

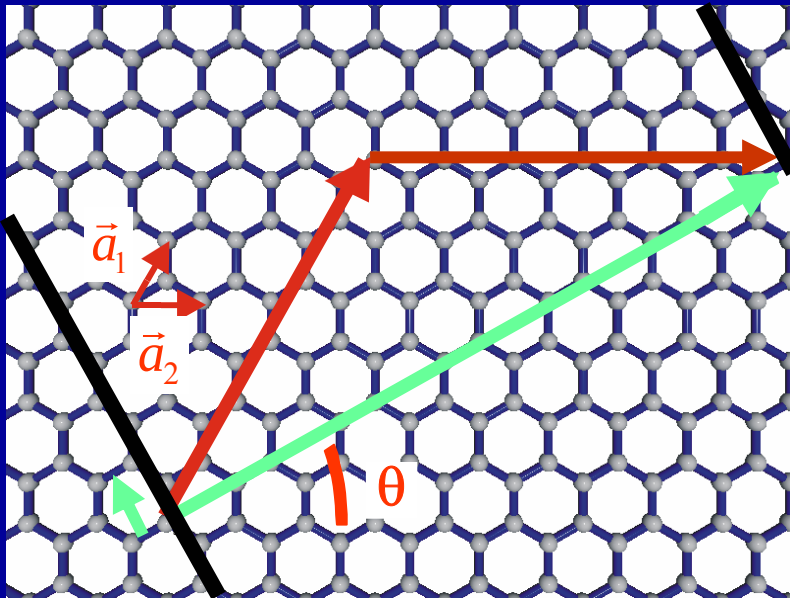
Unconventional Mesoscopic Transport in Carbon Nanotubes

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DSM/DRFMC/SPSMS



- Reminder - electronic properties of NTs
- Energy dependent transport length scales & quantum interferences
- Transport in Chemically doped nanotubes
- Transport features under molecular physisorption
- Anomalous conduction in defect-free MWNTs



Helicity

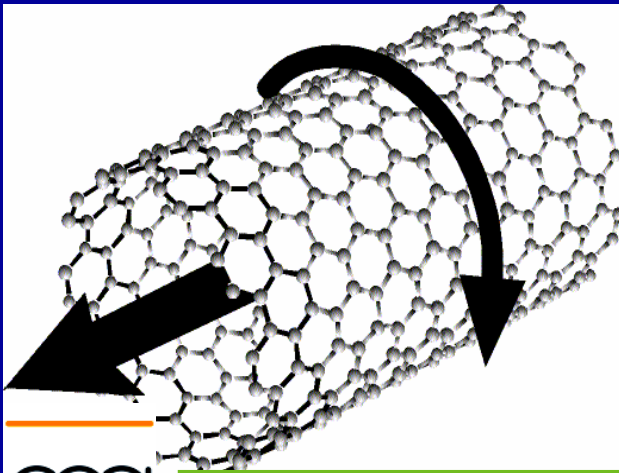
$$\vec{C}_{(l,m)} = l\vec{a}_1 + m\vec{a}_2$$

Diameter

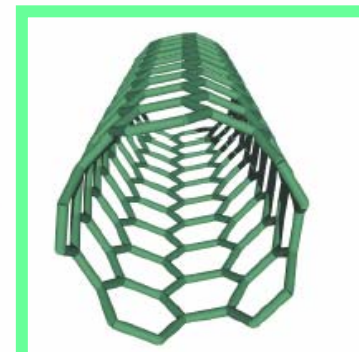
$$d_{(l,m)} = \frac{|\vec{C}_{(l,m)}|}{\pi}$$

Unit cell length

$$|\vec{T}_{(l,m)}|$$



(12,0)



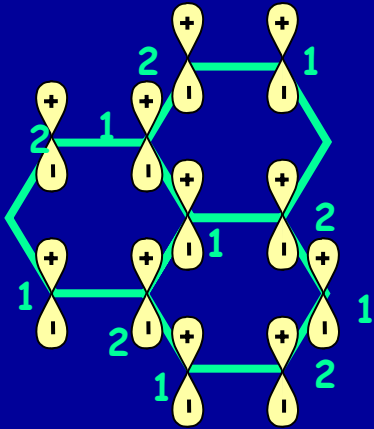
(6,6)



(6,4)



Effective π electrons-model



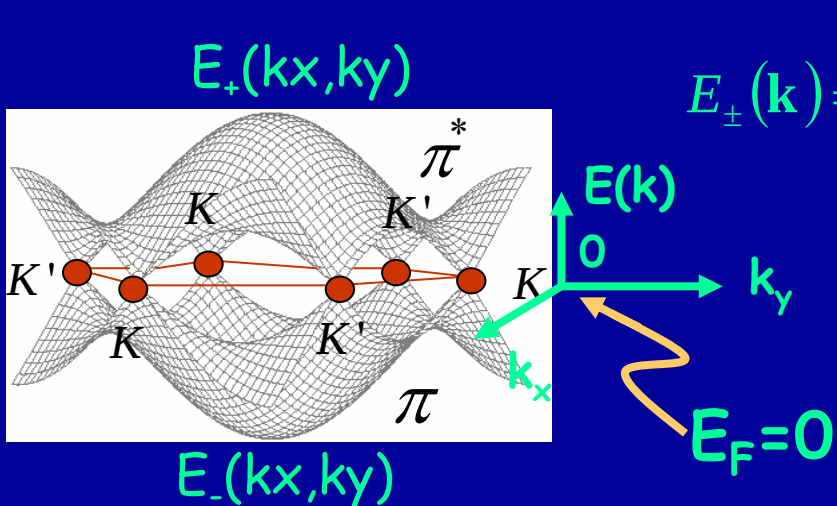
Hybrid molecular orbitals $sp_x p_y, p_z$

$sp_x p_y$ σ bonds (cohesion)

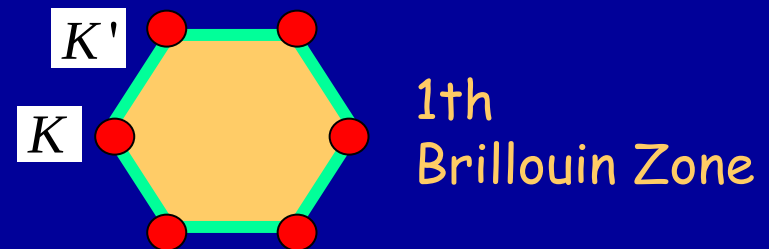
p_z π bonds (electronic properties close to E_F)

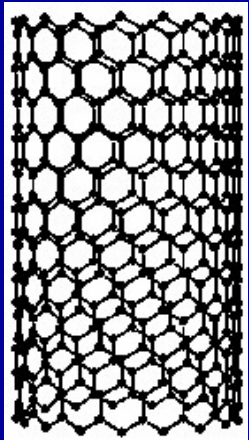
2 atoms/ unit cell - γ_0 coupling between p_z orbitals

$$H(\mathbf{k}) = \begin{pmatrix} 0 & f(\mathbf{k}) \\ f^*(\mathbf{k}) & 0 \end{pmatrix} \quad f(\mathbf{k}) = \gamma_0 \sum_{\alpha} \exp(i\mathbf{k} \cdot \boldsymbol{\tau}_{\alpha})$$



$$E_{\pm}(\mathbf{k}) = \pm \gamma_0 \left[3 + 4 \cos\left(\frac{\sqrt{3}}{2} k_x a\right) \cos \frac{k_y a}{2} + 2 \cos k_y a \right]^{1/2}$$





Periodic boundary conditions

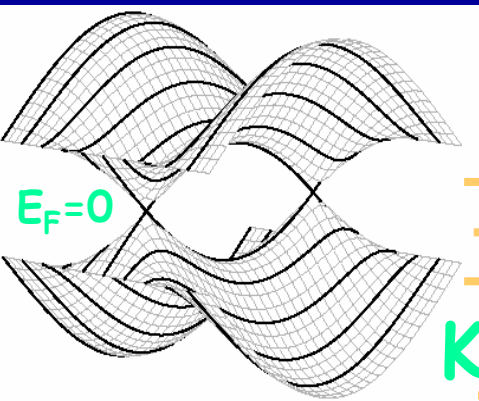
Helicity

$$\frac{-\pi}{|\vec{T}_{(l,m)}|} \leq k_y \leq \frac{\pi}{|\vec{T}_{(l,m)}|}$$

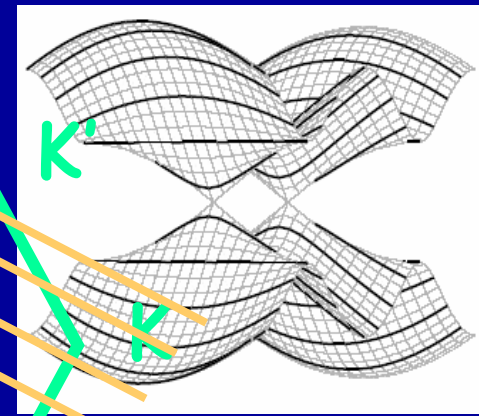
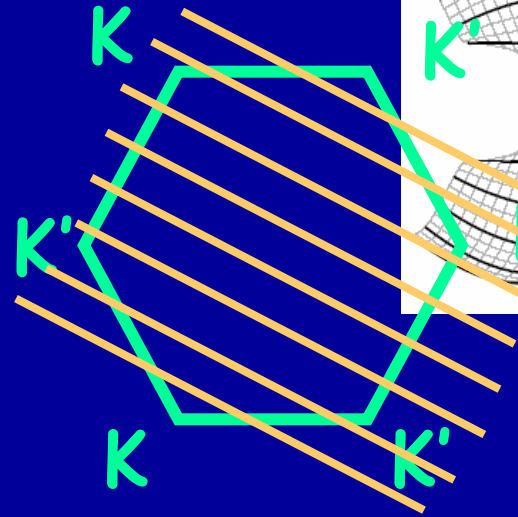
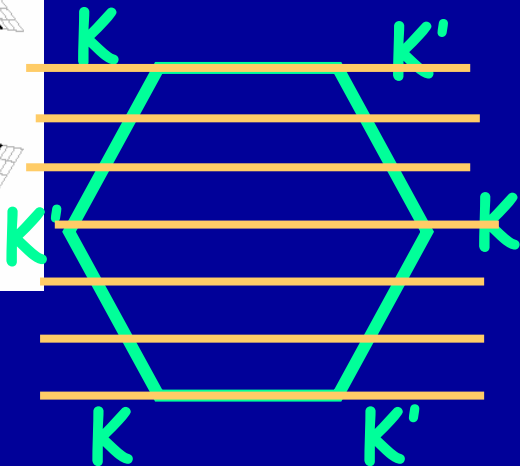
$$k_x = \frac{2\pi q}{|\vec{C}_{(l,m)}|} (q=1, N)$$

