Group seminar

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Regensburg 06.07.2005

Spin dependent transport in mesoscopic systems

Dario Bercioux

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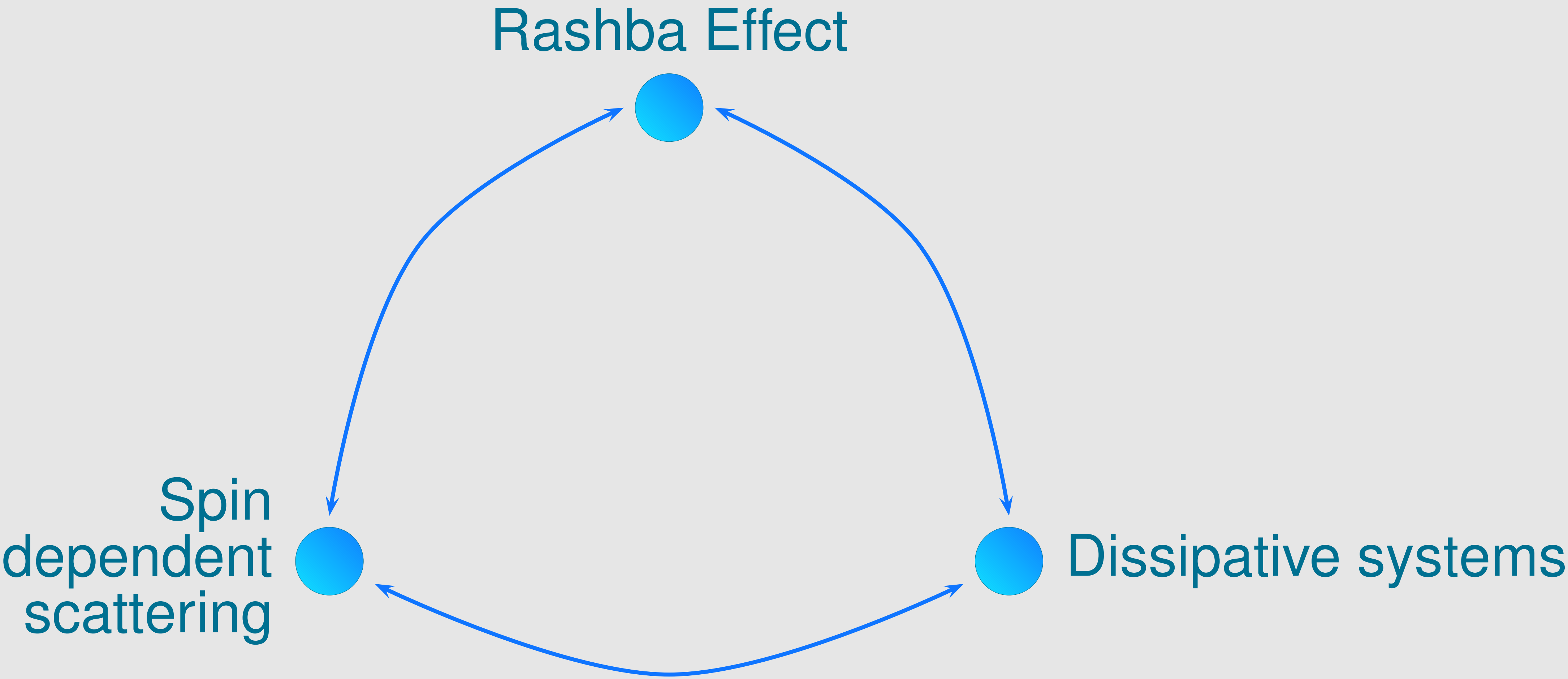
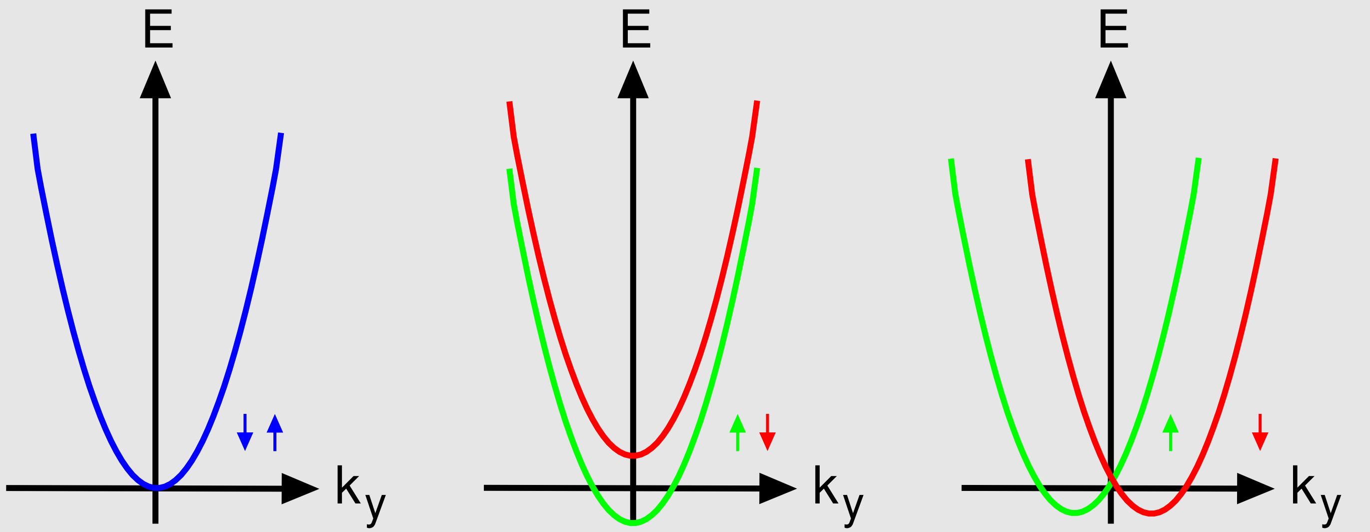


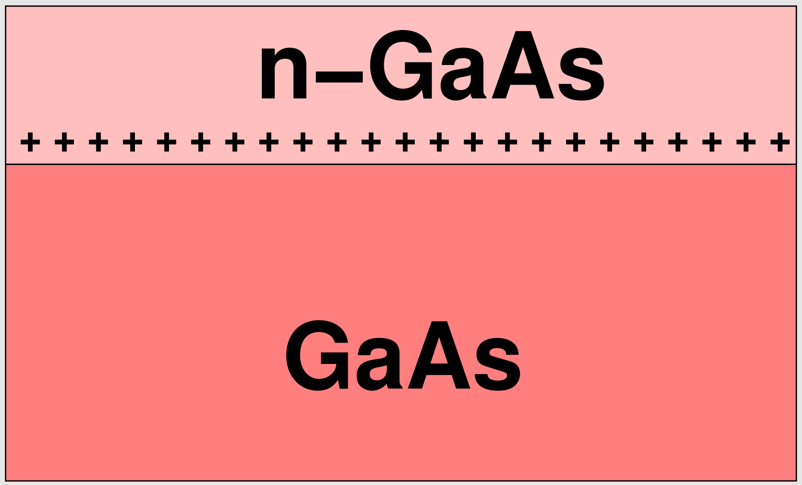
- Conclusions & Outlook

Introduction



spin degenerate **Zeeman splitting** **spin–orbit splitting**

Semiconductor Heterostructures



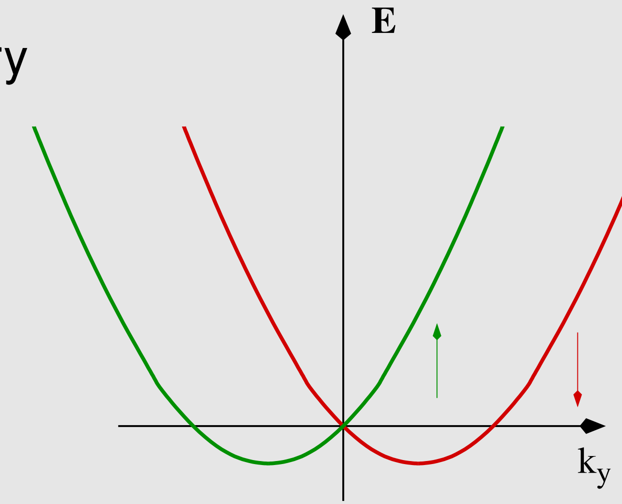
- ✓ two-dimensional electron gas (2DEG): low electron density and high mobility
- ✓ spin-orbit splitting due to bulk inversion asymmetry (BIA): *Dresselhaus effect*
- ✓ spin-orbit splitting due to structural inversion asymmetry (SIA): *Rashba effect*

Rashba spin-orbit coupling

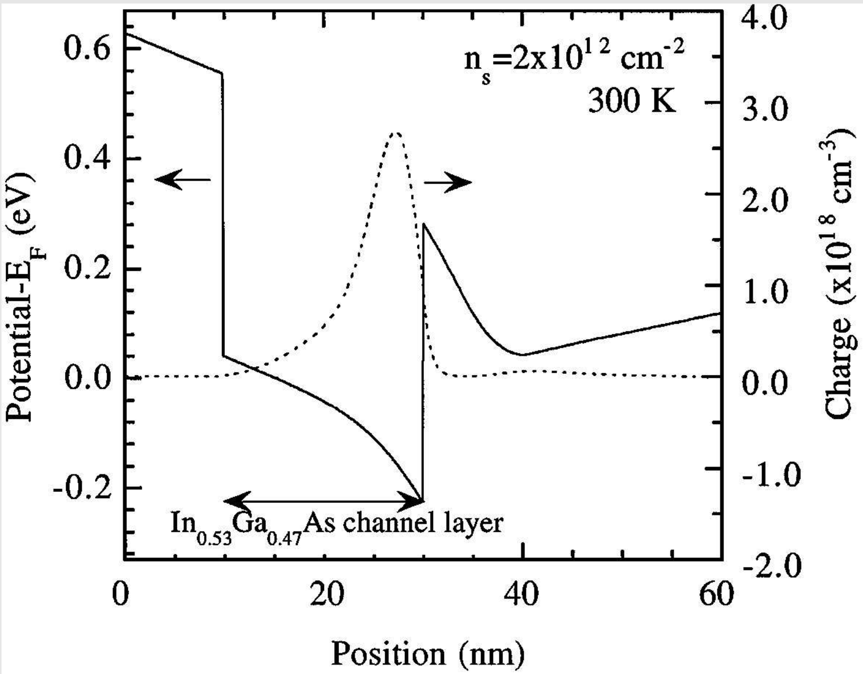
⋆ Quantum heterostructures: structural inversion asymmetry in growth direction $\hat{z}$ induces Rashba term:

