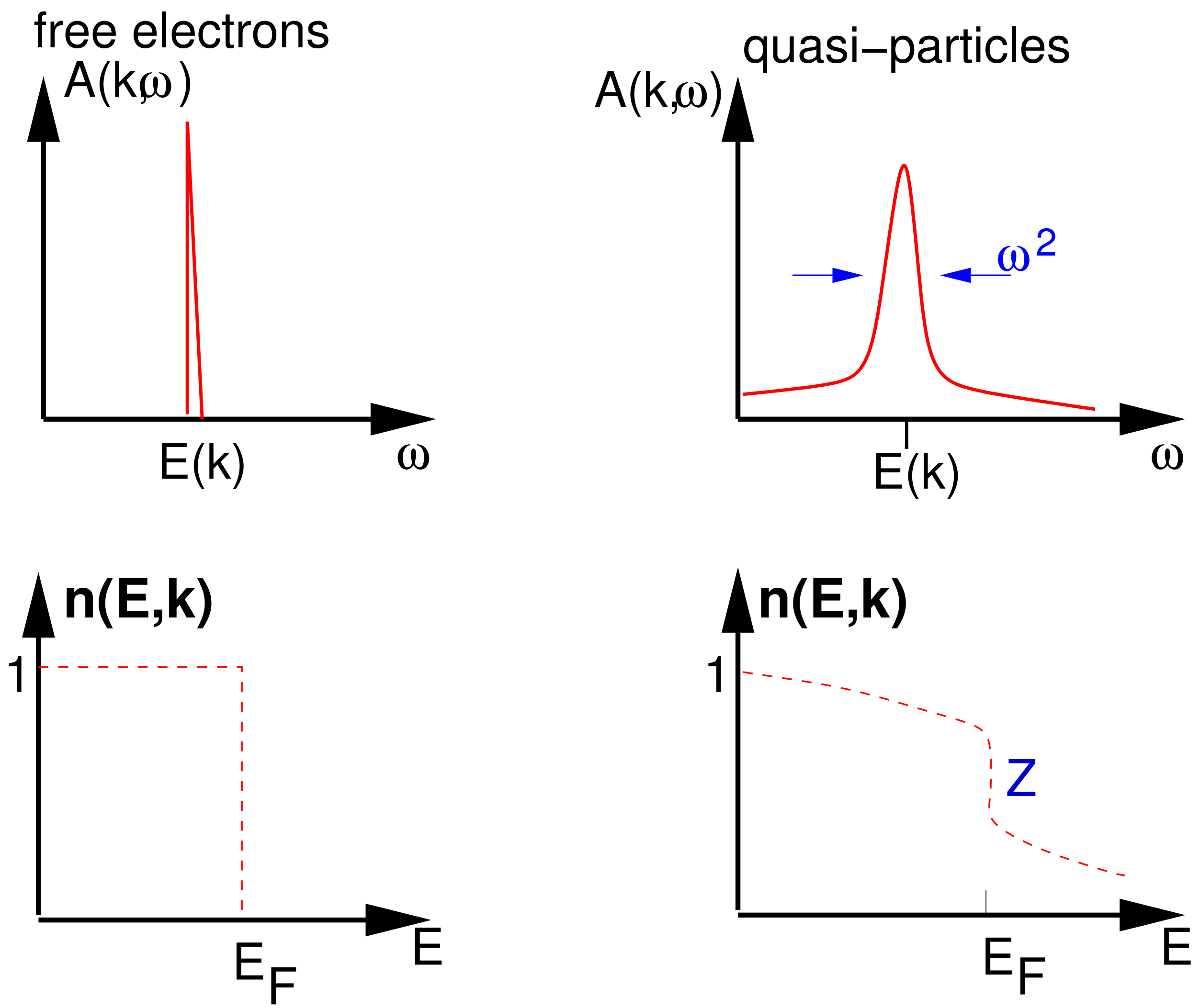
$$A(k,\omega) = -2Im\,G(k,\omega) \sim Z_k \times \text{Lorentzian}(\omega - E_k), \quad Z_k < 1$$

The Fermi liquid concept III



The Fermi liquid concept IV

Near $E_F \rightarrow v_F\,(|k| - k_F) + i\tau^{-1} sign(|k| - k_F)\,(|k| - k_F)^2$