$$\vec{C}_{(n,n)} = n(\vec{a}_1 + \vec{a}_2)$$

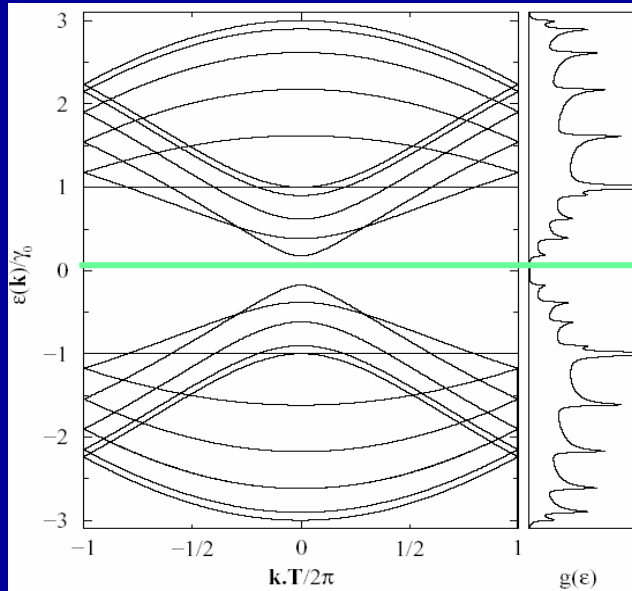
$$\vec{C}_{(l,m)} = (3p \pm 1) \vec{a}_1$$



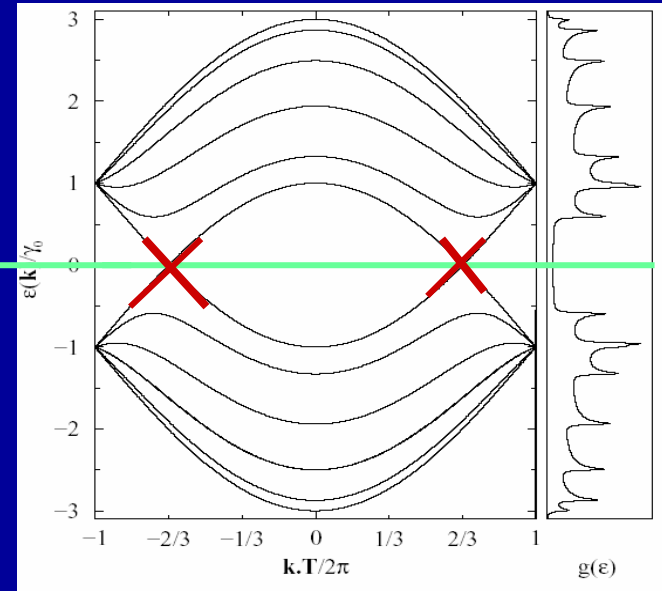
$$2\pi / |\vec{C}_{(l,m)}| \updownarrow$$



Semiconducting NT (10,0)



Metallic nanotube (5,5)



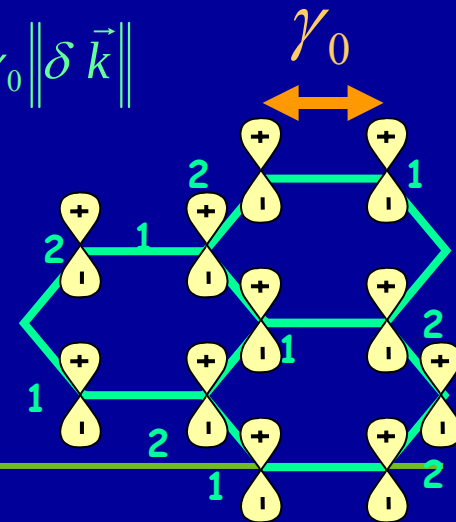
E_F

$$E_+(k=0) - E_-(k=0) \equiv \frac{2\pi a \gamma_0}{\sqrt{3} \|\vec{C}_h\|}$$

$$d_t = 1.4 \text{ nm} \rightarrow \Delta_g = 0.59 \text{ eV}$$

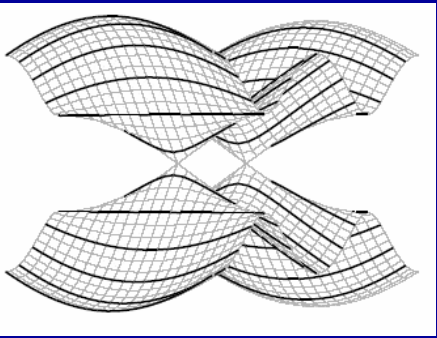
$$E_{\pm}(\delta k) \equiv \pm \frac{\sqrt{3}a}{2} \gamma_0 \|\delta \vec{k}\|$$

$$\hat{H} = \gamma_0 \sum_{\langle i,j \rangle} |p_{\perp}^j\rangle \langle p_{\perp}^i|$$

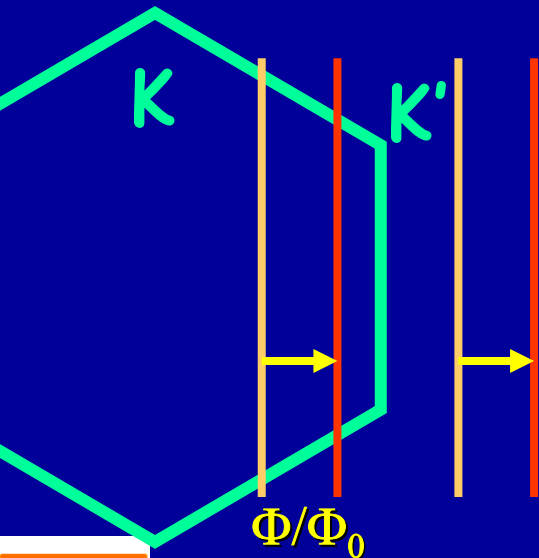
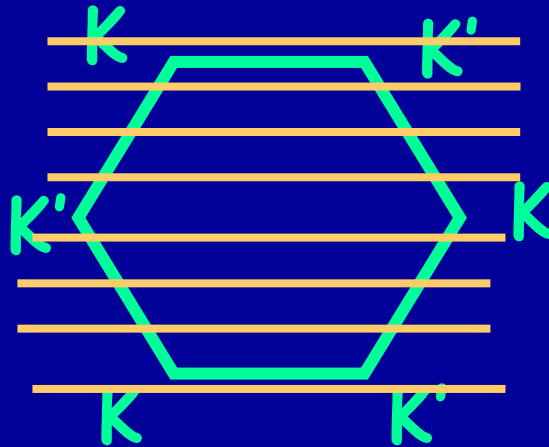




Nanotubes & Aharonov-Bohm

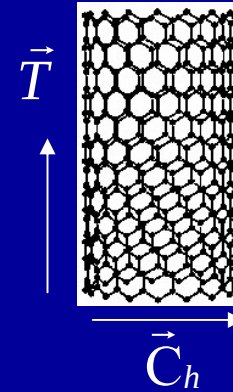


$$2\pi / |\vec{C}_{(l,m)}| \updownarrow$$



Gap opening at CNP
with Φ_0 -periodic modulation

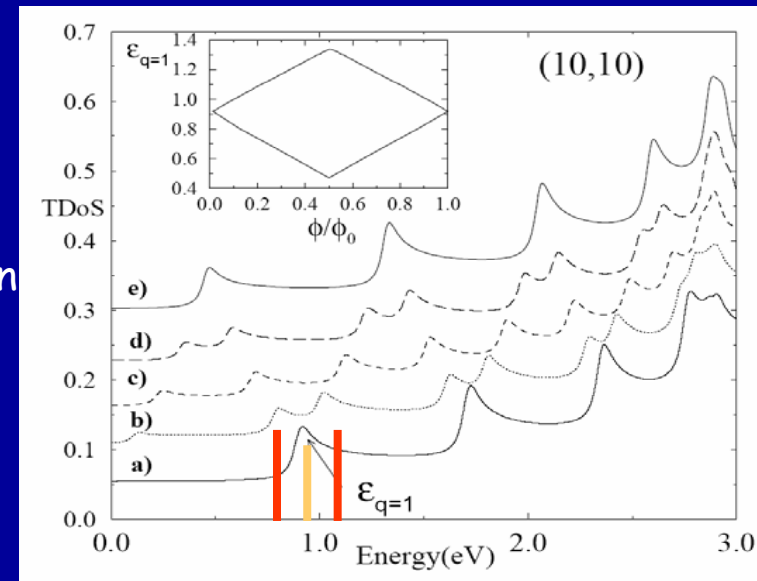
Splitting of VHS
proportional to B



Wavefunction phase

$$\Psi \approx e^{ik_y y} e^{i\left(k_x x + \frac{e}{h} \int \vec{B} dS\right)}$$