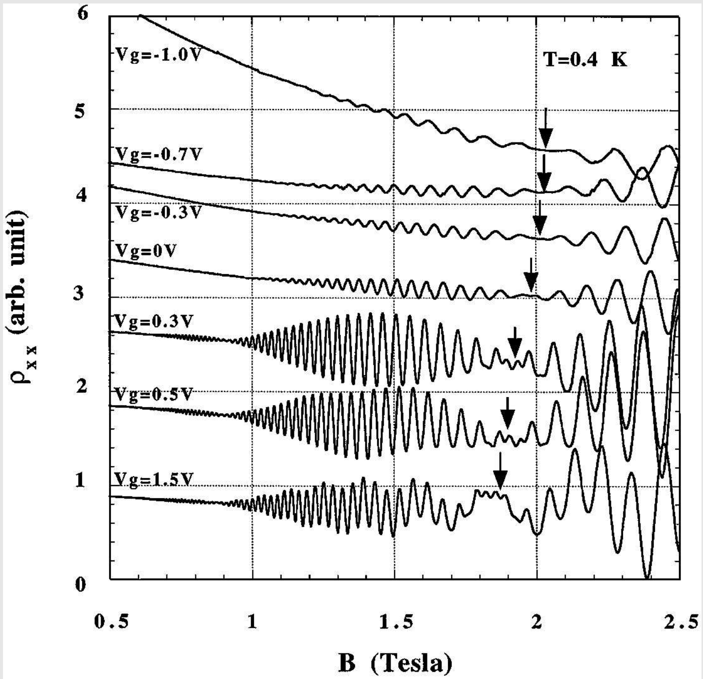
$$\mathcal{H}_{\mathrm{R}} = \frac{\hbar k_{\mathrm{SO}}}{m}\left[\vec{\sigma} \times \left(\vec{p} - \frac{e}{c}\vec{A}\right)\right] \cdot \hat{z}$$

Rashba, Fiz. Tverd. Tela **2**, 1224 (1960)
[Sov. Phys. Solid State **2**, 1109 (1960)]



⋆ Possibility of tuning k_{SO} by external gate voltages (de Andrada e Silva *et al.*, Phys. Rev. B **55**, 16293 (1997))





Nitta *et al.*, Phys. Rev. Lett. **78**, 1335 (1997)
Schäpers *et al.*, J. Appl. Phys. **83**, 4324 (1998)

Grundler, Phys. Rev. Lett. **84**, 6074 (2000)

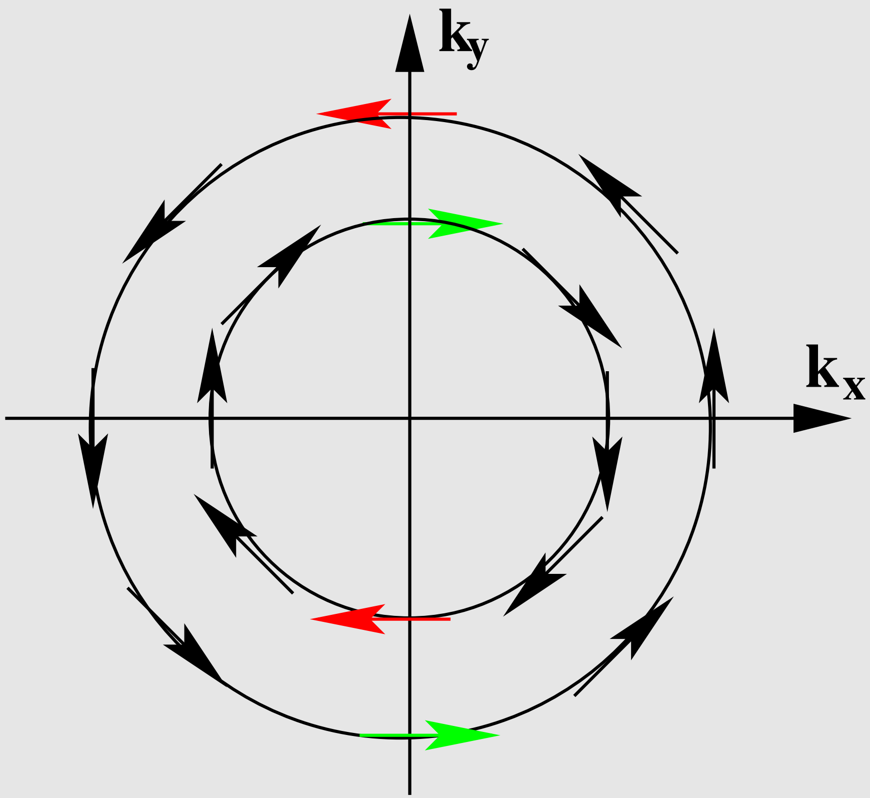
Rashba effect: an overview

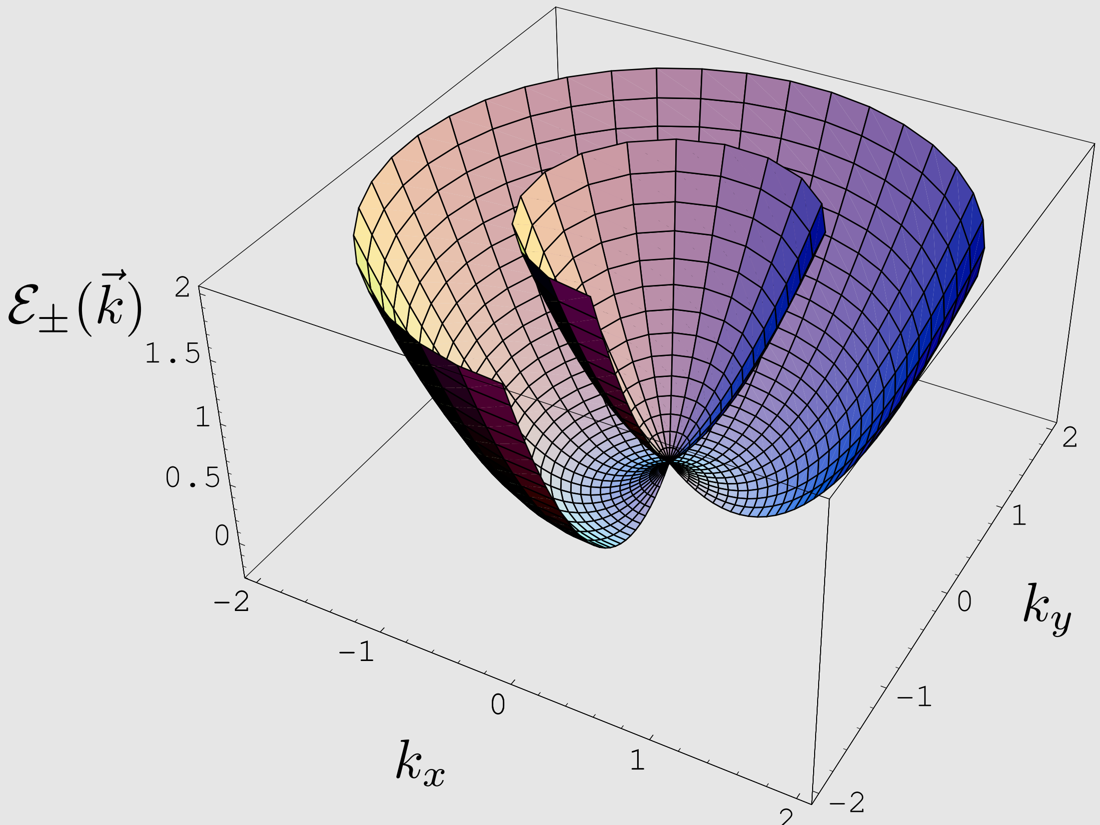
Hamiltonian of a two-dimensional gas (2DEG) with Rashba spin-orbit coupling

$$\mathcal{H} = \frac{p^2}{2m} + \frac{\hbar k_{\mathrm{SO}}}{m}(\sigma_x p_y - \sigma_y p_x) \quad (1)$$

Spectrum $$\mathcal{E}_\pm(k) = \frac{\hbar^2}{2m}(k \pm k_{\mathrm{SO}})^2 - \Delta_{\mathrm{SO}} \quad (2)$$

Wavefunctions $$\Psi_\pm(x,y) = e^{i\vec{k}\cdot\vec{r}}\begin{pmatrix}1\\ \pm i e^{-i\theta}\end{pmatrix} \quad \text{with} \quad \theta = \arctan\left(\frac{k_y}{k_x}\right) \quad (3)$$