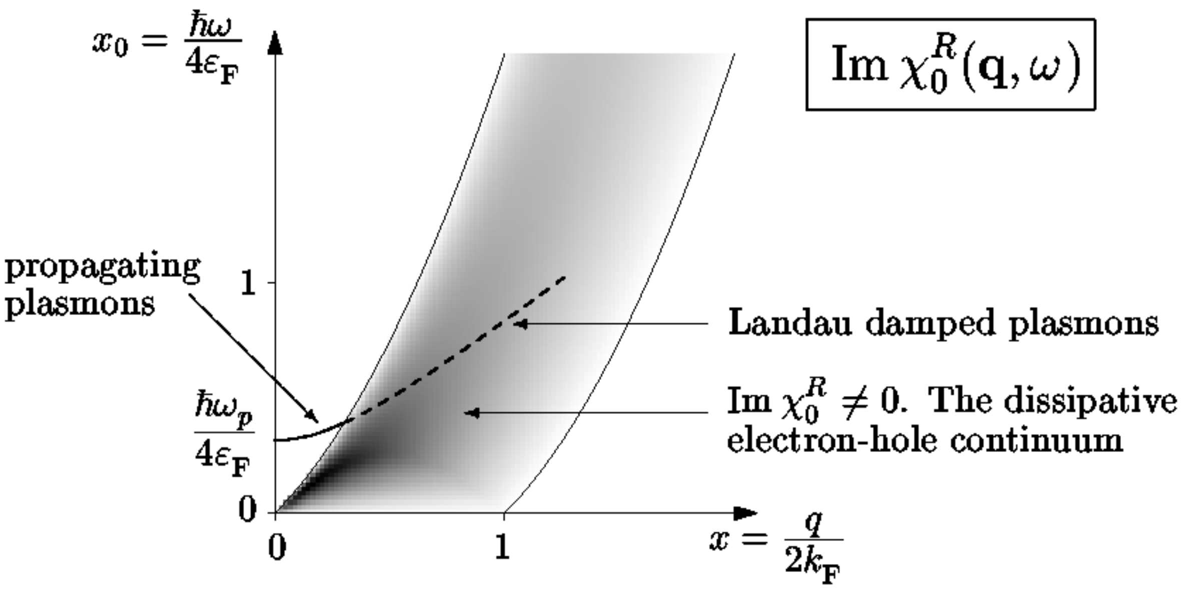


Elementary excitations in 3D electron gas

- Dynamical renormalization of Coulomb interaction $\rightarrow$ screening

$\rightsquigarrow V(q) \rightarrow V(q)/\epsilon(q,\omega)$

- Random Phase Approximation: $\epsilon_{RPA}(q,\omega) = 1 - V(q)\chi_0(q,\omega)$



Elementary excitations:

(i) Electron-hole pairs

(ii) Collective modes:

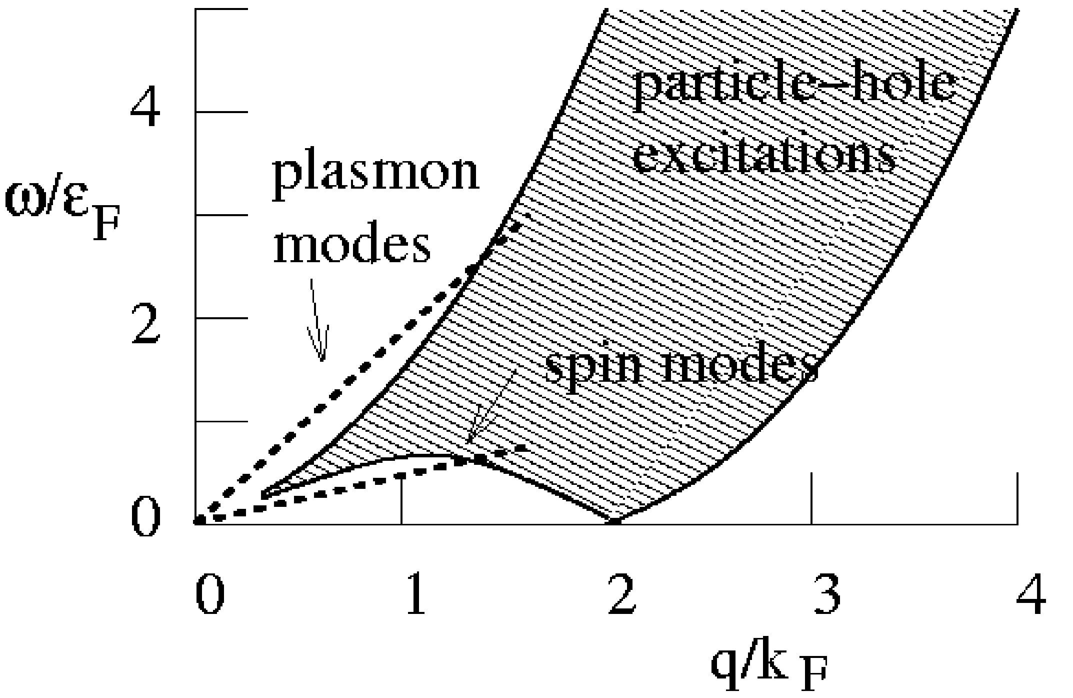
charge density fluctuations

$$\epsilon_{3D}(q,\omega) \sim 1 - (\frac{\omega_P}{\omega})^2[1 + O((\frac{qv_F}{\omega})^2)]$$

$$\omega_P \sim 10 - 15\ eV$$

$$\omega_P^2(q) = \omega_P^2 + const. \times q^2$$

Breakdown of the Fermi liquid in 1D



(i) $\omega_P(q) \sim q\, v_F$ (no gap)

(ii) QP-damping $\gamma = \tau_k^{-1} \sim |k| - k_F$

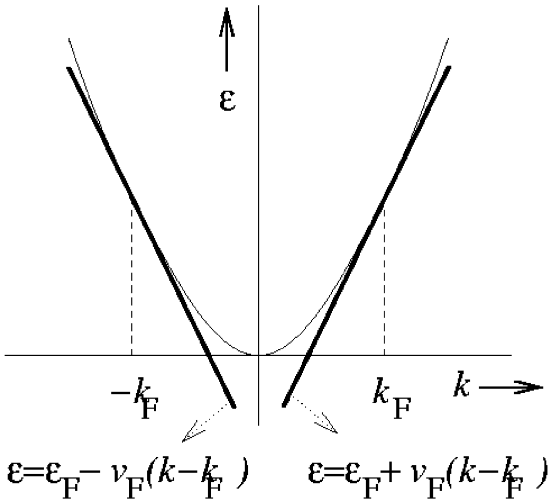
(iii) QP weight $Z_{k\sim k_F} \sim \omega - E_F$

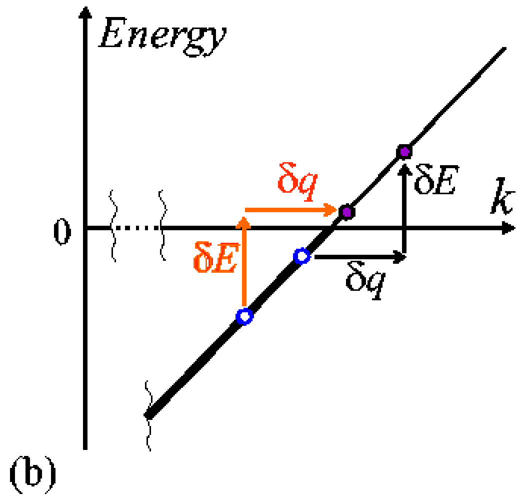
$\rightsquigarrow$ There are **NO** QP in 1D in the low-energy sector! They decay much faster than adiabatic switch-on of interactions

$\rightsquigarrow$ Density fluctuations are the relevant low-energy excitations !

Fermi liquid breaks down $\cdots\dagger$

The TLL I: Linearization





- "Fermi surface"$=\pm k_F$, linearize near k_F, i.e. $E_k = \epsilon_k - E_F \sim \hbar v_F(|k| - k_F)$
- right- (k>0) and left (k<0)-moving electrons

- Density operators as linear combinations of particle-hole excitations: $\rho_{L/R}(q) = \sum_k c^\dagger_{k,L/R} c_{k+q,L/R}$
- $\rho_{L/R}(q)$ satisfy approx. bosonic commutation relations
- Can we map (at low energies) an electronic Hamiltonian onto a bosonic one?