$$k_x = \frac{2\pi q}{C_h} + \frac{2\pi \Phi}{\Phi_0}$$



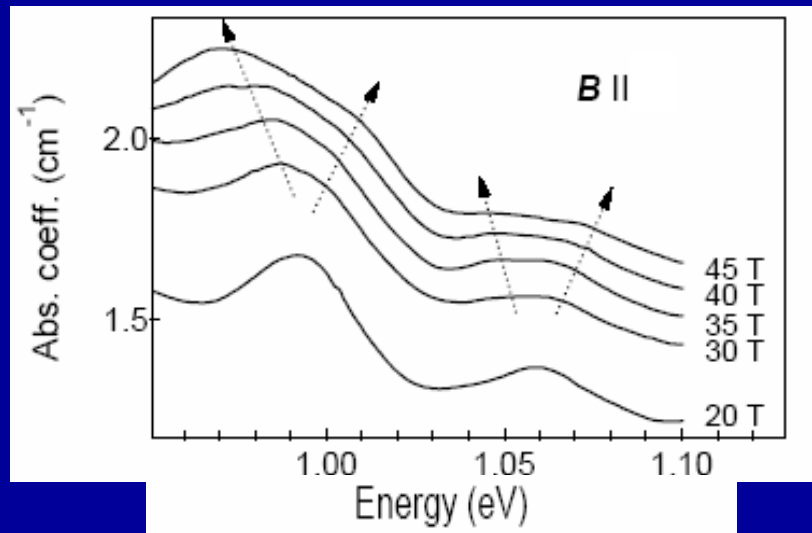
H. Ajiki and T. Ando, J. Phys. Soc. Jpn 62, 1255 (1993)

S. R., G. Dresselhaus, M. Dresselhaus & R. Saito, Phys. Rev. B 62, 16092 (2000)



Magneto-optical spectrum for a bundle of
SWNT (diameter 1nm)
 $B_{\text{max}} = 45$ Tesla

Shifting and splitting of absorption peaks and photoluminescence
Linearly with Φ/Φ_0



30-40meV at 45T

Junichiro Kono et al.
Rice Quantum Institute (USA)

S. Zaric et al., *Science* 304, 1129 (2004)

Kubo-framework (linear resp. ohmic contact/coherent reg.)

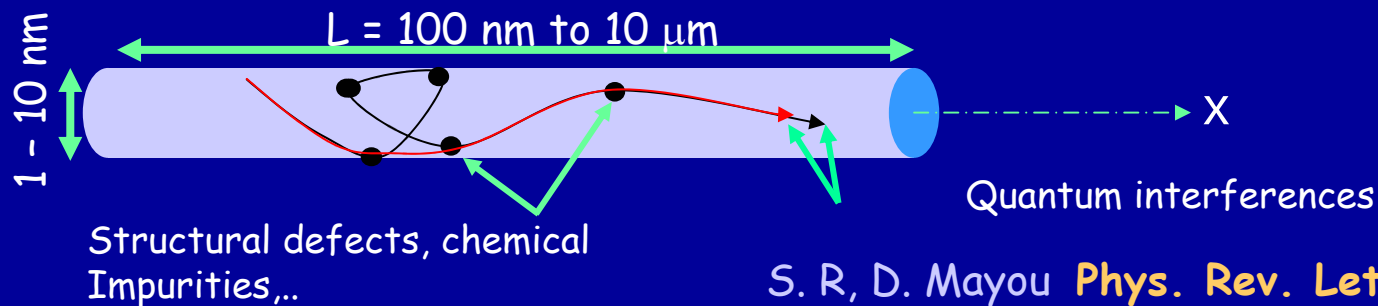
Solving time dependent Schrödinger equation

$$|\Psi(t)\rangle = \exp\left(-\frac{i\hat{H}t}{\hbar}\right)|\Psi_0\rangle \approx \sum_{n=0}^N \left(\int n(E) e^{\frac{-iEt}{\hbar}} Q_n(E) dE \right) Q_n(\hat{H}) |\Psi_0\rangle$$

Transport coefficient computing (diffusion constant)

$$\frac{e^2 h}{\Omega} \text{Tr} [\hat{V}_x \delta(E - \hat{H}) \hat{V}_x \delta(E - \hat{H})] \approx \text{Tr} \left[\delta(E_F - \hat{H}) \frac{(\hat{X}(t) - \hat{X}(0))^2}{t} \right]$$

Quantum dynamics of propagating wavepackets $D_E(t) = \frac{1}{t} \langle \{X(t) - X(0)\}^2 \rangle_E$



S. R, D. Mayou **Phys. Rev. Lett.** 79, 2518 (1997)
S. R, R. Saito **Phys. Rev. Lett.** 87, 246803 (2001)

Transport regimes

$$D_E(t) = \frac{1}{t} \left\langle \{X(t) - X(0)\}^2 \right\rangle_E$$

Conductance scaling

$$G_E = \frac{2e^2}{L} \lim_{t \rightarrow \tau_L} \text{Tr}[\delta(E - H) D_E(t)]$$

τ_L : propagation time over L

Ballistic regime & Conductance quantization

$$D(t) \approx v_F^2 t \Rightarrow G_E = \frac{2e^2}{h} N_E$$

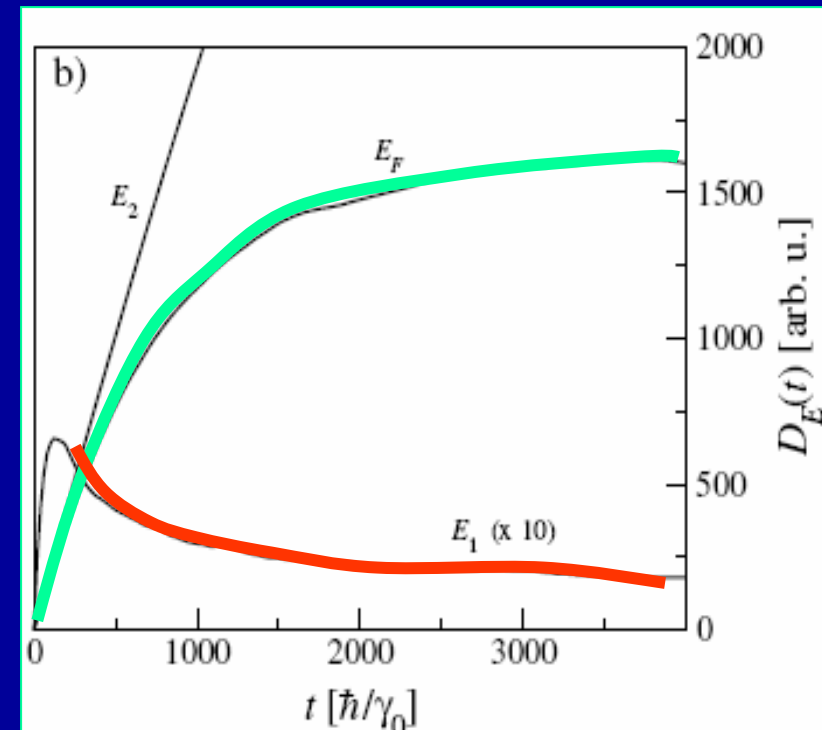
Diffusive regime & mean free path

$$D(t > \tau) \approx v_F l_e \Rightarrow G_E = \frac{2e^2}{h} \frac{l_e}{L_{\text{sys}}}$$

Quantum interference effects

Weak & Strong localization effects
Correction & exponential decrease of cond.

$$G_E = \frac{2e^2}{h} \left(\frac{l_e}{L_{\text{sys}}} - \delta\sigma \right) \quad G_E = \frac{2e^2}{h} \exp\left(-\frac{l_e}{L_{\text{sys}}}\right)$$

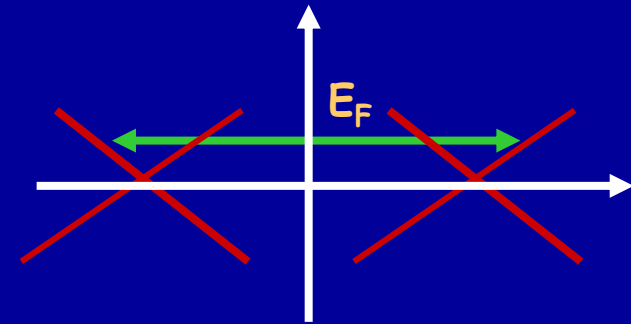




Elastic scattering time and Fermi golden rule

$$\frac{1}{2\tau(E_F)} = \frac{2\pi}{\hbar} \left| \langle \Psi_{n1}(k_F) | \hat{U} | \Psi_{n2}(-k_F) \rangle \right|^2 \rho(E_F) N_c N_{ring}$$

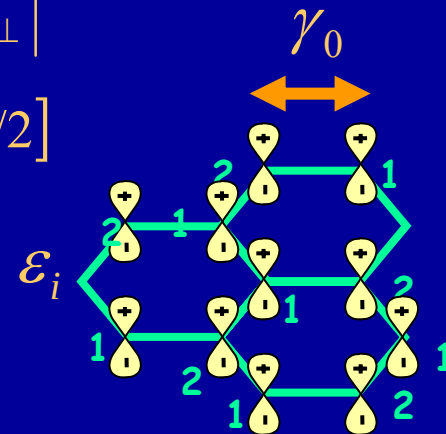
Mean free path $\ell_e = v_F \tau$ ($v_F = \sqrt{3} a \gamma_0 / 2 \hbar$)



Anderson-type disorder $\hat{H} = \gamma_0 \sum_{\langle i,j \rangle} |p_{\perp}^j\rangle \langle p_{\perp}^i| + \sum_{\langle i \rangle} \varepsilon_i |p_{\perp}^i\rangle \langle p_{\perp}^i|$