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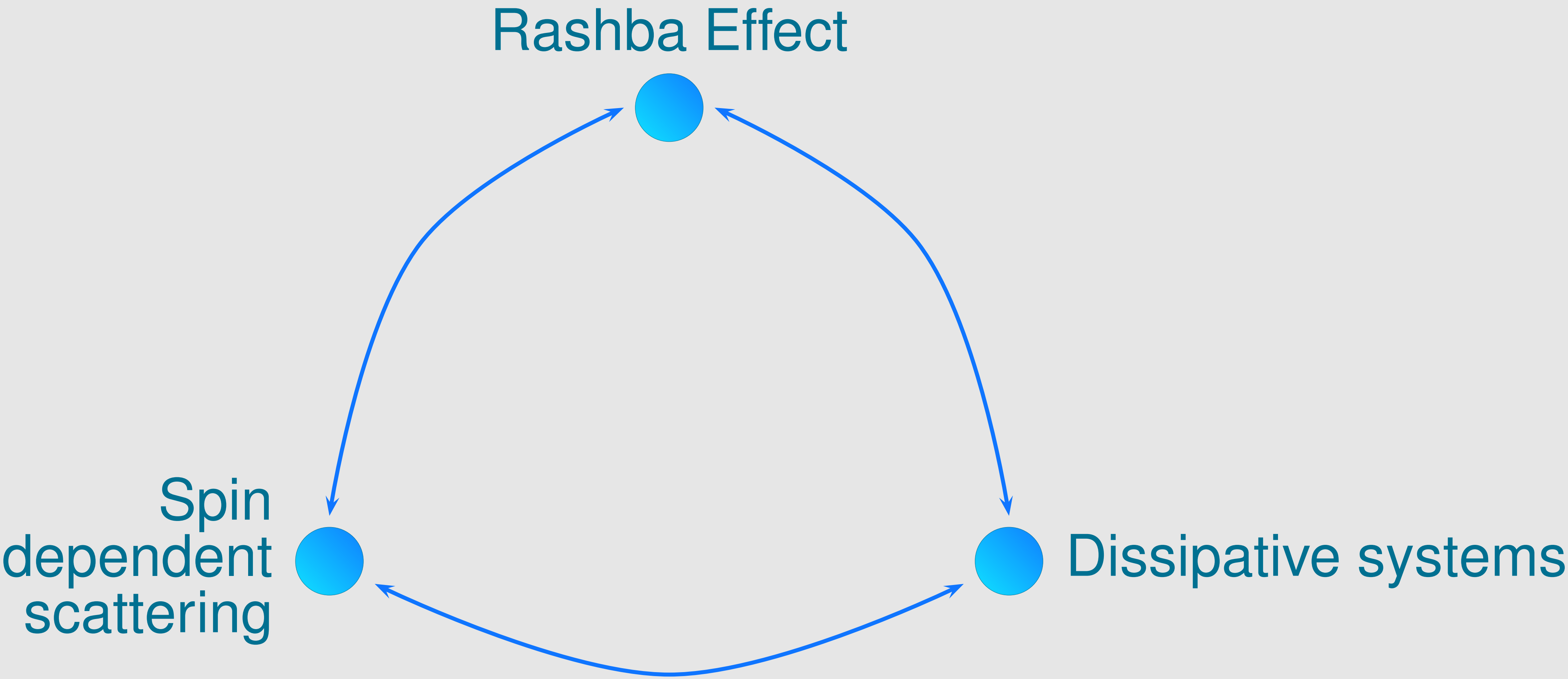
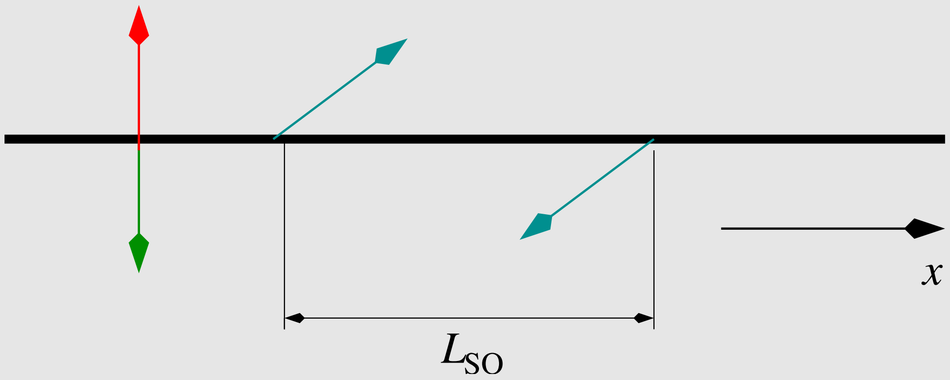


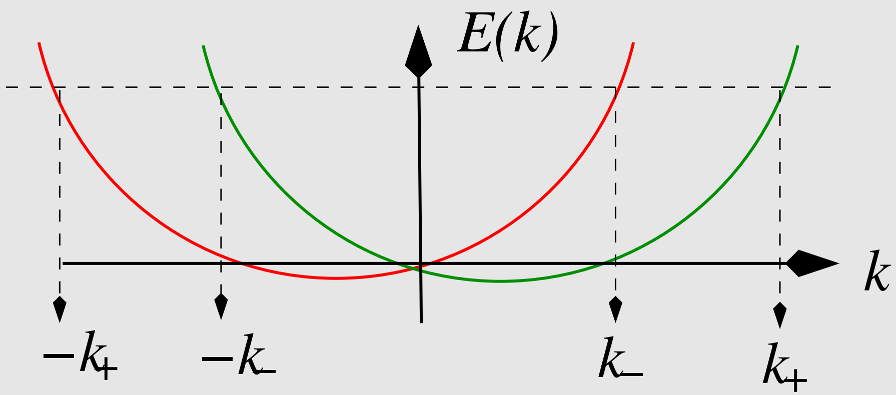
Conclusions & Outlook

Rashba effect in quantum wires (QWs) (I)



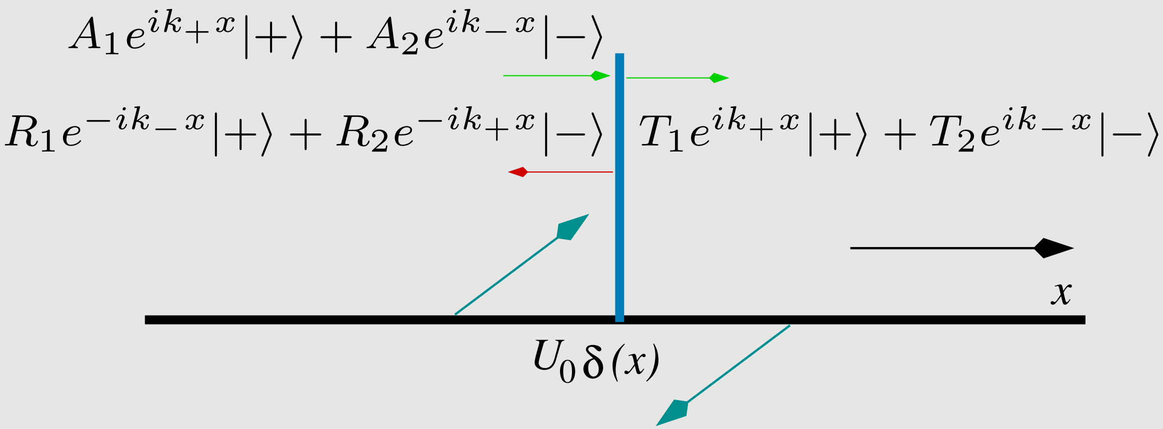
$$\mathcal{H} = \frac{p_x^2}{2m} - \frac{\hbar k_{\text{SO}}}{m}\sigma_y p_x$$

$$L_{\text{SO}} = \frac{\pi}{k_{\text{SO}}}$$



$$\bar{E} = \frac{\hbar^2}{2m}\bar{k}^2 = \frac{\hbar^2}{2m}(k \pm k_{\text{SO}})^2$$

$k_+ = \bar{k} - k_{\text{SO}}$ and $k_- = \bar{k} + k_{\text{SO}}$

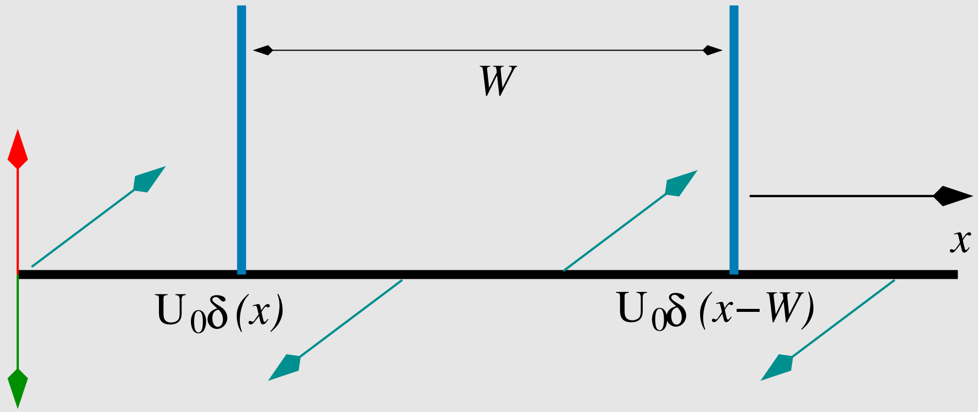


$$T_\pm(\bar{k}) = \left|\frac{i(k_+ + k_-)}{\Delta + i(k_+ + k_-)}\right|^2 = \left|\frac{2i\bar{k}}{\Delta + 2i\bar{k}}\right|^2$$

$$R_\pm(\bar{k}) = \left|\frac{\Delta}{\Delta + i(k_+ + k_-)}\right|^2 = \left|\frac{\Delta}{\Delta + 2i\bar{k}}\right|^2$$