YES $\rightarrow$ BOSONIZATION

The TLL II: Bosonization

- Effective Hamiltonian, bilinear in the charge density

$$H_c = \frac{2\pi v_F}{\mathcal{L}} \sum_{q>0,\ell=L,R} \rho_\ell(q)\rho_\ell(-q) + \frac{V_1}{2\mathcal{L}} \sum_{q>0} [\rho_R(q) + \rho_L(q)][\rho_R(-q) + \rho_L(-q)]$$

- Express $\rho_{L,R}(q)$ through bosonic operators $\{a_q, a_q^\dagger\}$ and diagonalize the resulting H_c.

$$\begin{aligned} H_c &= \sum_q E(q) B_q^\dagger B_q \\ E(q) &= q\, v_F \sqrt{1 + \frac{V_1}{\pi v_F}} = q\, v_F K_c^{-1} = q\, v_c(>,<) v_F \end{aligned}$$

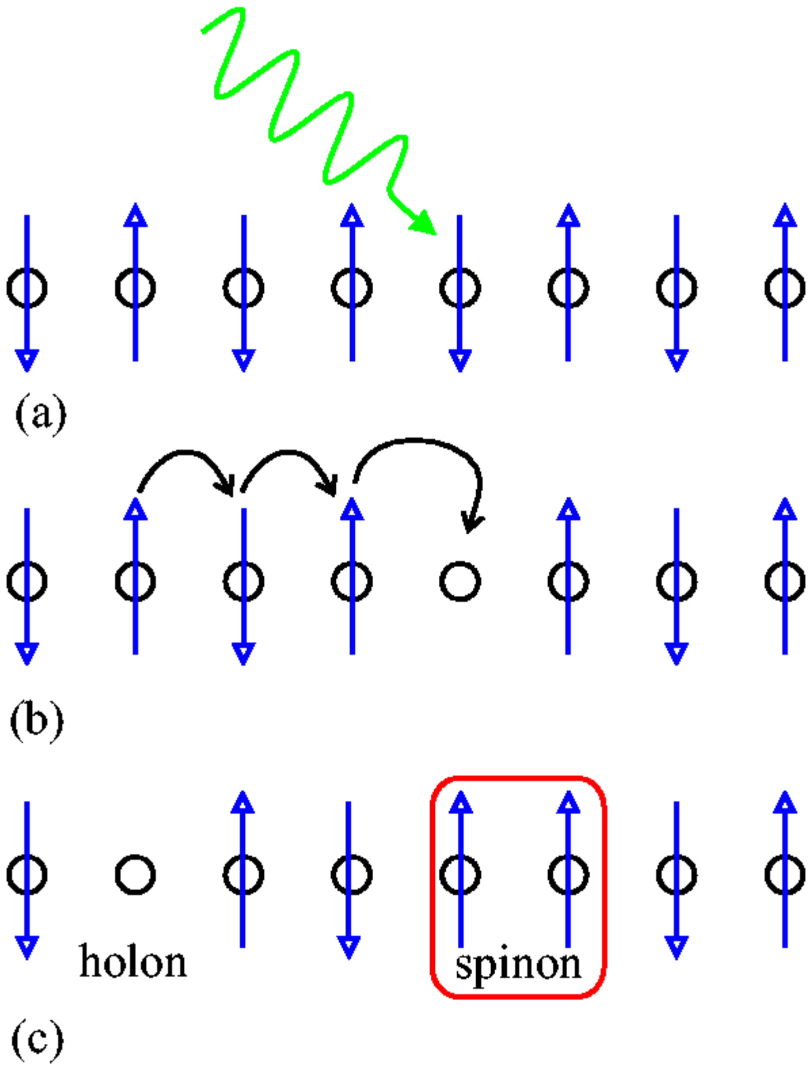
- Real space representation (Hamiltonian density of a string !)

$$H_c = \frac{v_c}{2} \int dx \, \{K_c \Pi^2(x) + K_c^{-1} (\partial \Phi(x))^2\}$$

$$\partial_t^2 \Phi(x) - v_c^2 \partial_x^2 \Phi(x) = 0 \rightsquigarrow \omega(q) = v_c\, q$$

Note !: All this is a low-energy theory, $\exists$ high-energy cut-off q_0 everywhere

The TLL III: Spin-charge separation



$$H = H_{charge}(g_c) + H_{spin}(g_s)$$

$$[H, H_c] = [H, H_s] = 0$$

$\rightsquigarrow$ H_c and H_s can be diagonalized independently

$\longrightarrow$Charge density mode with velocity v_c

$\longrightarrow$Spin density mode with velocity v_s

Depending on the strength and sign (attractive or repulsive) of the el-el interaction $\rightsquigarrow$ $v_c > v_s$ or $v_c < v_s$