Vicinity of CNP $\varepsilon_i \in [-W/2, W/2]$

$$\ell_e = \frac{18}{\sqrt{3}} \left(\frac{\gamma_0}{W} \right)^2 \parallel \vec{C}_h \parallel$$



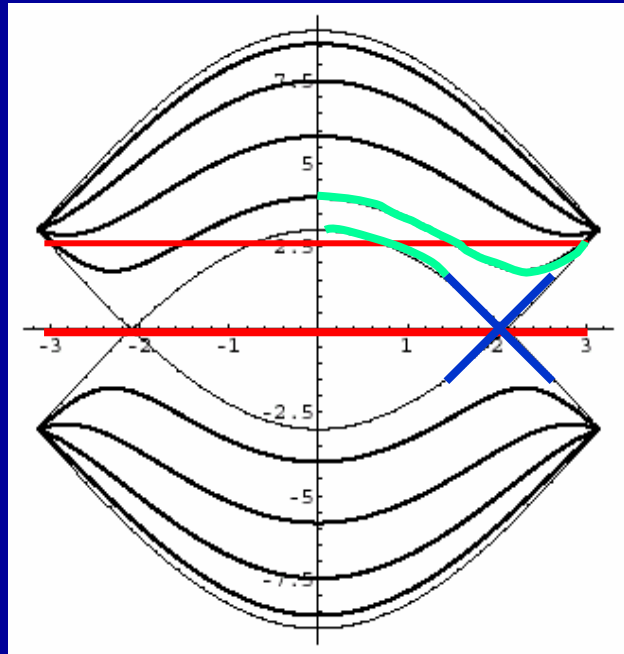
For a given disorder strength, the mean free path upscales with diameter!!!

C. White, T.N.Todorov, **Nature** **393**, 240 (1998)

S. R., G. Dresselhaus, M. Dresselhaus & R. Saito, **Phys. Rev. B** **62**, 16092 (2000)

(5,5)

Energy



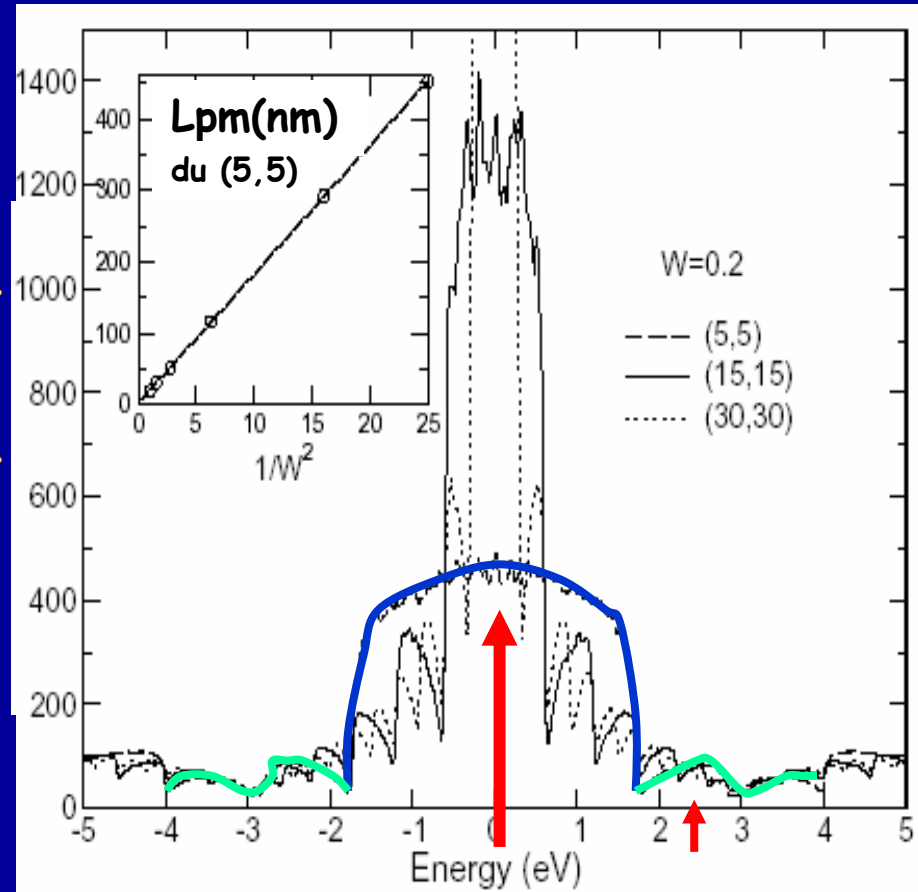
k

Charge neutrality point

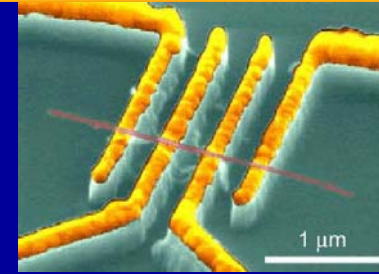
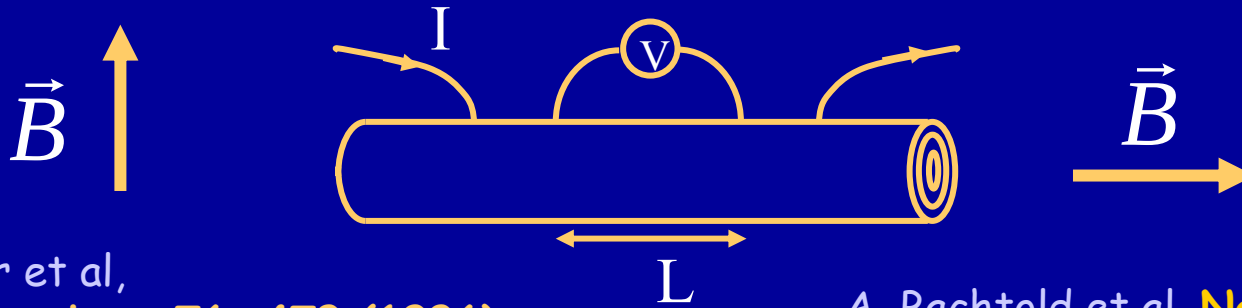
$$\ell_e = \frac{18}{\sqrt{3}} \left(\frac{\gamma_0}{W} \right)^2 \parallel \vec{C}_h \parallel$$

Under Fermi level shift

Mean free path moyen (nm)

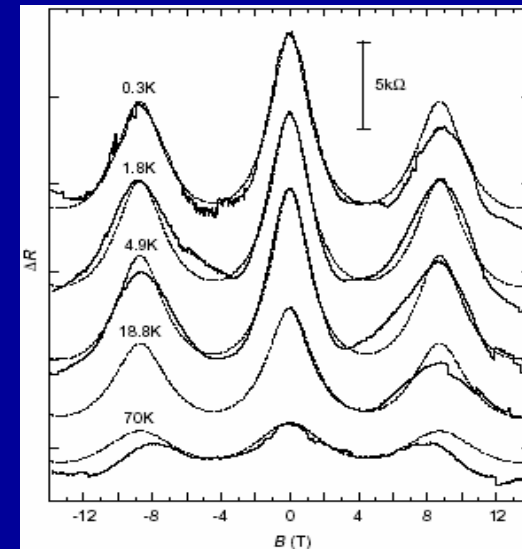
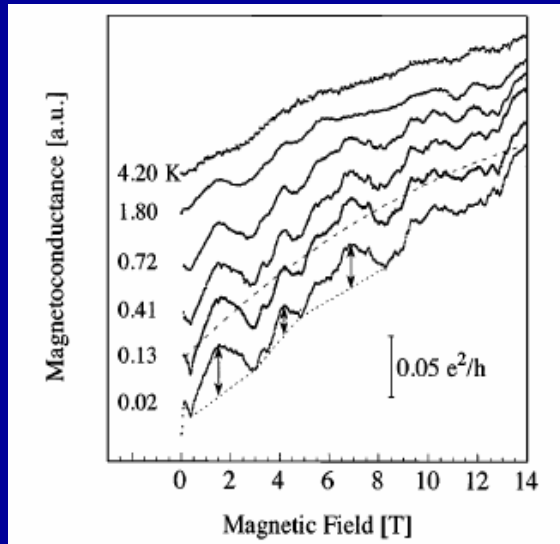


Energy (eV)