$\Delta = \dfrac{2mU_0}{\hbar^2}$ and $k_+ + k_- = 2\bar{k}$

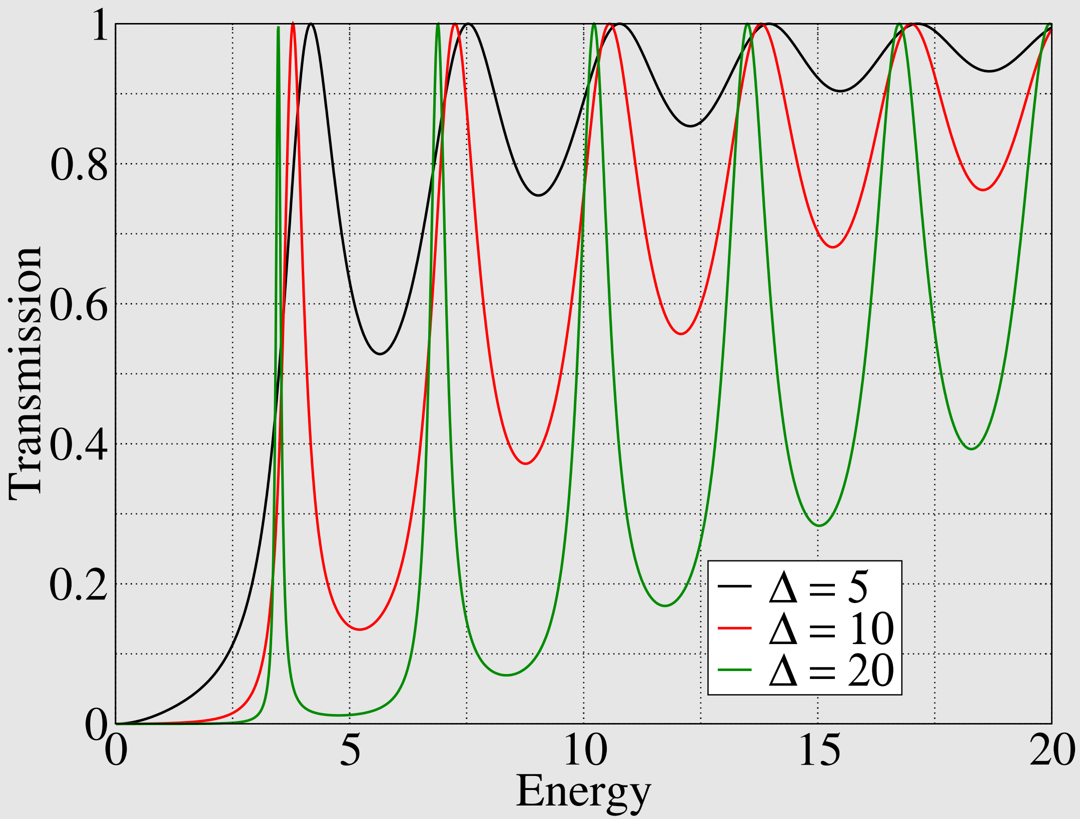
Rashba effect in quantum wires (II)



$$\mathcal{H} = \frac{p_x^2}{2m} - \frac{\hbar k_{\mathsf{SO}}}{m}\sigma_y p_x + U_0[\delta(x)+\delta(x-W)]$$

$$L_{\mathsf{SO}} = \frac{\pi}{k_{\mathsf{SO}}} \quad \Leftrightarrow \quad W$$

The total transfer matrix is $\mathbf{M}_{\mathsf{T}} = \mathbf{M}_{\mathsf{L}} \cdot \begin{pmatrix} e^{-ik_+W} & 0 & 0 & 0 \\ 0 & e^{-ik_-W} & 0 & 0 \\ 0 & 0 & e^{ik_+W} & 0 & 0 \\ 0 & 0 & 0 & e^{ik_+W} \end{pmatrix} \cdot \mathbf{M}_{\mathsf{R}}$



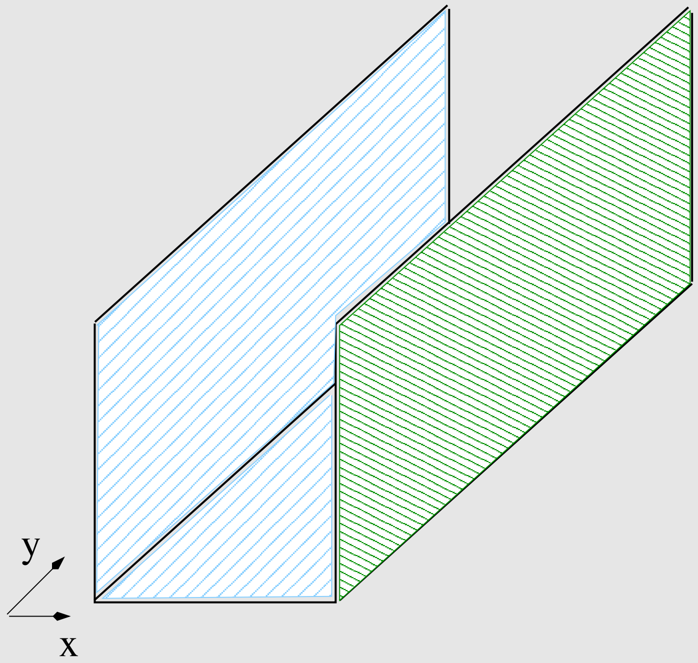
The total transmission is

$$T_{\mathsf{T}} = \frac{T_1^2}{1 + R_1^2 + 2F(k)T_1^2}$$

with

$$F(k) = \Re[M_{\mathsf{L}}^{(11)} M_{\mathsf{R}}^{(11)} M_{\mathsf{L}}^{(13)*} M_{\mathsf{R}}^{(31)*} e^{i(k_+ + k_-)W}]$$

Rashba effect in quantum wires (III)



The Hamiltonian of the system is

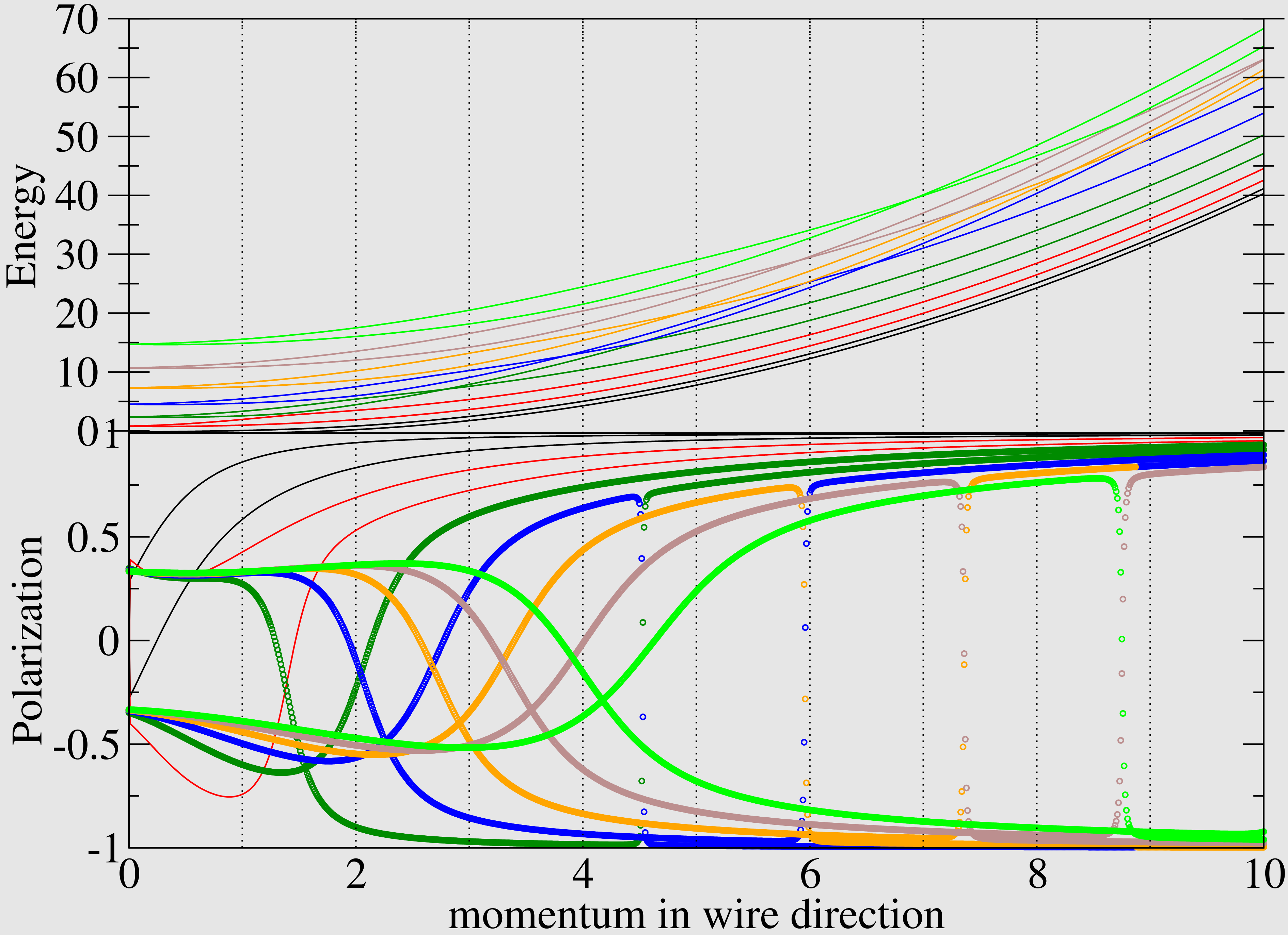
$$\mathcal{H} = \frac{p^2}{2m} + V(x) + \frac{\hbar k_{\mathrm{SO}}}{m}(\sigma_x p_y - \sigma_y p_x) \qquad (4)$$

This can be diagonalized in a reduced Hilbert space to take account of the subband hybridization due to $-\sigma_y p_x$.