The TLL IV: Scaling laws and Transport

- General feature: power-law scaling of correlation functions with non-universal exponents
- DOS: $\nu(\omega) \sim max(\omega, k_B T)^{\chi(K)-1}$
- Zero-temperature conductance G of infinite TLL $\sim K\dfrac{e^2}{h}$
- Single-impurity "cuts" the TLL in two pieces

  $\rightarrow$ for repulsive interactions (at $k_B T = 0$ and $L \rightarrow \infty$), transmission=0, reflection=1

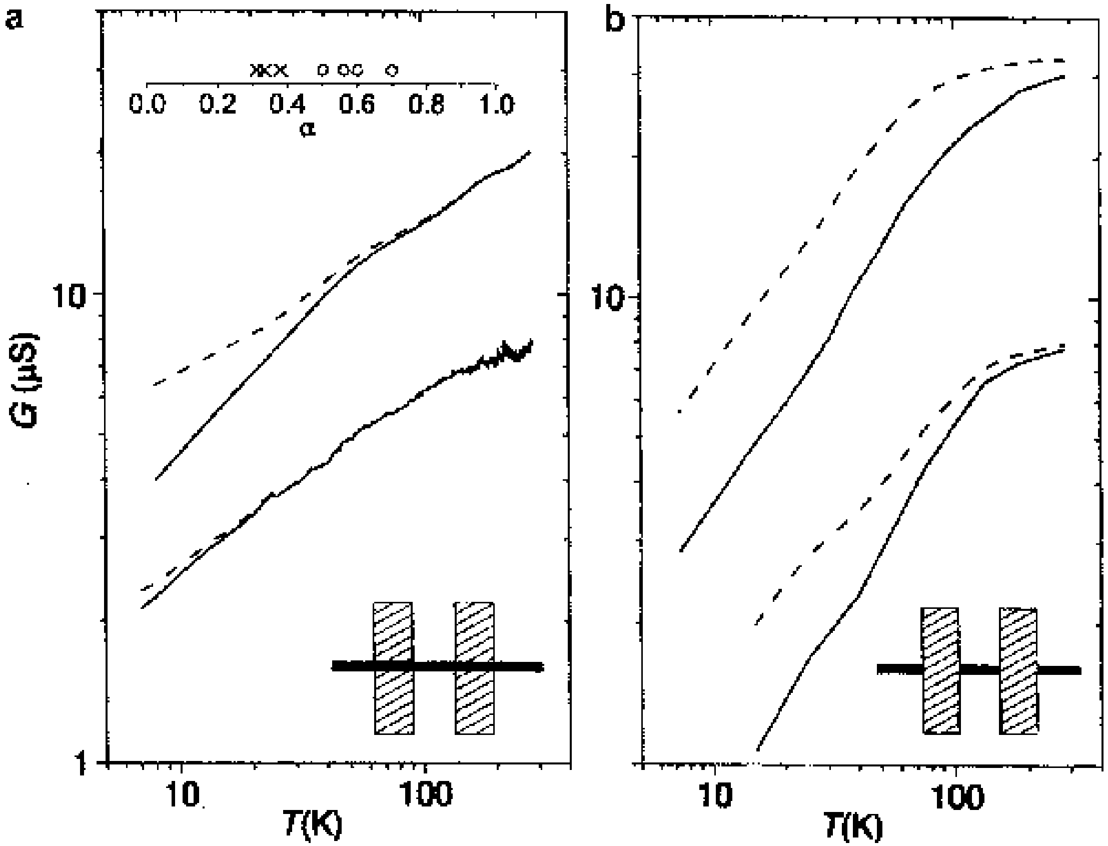
  $\rightarrow$ DOS has power-law behaviour and vanishes for $E \rightarrow E_F$
- TLL between two barriers

  $\rightsquigarrow$ infinitely narrow resonance at $k_B T = 0 \leftrightarrow$ vanishing DOS for $E \rightarrow 0$
- Tunnel experiments

$$I_{LL-LL} \sim max(eV, k_B T)^{\alpha(K)}$$

$$G \sim max(eV, k_B T)^{\alpha}$$

Carbon nanotubes: a Luttinger liquid?