L. Langer et al,
Phys. Rev. Lett 76, 479 (1996)

A. Bachtold et al, **Nature** 397, 673 (1999)



Evidences for a weak localization regime

Negative Magnetoresistance
Universal Fluctuations of conductance

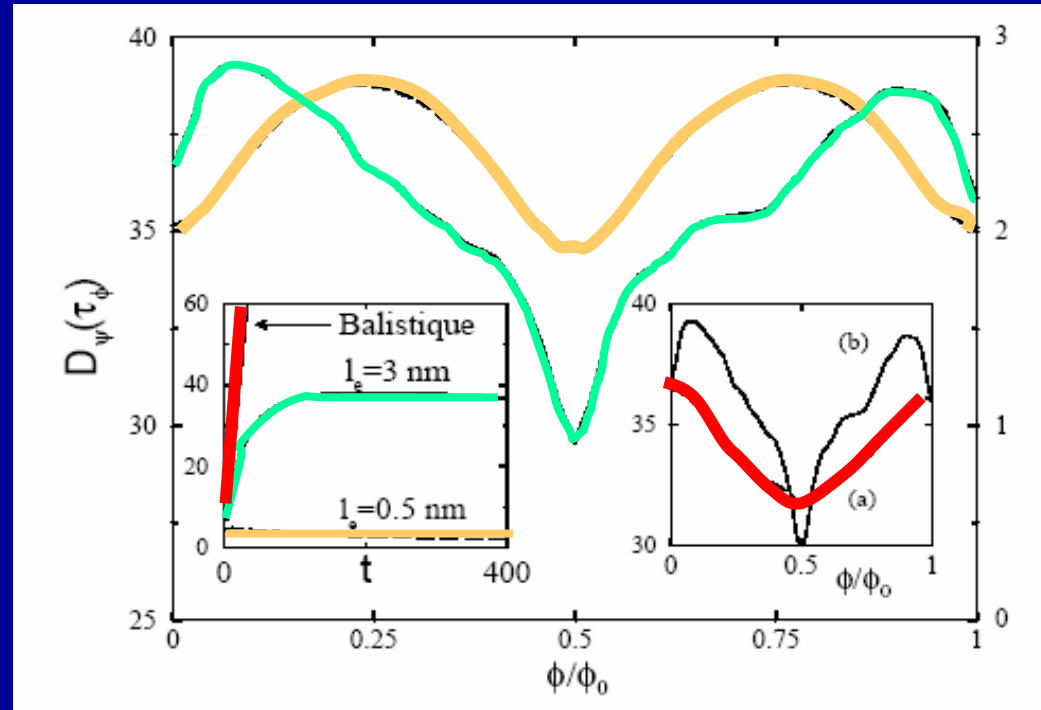
Negative MR
 $\Phi_0/2$ -periodic AAS oscillations

Approximation : No effect of B on density of states

$l_e < \text{tube circumference}$
 Negative MR
 $\Phi_0/2$ - periodic AB oscillations

$L_{\text{tube}} > l_e > \text{tube circumference}$
 Negative MR
 Φ_0 -periodic AB oscillations

$l_e > L_{\text{tube}}$
 Positive MR
 Φ_0 -periodic AB oscillations



S. R, F. Triozon, A. Rubio, D. Mayou,
 Phys. Rev. B 64, 121401 (2001)

Incorporation of N_2 (gas) during the synthesis

Defected multiwall nanotubes obtained (bamboo-shape)



R. Czerw *et al.* Nanoletters 1, 457 (2001)

Bundles of Nitrogen-doped Single-wall nanotubes

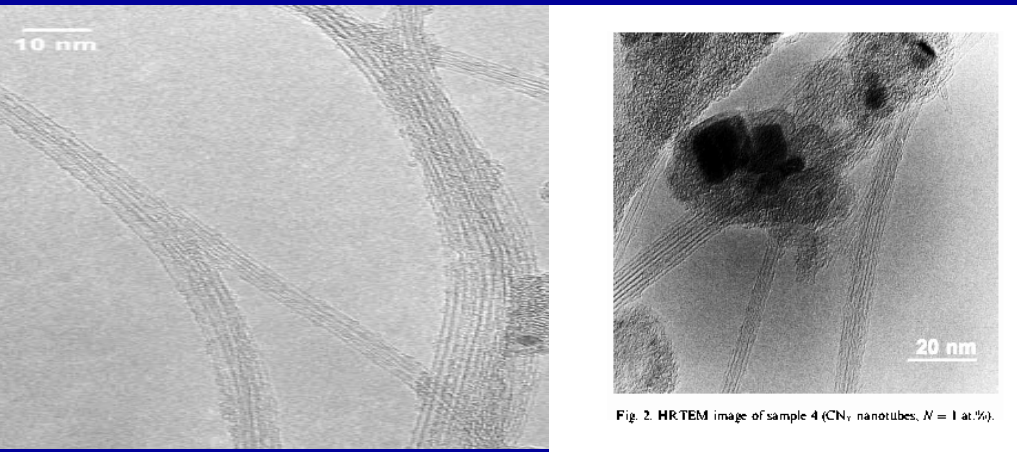


Fig. 2. HRTEM image of sample 4 (CN_x nanotubes, $N = 1$ at.%).

EELS spectrum

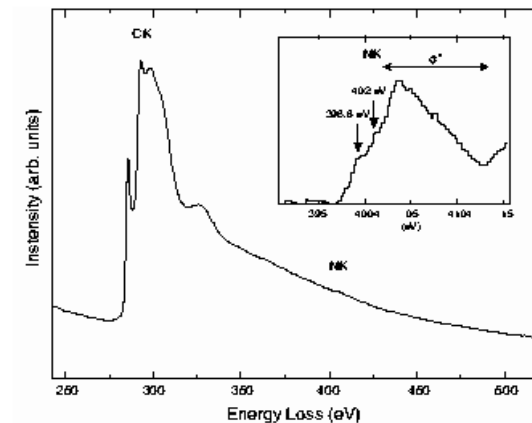
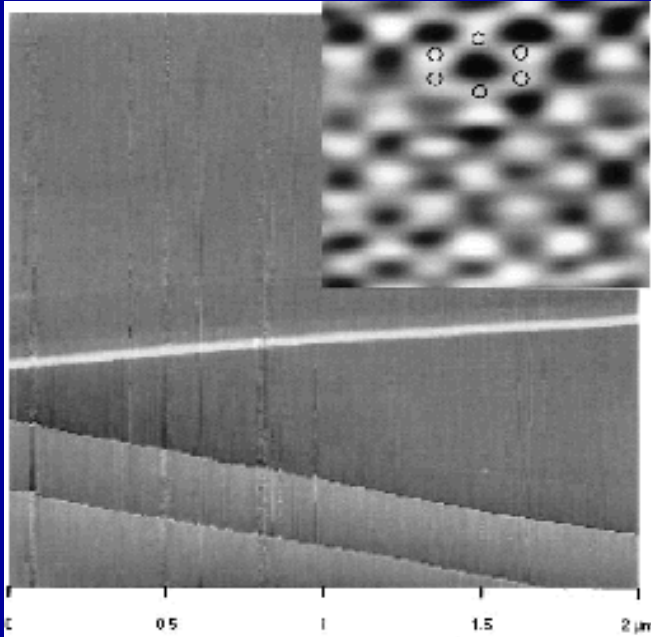


Fig. 3. EEL core electron K-shell spectra of CN_x nanotube bundles (sample 4). The nanotubes are doped with around 1 at.% nitrogen. For the C-K edge well defined π^* and σ^* fine structure features are observed which are evidences of sp^2 -hybridisation in graphitic structures. The inset is a magnification of the N-K edge.

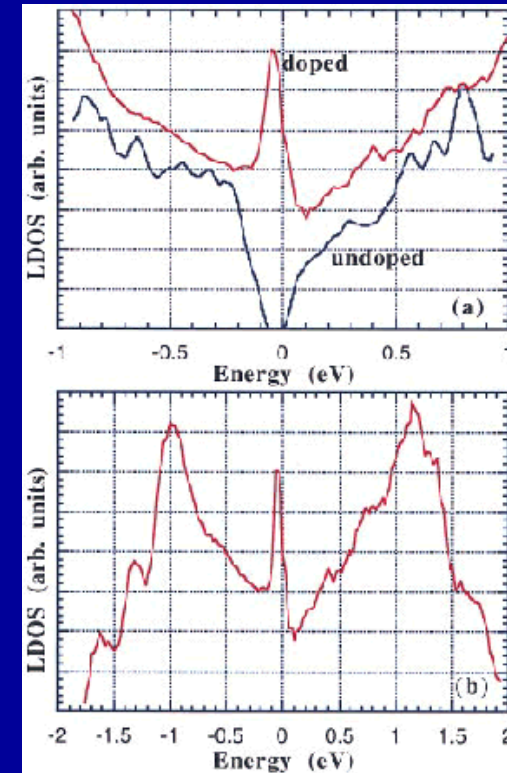
M. Glerup *et al.* Chem. Phys. Lett. 387, 193 (2004).

Incorporation of Boron (% 1 à 1.5) during synthesis

Long MWNTs (diameter $\sim 10\text{-}20\text{ nm}$)
with low disorder



Density of states by tunneling spectroscopy (STS)



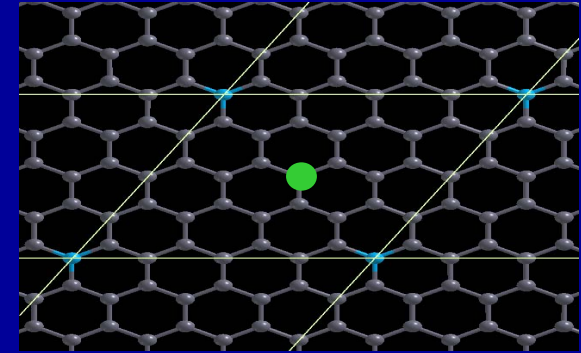
D.L. Carroll *et al.* *Phys. Rev. Lett.* **81**, 2332 (1998),

X. Blase *et al.* *Phys. Rev. Lett.* **83**, 5078 (1999)

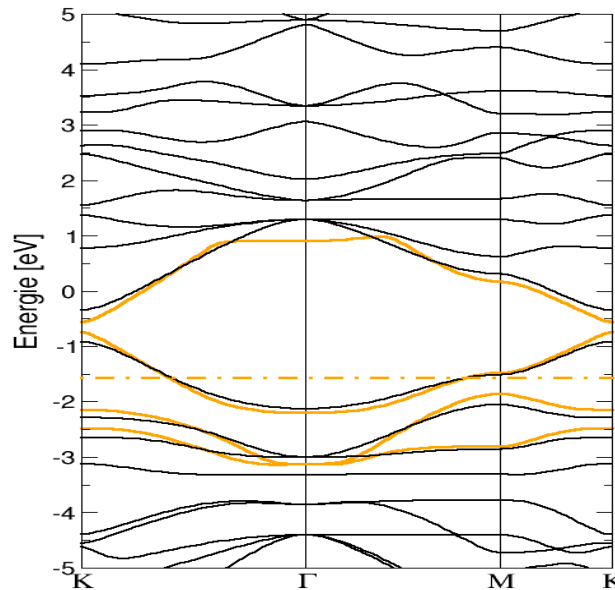
Doped nanotubes - case of Boron

Bandstructure (DFT) for a unit cell with 31 atoms C and 1 impurity (B or N)