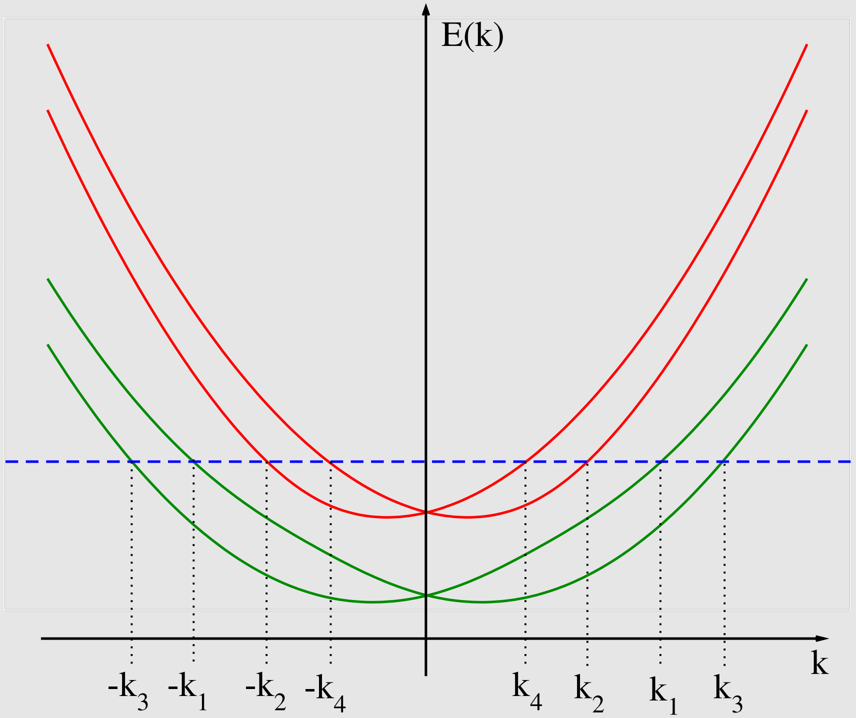
$$\begin{pmatrix} E_n^0 - E & \frac{\hbar^2 k_{\mathrm{SO}}}{m} & 0 & -\frac{\hbar^2 k_{\mathrm{SO}}}{m}\Delta_{\mathrm{SO}} \\ \frac{\hbar^2 k_{\mathrm{SO}}}{m} & E_n^0 - E & \frac{\hbar^2 k_{\mathrm{SO}}}{m}\Delta_{\mathrm{SO}} & 0 \\ 0 & \frac{\hbar^2 k_{\mathrm{SO}}}{m}\Delta_{\mathrm{SO}} & E_{n+1}^0 - E & \frac{\hbar^2 k_{\mathrm{SO}}}{m} \\ -\frac{\hbar^2 k_{\mathrm{SO}}}{m}\Delta_{\mathrm{SO}} & 0 & \frac{\hbar^2 k_{\mathrm{SO}}}{m} & E_{n+1}^0 - E \end{pmatrix} \begin{pmatrix} a_{n\uparrow} \\ a_{n\downarrow} \\ a_{n+1\uparrow} \\ a_{n+1\downarrow} \end{pmatrix} = 0$$

F. Mireles and G. Kirczenow, Phys. Rev. B **64**, 24426 (2001)

Rashba effect in quantum wires (IV)



Rashba effect in quantum wires (V)



The Hamiltonian of the system is

$$\mathcal{H} = \frac{p^2}{2m} + V(x) + \frac{\hbar k_{\text{SO}}}{m}(\sigma_x p_y - \sigma_y p_x) + U_0\delta(y)$$

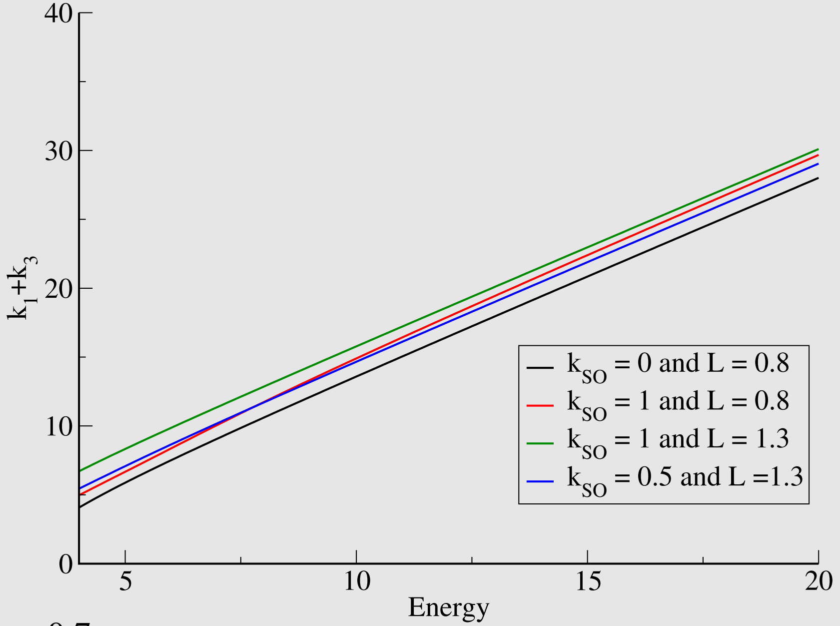
The transmission function is evaluated in the truncated subspace containing the first two subbands.

$$E_{1,2,3,4}(k) = \frac{E_1 + E_2}{2} \pm \frac{1}{2}\sqrt{\left[(E_2 - E_1) \pm 2\frac{\hbar^2 k_{\text{SO}}}{m}k\right]^2 - \left(\frac{2\hbar^2 k_{\text{SO}}}{m}\Delta_{\text{SO}}\right)^2}$$

with $\Delta_{\text{SO}} = \langle 1|p_x|2\rangle$ subbands mixining term.
The trasmission probabilities are

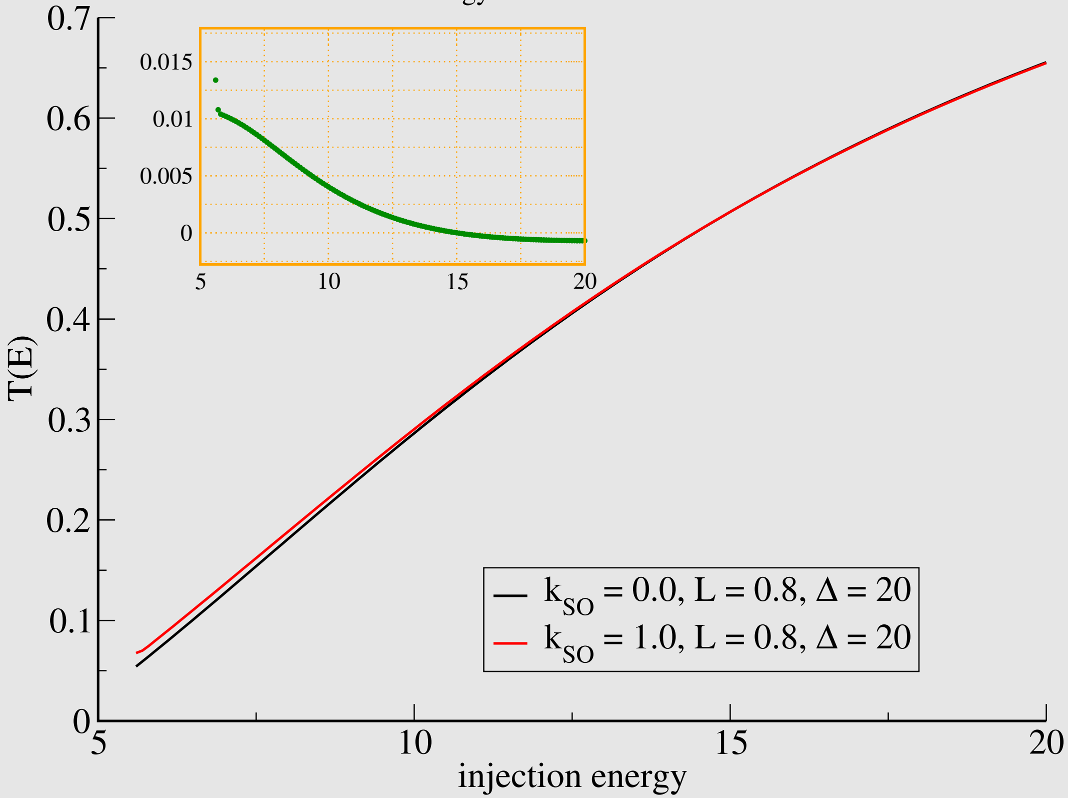
$$T_{\text{lower}}(E) = \left|\frac{i(k_1 + k_3)}{\Delta + i(k_1 + k_3)}\right|^2 \quad \text{and} \quad T_{\text{upper}}(E) = \left|\frac{i(k_2 + k_4)}{\Delta + i(k_2 + k_4)}\right|^2.$$

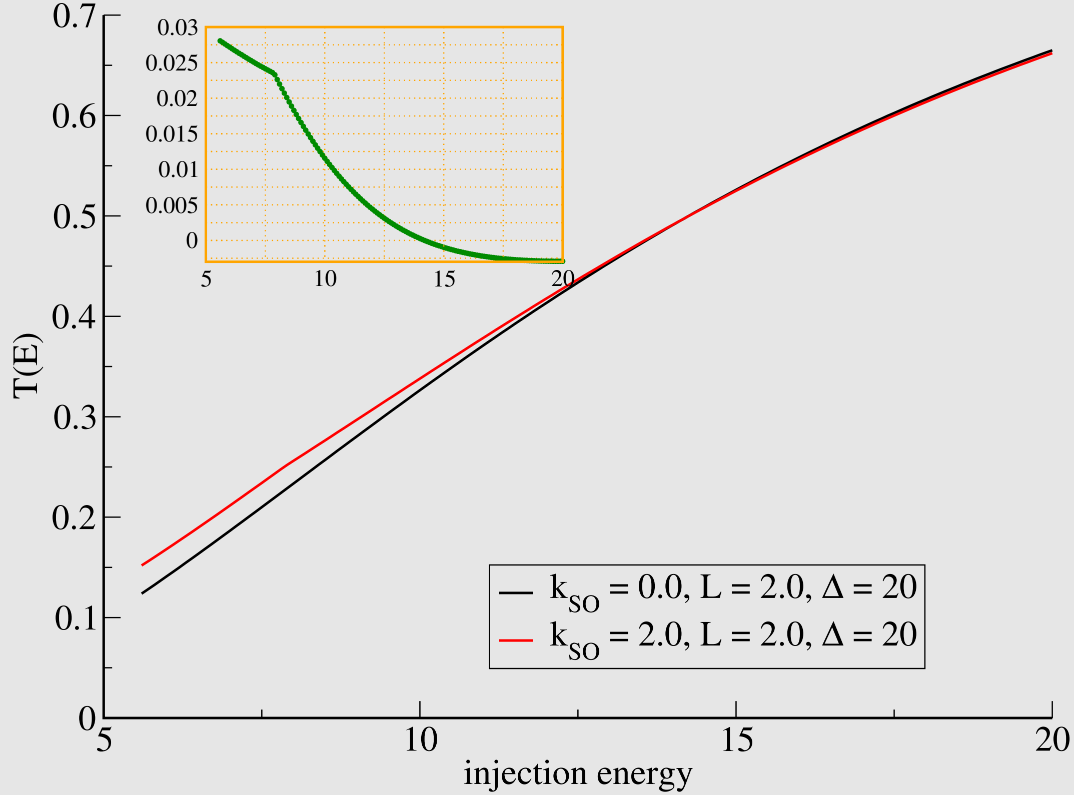
Rashba effect in quantum wires (VI)