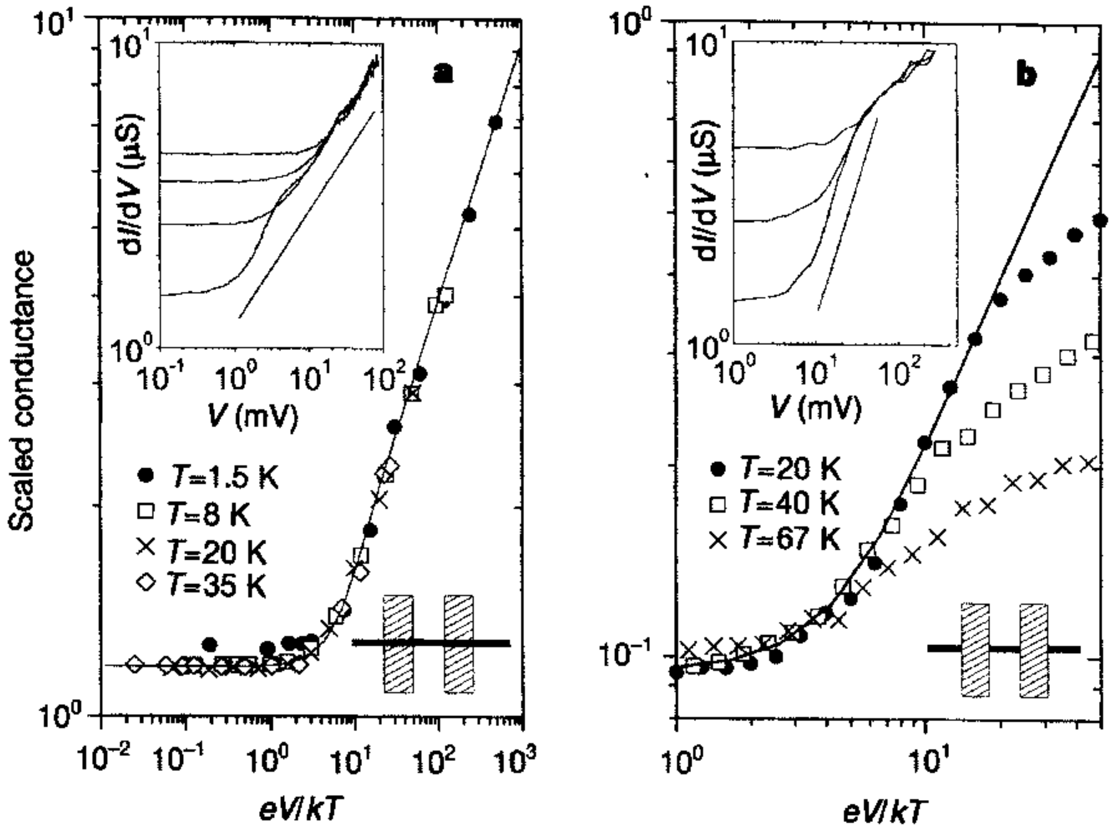
M. Bockrath *et al.*, Nature **397**, 598 (1999)

"Here we present measurements of the conductance of individual ropes of such SWNTs as a function of temperature and voltage. Power law behavior as a function of temperature or bias voltage is observed: G$\sim T^\alpha$ and dI/dV$\sim V^\alpha$. Both the power-law functional forms and the inferred exponents are in good agreement with theoretical predictions for tunneling into a LL."

$$G \sim max(eV, k_B T)^\alpha$$

$$\alpha \rightarrow \alpha_{\text{end}}(K), \alpha_{\text{bulk}}(K), \quad K \sim 0.28$$

Carbon nanotubes: a Luttinger liquid?



$$\frac{dI}{dV} \sim T^{\alpha} ch(\gamma \frac{eV}{2k_B T}) |\Gamma(\frac{1+\alpha}{2} + \gamma \frac{ieV}{2\pi k_B T})|$$

$T^{-\alpha}\frac{dI}{dV} \longrightarrow$ universal curve

Summary

- Fermi liquid is unstable in 1D, no QP at low energies $\sim$ Tomonaga-Luttinger liquid
- The TLL is characterized by:

  (i) zero excitation gap in the charge and/or spin sector

  (ii) low-energy excitations $\rightsquigarrow$ coherent fluctuations of spin and charge degrees of freedom

  (iii) spin-charge separation ($v_c \neq v_s$)

  (iv) power-law scaling in the observables (DOS, I(V), G)

- Haldane's conjecture (1980-81): Given a 1D interacting system, let there be a branch of gapless excitations. Then, the TLL is the stable low-energy fixed point of the system $\rightsquigarrow$ asymptotic low-energy properties are described by the TLL with renormalized parameters (similar to Landau's Fermi liquid).