Adjustement (fit) of tight-binding parameters up to second neighbors (for B or N)



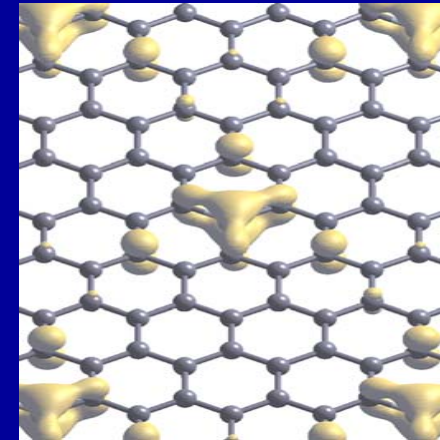
Bandstructure



$$H = \sum_i \varepsilon_i^{q=1,P} |p_\perp^i\rangle \langle p_\perp^i| + \sum_{ij} \gamma_0^{k=1,M} |p_\perp^i\rangle \langle p_\perp^j|$$

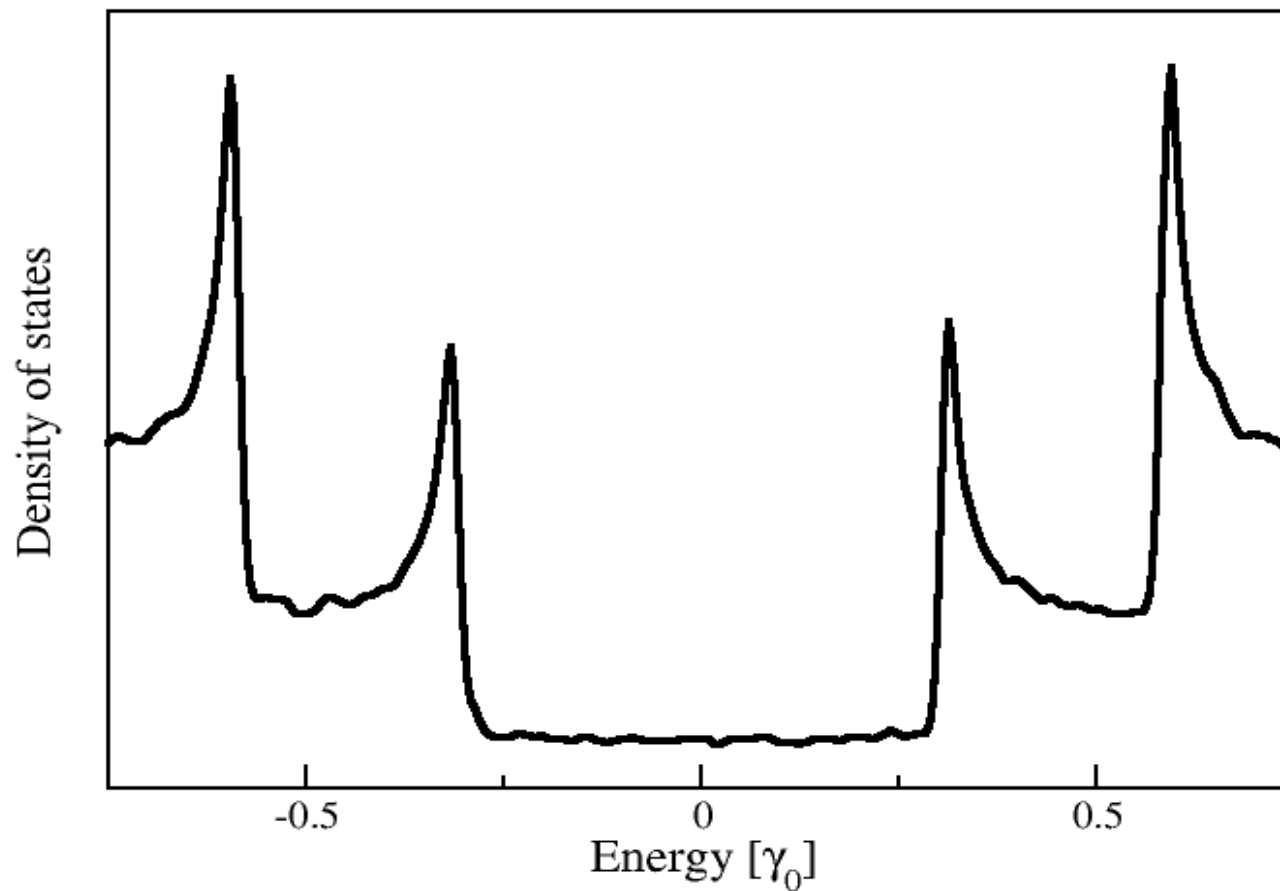
— Ab initio
— Tight-binding

Quasi-bonded states
of Boron
(energy-dependent
backscattering)



Nanotube (10,10)

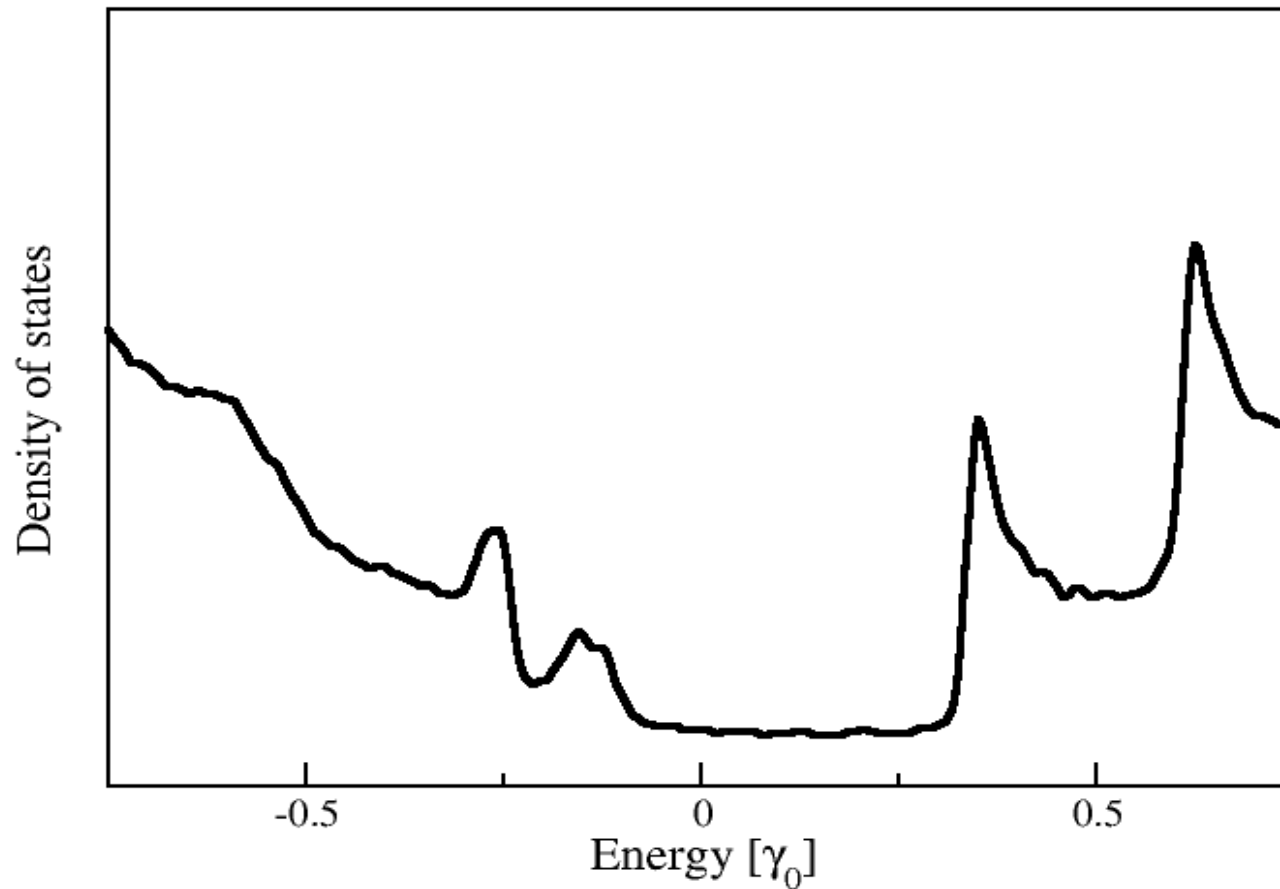
BORON: 0.01 %

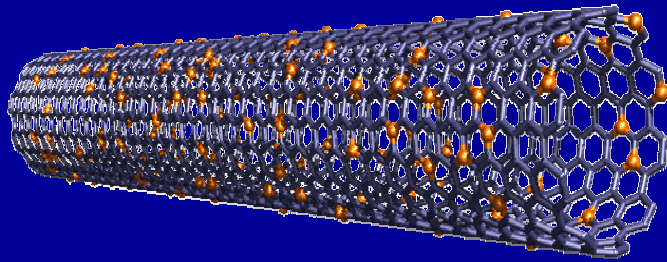




Nanotube (10,10)