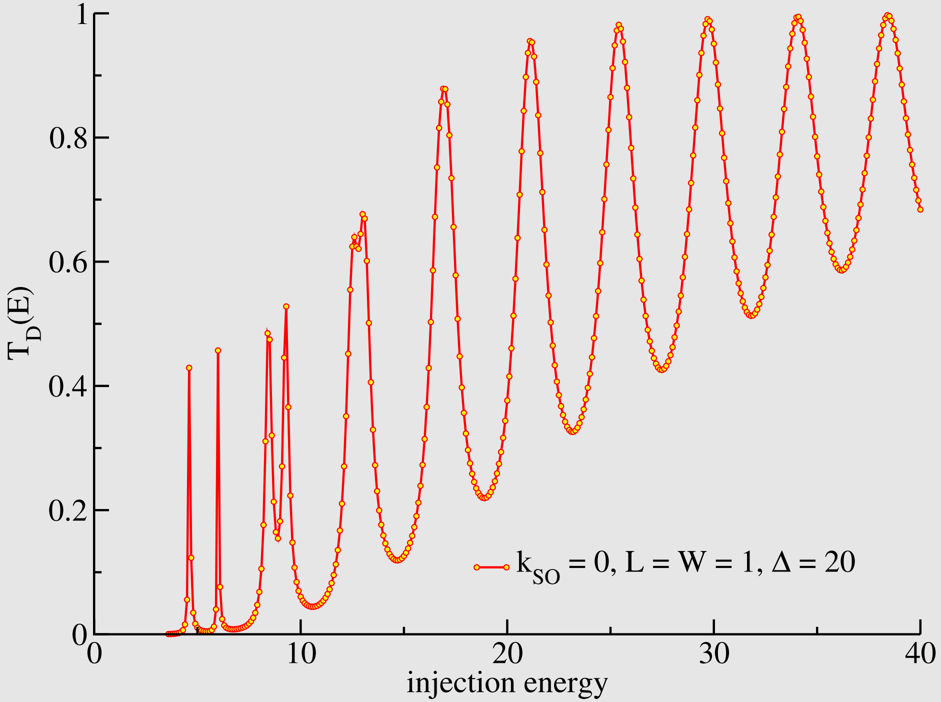
$k_1 + k_3$ and $k_2 + k_4$ now depend on the spin-orbit coupling and on the wire width.

The difference is bigger to low energy.





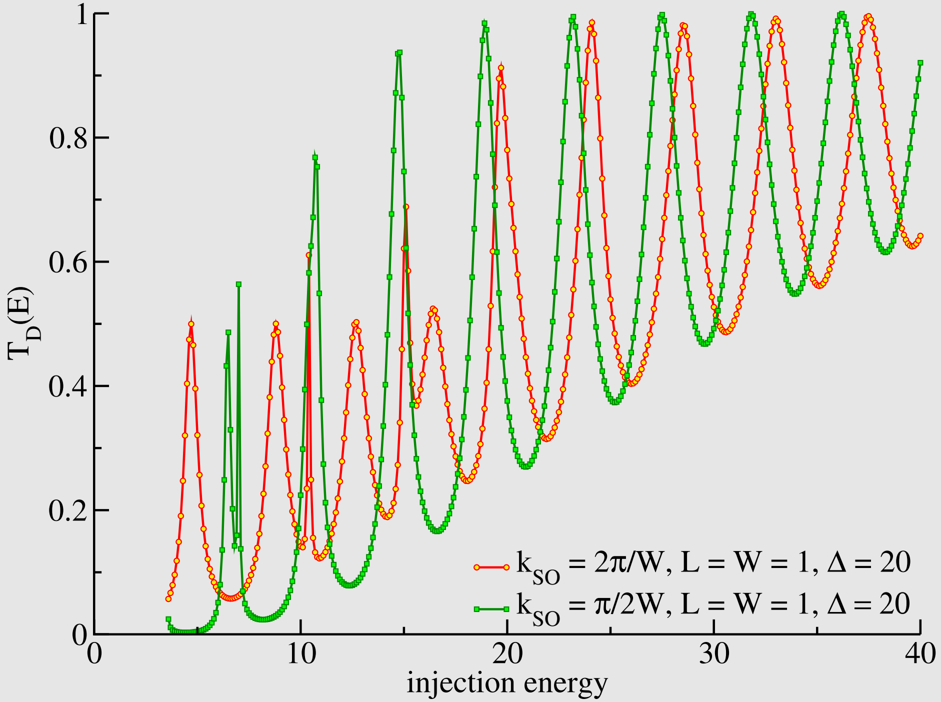
Rashba effect in quantum wires (VII)

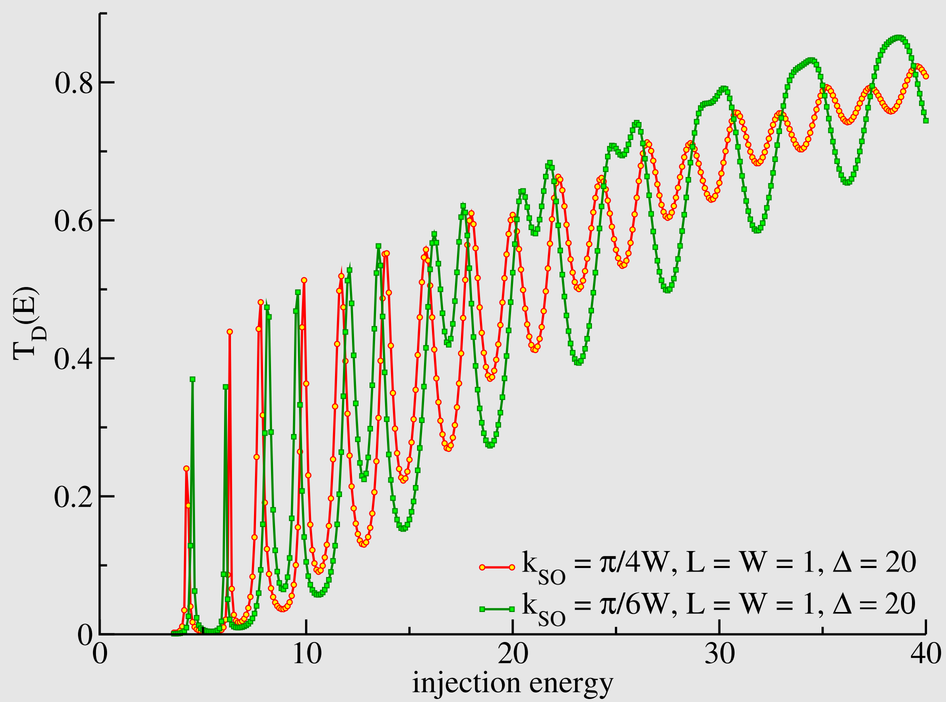


In the case of the double barrier we combine the effects due to the spin-orbit dependent scattering plus spin-precession plus subbands hybridization.

$$T_{\mathsf{T}} = \frac{T_1^2(k_{13})}{1 + R_1^2(k_{13}) + 2F(k_{13})T_1^2(k_{13})} + \dots$$