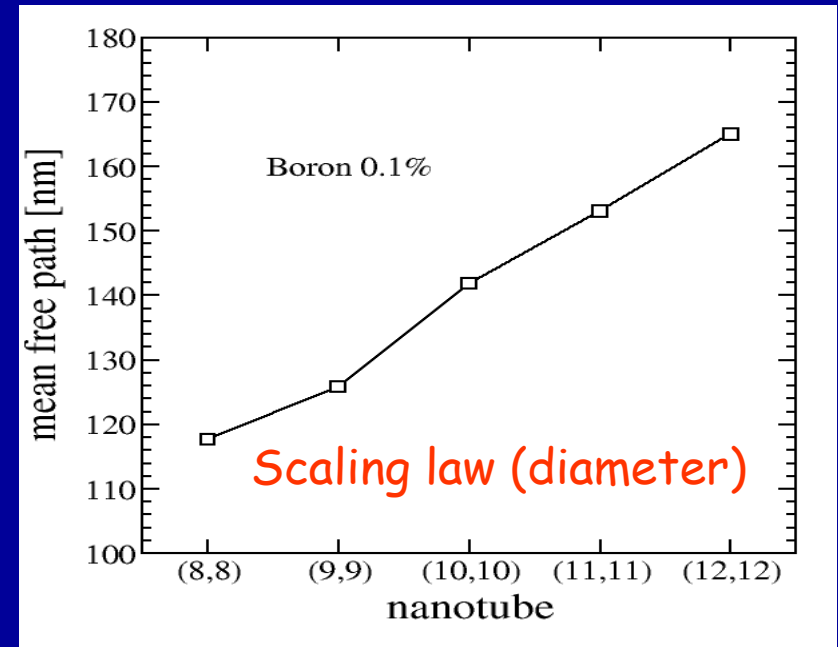
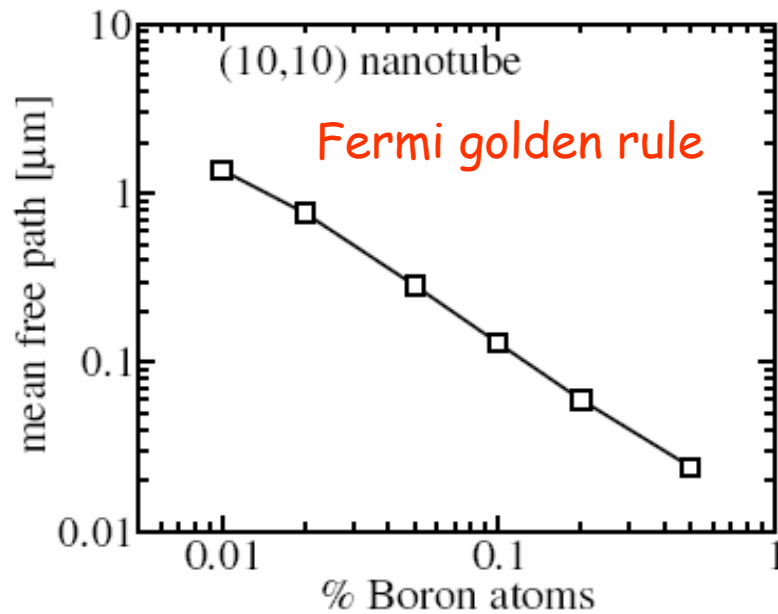
BORON: 0.5 %





$$H = \sum_i \varepsilon_i^{q=1,P} |p_\perp^i\rangle \langle p_\perp^i| + \sum_{ij} \gamma_0^{k=1,M} |p_\perp^i\rangle \langle p_\perp^j|$$

Feature close to
Charge neutrality point

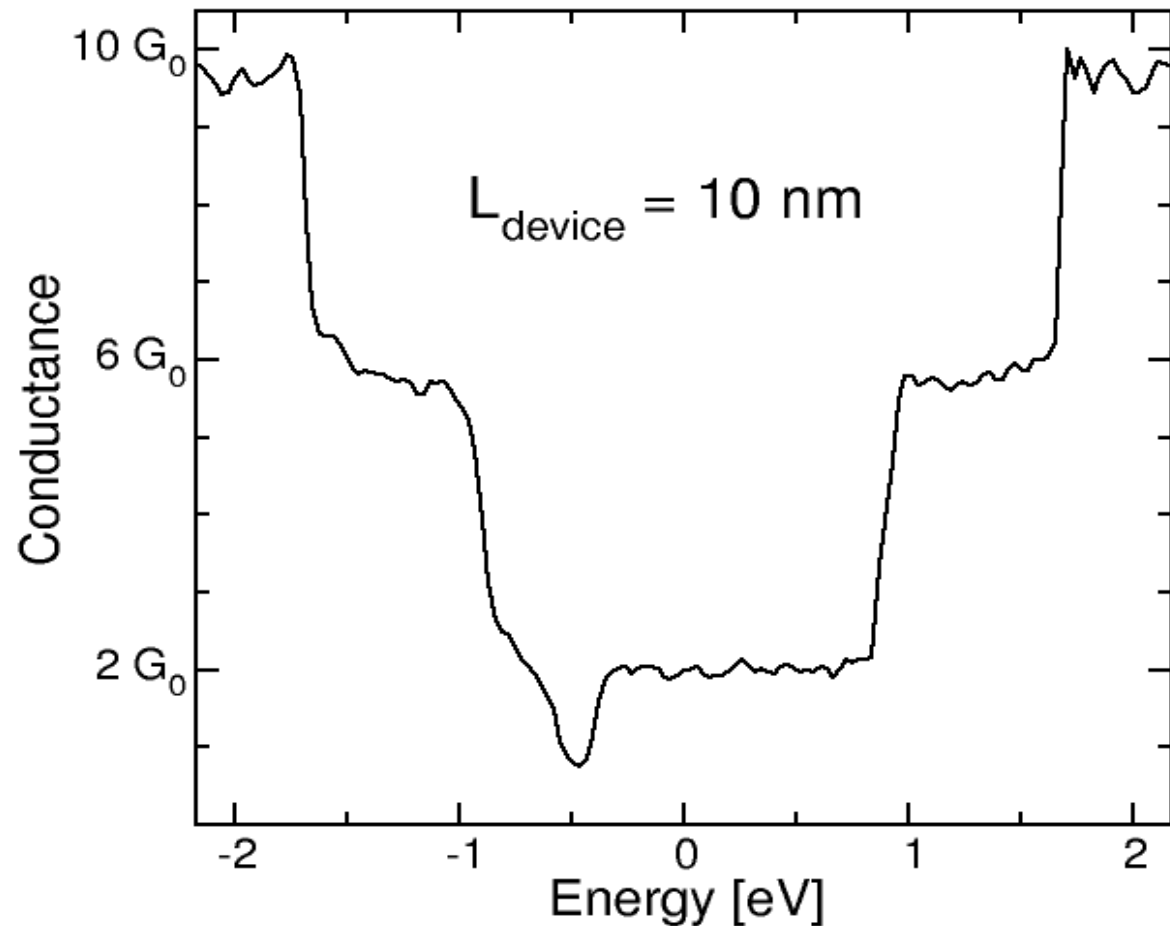
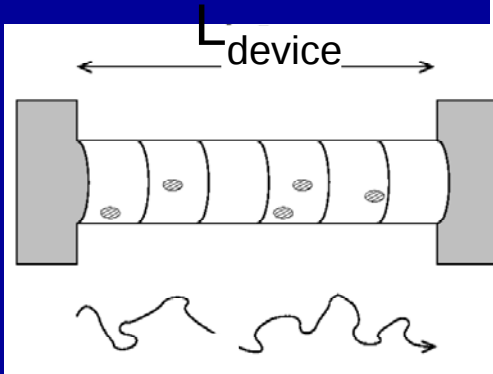


Good agreement with experimental
Estimates [0.1% Bore, 20nm diameter]
V. Krstic et al., PRB 67, 041401 (2003)

S. Latil, S. R, D. Mayou, J.C. Charlier,
Phys. Rev. Lett 92, 256805 (2004)

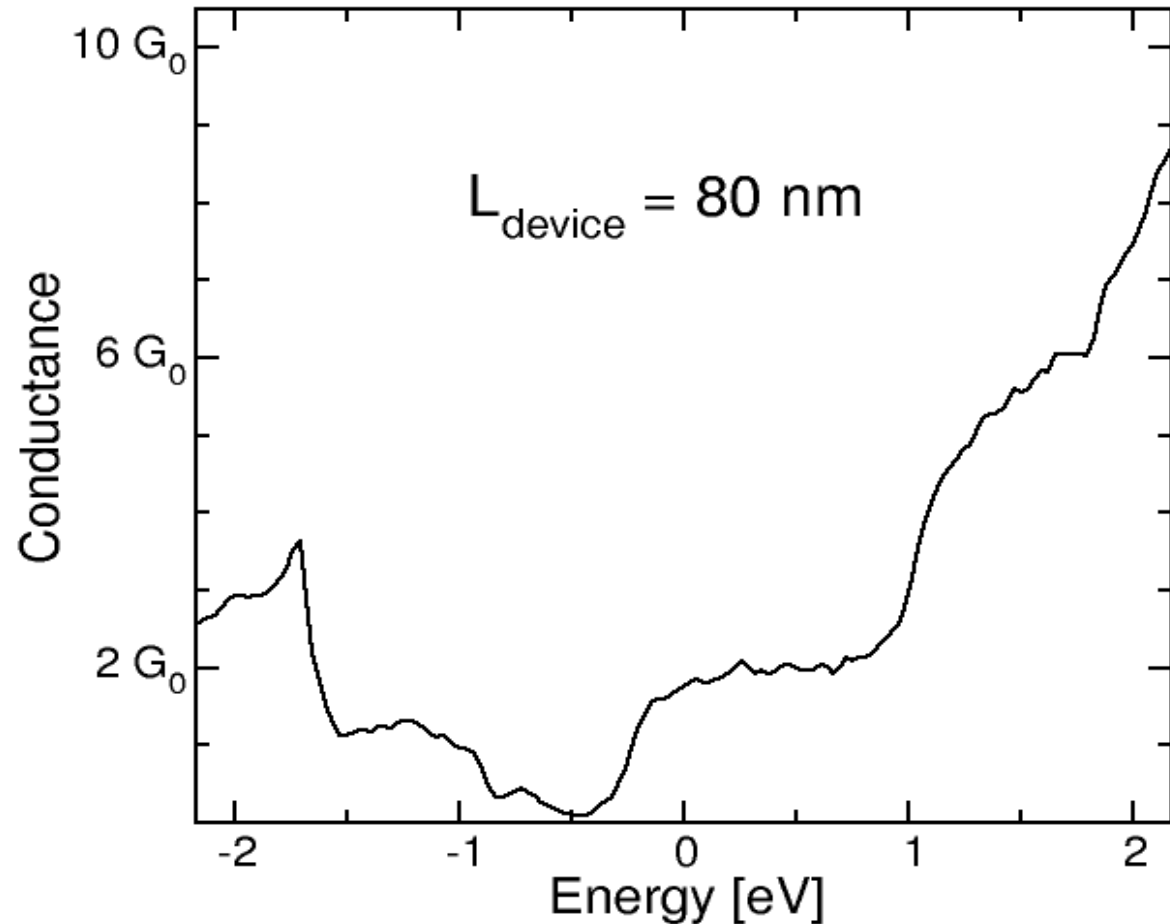
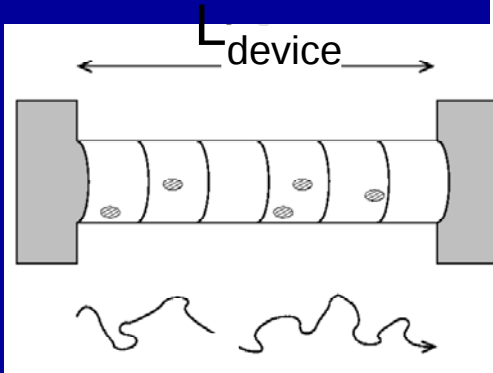
Nanotube (10,10)