$$\dots + \frac{T_2^2(k_{24})}{1 + R_2^2(k_{24}) + 2F(k_{24})T_2^2(k_{24})}$$





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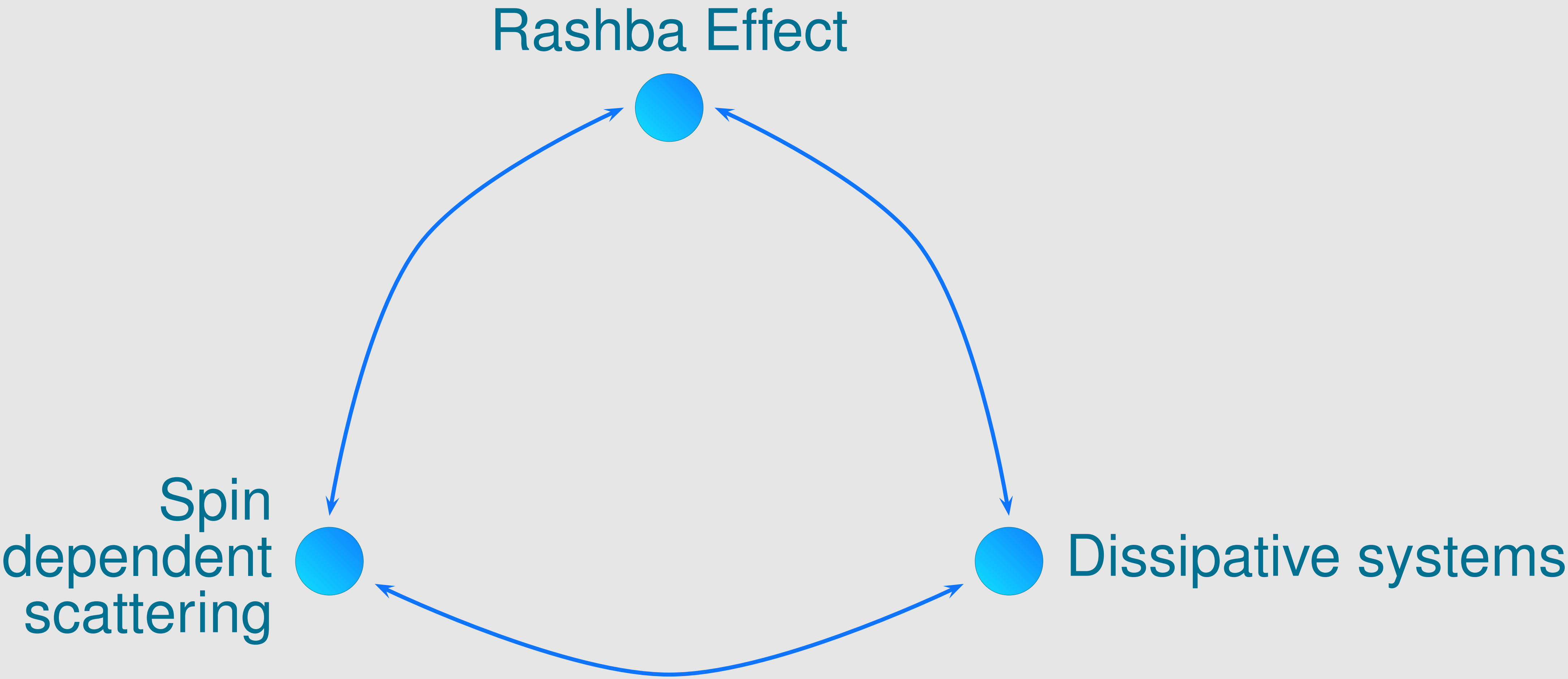
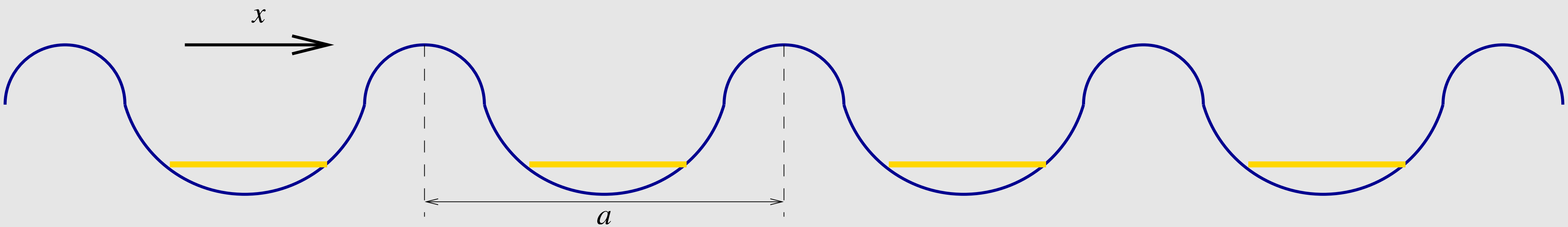


Conclusions & Outlook

1D periodic potential with Rashba effect



The Hamiltonian of the periodic system is

$$\mathcal{H} = \left(\frac{p_x^2}{2m} + U(x) \right) \otimes \mathbb{I}_2 + \frac{k_{\mathsf{SO}}\hbar}{m} \sigma_y p_x \quad \text{with} \quad U(x) = U(x+a)$$

In tight-binding this can be written

Space representation in σ_y eigenstates

$$\mathcal{H}_{\mathsf{tb}} = \sum_{\substack{j=-\infty \\ \chi=\{+,-\}}}^{+\infty} \left[\Delta_\chi |\chi, j\rangle\langle\chi, j+1| + \Delta_\chi^* |\chi, j+1\rangle\langle\chi, j| \right] \quad \begin{cases} \Delta_+ = t_0 - \mathrm{i} t_{\mathsf{SO}} \\ \Delta_- = t_0 + \mathrm{i} t_{\mathsf{SO}} \end{cases}$$

Space representation in σ_z eigenstates

$$\mathcal{H}_{\mathsf{tb}} = \sum_{\substack{\langle i,j\rangle \\ \sigma,\sigma'}} |i,\sigma\rangle \left(t_0 \mathbb{I}_2 + \mathrm{i}\sigma_y t_{\mathsf{SO}} \right)_{\sigma,\sigma'} \langle j,\sigma'| + hc \quad \begin{cases} t_0 = \Delta \cos(k_{\mathsf{SO}} a) \\ t_{\mathsf{SO}} = \Delta \sin(k_{\mathsf{SO}} a) \end{cases}$$

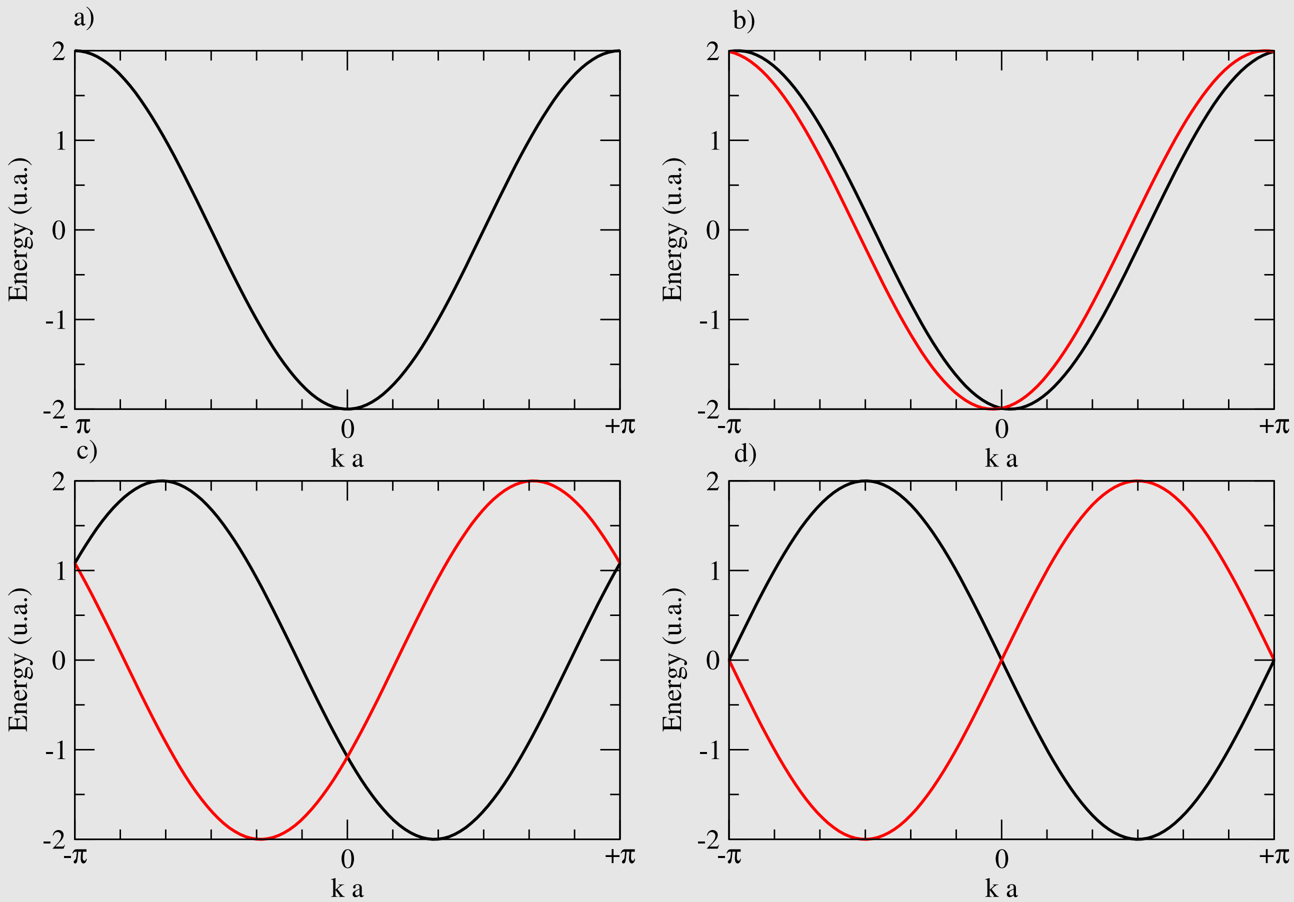
1D periodic potential with Rashba effect (II)

Momentum representation

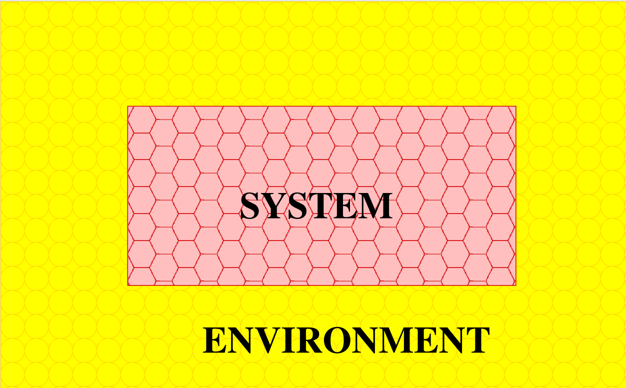
$$\mathcal{H}_{\text{tb}} = \sum_k \mathcal{E}_+(k)|k,+\rangle\langle k,+| + \mathcal{E}_-(k)|k,-\rangle\langle k,-|$$

with spectrum

$$\mathcal{E}_\pm(k) = 2\cos(ka)t_0 \pm 2\sin(ka)t_{\text{SO}}$$



1D periodic potential with Rashba effect (III)



The environment is represented by a bath of harmonic excitation above a stable ground state. The coupling with the system is due to a *linear* function of the bath coordinates.

The Hamiltonian of the global system is

$$\mathcal{H} = \mathcal{H}_0 + \frac{1}{2}\sum_{\alpha=1}^{N}\left[\frac{p_\alpha^2}{2m_\alpha} + m_\alpha\omega_\alpha^2\left(x_\alpha - \frac{c_\alpha}{m_\alpha\omega_\alpha^2}q\right)\right]$$

The environmental is characterized by its spectral function

$$J(\omega) = \frac{\pi}{2}\sum_\alpha \frac{c_\alpha^2}{m_\alpha\omega_\alpha}\delta(\omega-\omega_\alpha) \propto \eta_s\omega_{\text{ph}}^{1-s}\omega^s e^{-\omega/\omega_c} \quad \begin{cases} 0 < s < 1 & \textit{sub-Ohmic case} \\ s = 1 & \textit{Ohmic case} \\ s > 1 & \textit{super-Ohmic case} \end{cases}$$

U. Weiss, *Quantum dissipative systems*, World Scientific

1D periodic potential with Rashba effect (IV)

It is important to study the operator coupling the system to the environment

$$\hat{q} = \sum_{j=-\infty}^{+\infty} ja\left(|j,+\rangle\langle j,+| + |j,-\rangle\langle j,-|\right)$$

The particle is in $|\chi, n = 0\rangle$ at $t_0 = 0$, $P_{n\chi}(t)$ is the probability of finding the particle at site n with spin χ at time t.

- Position's expectation value

$$P(t) = \langle \hat{q} \rangle = a \sum_n n\left(P_{n+}(t) + P_{n-}(t)\right) = \langle \hat{q}_+(t) \rangle + \langle \hat{q}_-(t) \rangle$$

- Quantum charge current

$$J_{\text{charge}} = e \lim_{t\to\infty} \frac{d}{dt} \langle \hat{q} \rangle = e \lim_{t\to\infty} \frac{d}{dt} \left(\langle \hat{q}_+(t) \rangle + \langle \hat{q}_-(t) \rangle\right)$$

- Quantum spin current

$$J_{\text{spin}} = e \lim_{t\to\infty} \frac{d}{dt} \left(\langle \hat{q}_+(t) \rangle - \langle \hat{q}_-(t) \rangle\right)$$

1D periodic potential with Rashba effect (V)

☛ Diffusive variance

$$S(t) = \langle \hat{q}^2 \rangle - \langle \hat{q} \rangle^2 = a^2 \sum_n n^2 \left(P_{n+}(t) + P_{n-}(t) \right) - P(t)^2$$

☛ Quantum diffusion coefficient

$$D = \frac{1}{2} \lim_{t \to \infty} a^2 \frac{d}{dt} \sum_n n^2 \left(P_{n+}(t) + P_{n-}(t) \right) - P(t)^2$$

☛ The populations $P_{n\chi}$ are the diagonal elements of the reduced density matrix (RDM). Using the Feynman-Vernon method they can be written as a double path integral for the propagator function

$$P_{n,\chi}(t) = \rho(nn, \chi\chi, t; 00, \chi\chi, 0) = \int \mathcal{D}q \sum_{\sigma} \int \mathcal{D}q' \sum_{\sigma'} \mathcal{A}[q\sigma] \mathcal{A}^*[q'\sigma'] \mathcal{F}[q, \sigma, q', \sigma']$$

$\mathcal{A}[q\sigma]$ is the propability amplitude to go from $(q = 0, \sigma)$ to $(q = na, \sigma)$ along $q_\sigma(t')$. The environment is in the influence integral $\mathcal{F}[q, \sigma, q', \sigma']$.

1D periodic potential with Rashba effect (VI)

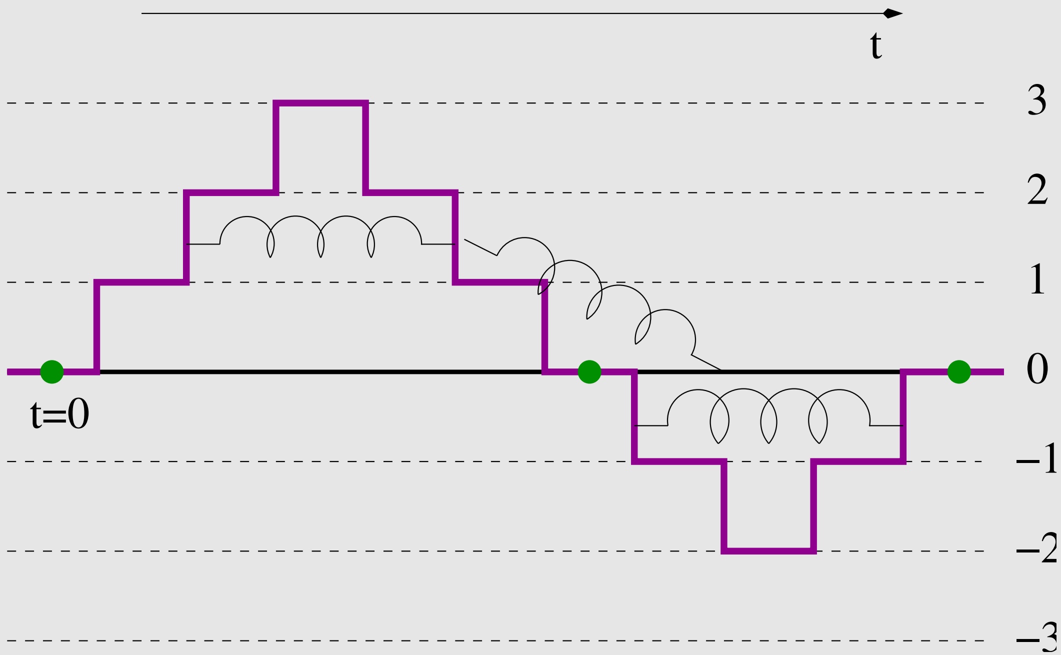
For each spin carrier we can define a double path in the RDM

$$q_\sigma^{(k)}(\tau) = a \sum_{j=1}^{k} u_{j,\sigma} \Theta(\tau - t_j) \quad q_\sigma'^{(l)}(\tau) = a \sum_{i=1}^{l} v_{i,\sigma} \Theta(\tau - t_i')$$

where $u_{j,\sigma} = \pm 1$ and $v_{i,\sigma} = \pm 1$. For double path from $(0,0)$ to (n,n) there is the constraint

$$\sum_{j=1}^{k} u_{j,\sigma} = n \quad \sum_{k=1}^{l} v_{i,\sigma} = n$$

☛ Nonintercating-cluster approximation (NICA)



R. Egger, C.H. Mak, and U. Weiss, Phys. Rev. E **50**, R655 (1994)

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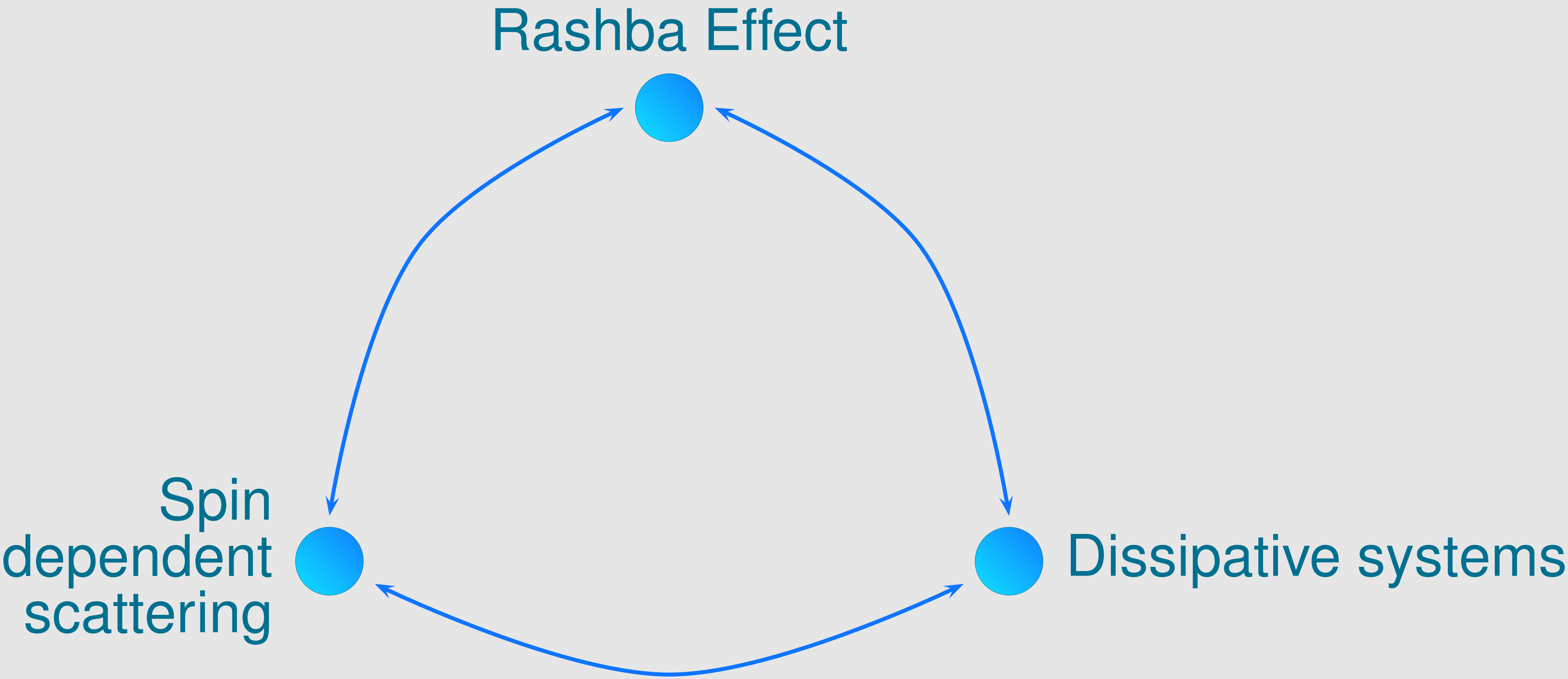

Conclusions & Outlook

Conclusions and Outlook

☛ In 1D quantum system: Rashba effect gives rise only to spin precession.

☛ In quasi-1D quantum system: polarization due to Rashba effect is dependent by the subband.

☛ In quasi-1D quantum system: Rashba effect gives rise to spin dependent quantum tunneling.

☛ In the simples tight binding model for 1D Rashba Hamiltonian the dynamics for the two spin branches are independent.

☛ ☛ To finish calculation for the single band model and move to multibands model.