BORON: 0.1 %

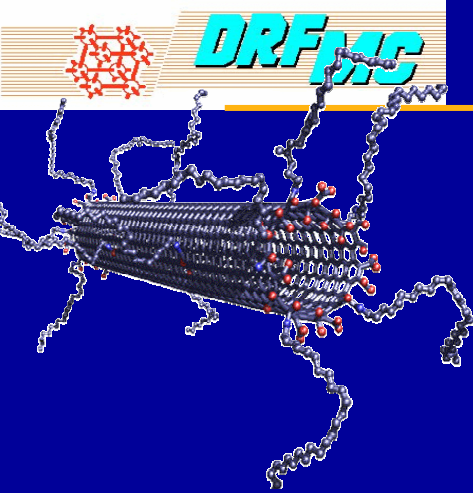


Nanotube (10,10)

Boron : 0.1 %



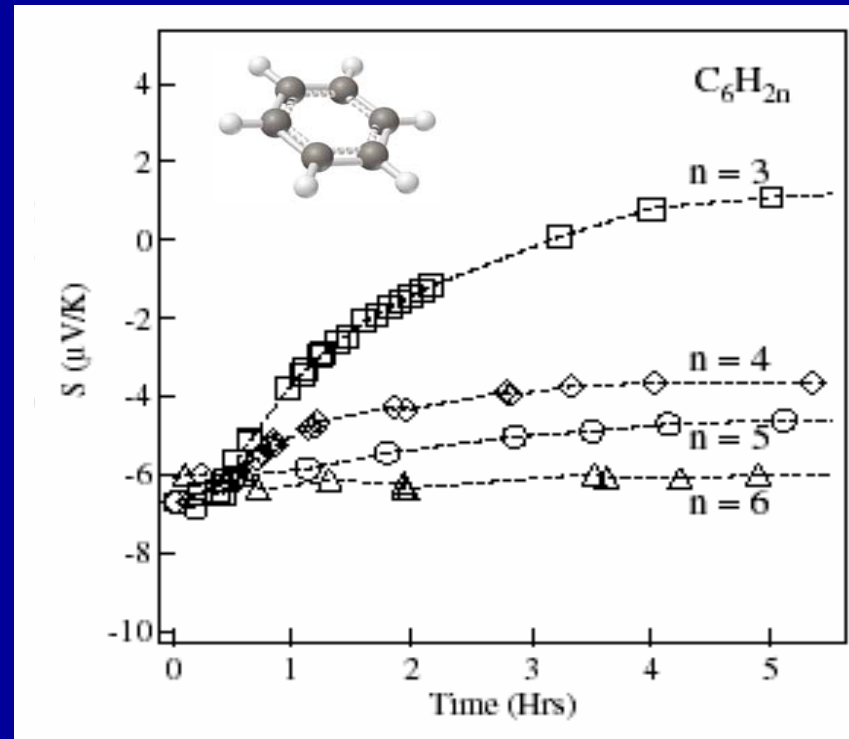
Molecular Physisorption



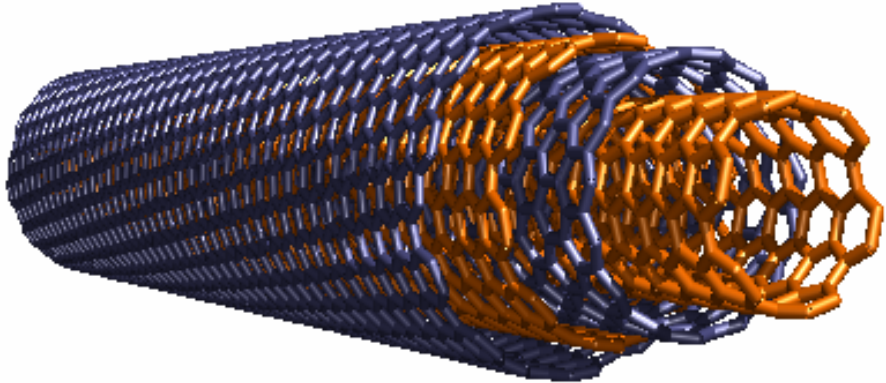
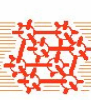
(Bio)-Chemical Sensitivity and Selectivity of electronic transport of Carbon nanotubes ??

Giant variations of the Thermopower due to Benzene physisorption, ..

G. Sumanakera et al.
Phys. Rev. Lett. 89, 166801
(2002)



$$S = - \frac{\pi^2 k_B^2 T}{3eG} \left\{ \frac{d G(E)}{d E} \right\}_{E_F}$$



Tight-binding model

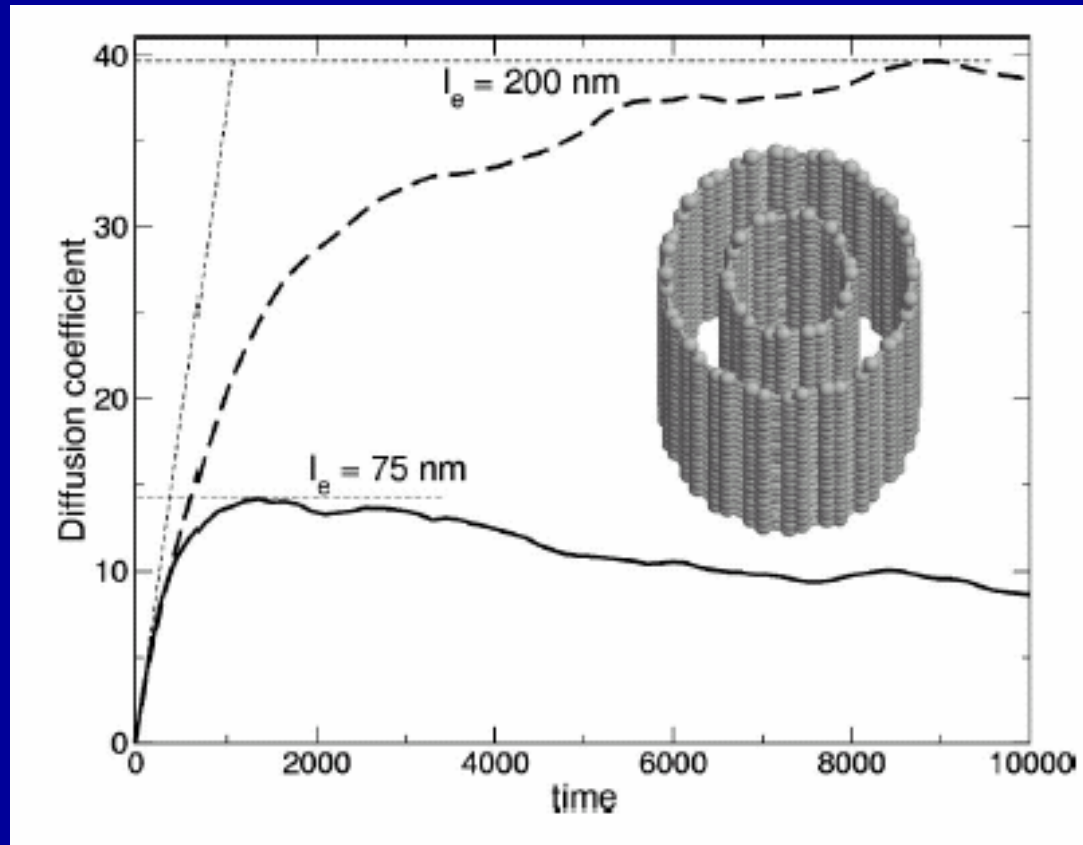
γ_0 : intrashell coupling strength

β : intershell coupling strength

J.C. Charlier & J.P. Michenaud,
Phys. Rev. Lett. 70, 1858 (1993)

$$\hat{H} = \gamma_0 \sum_{\langle i,j \rangle} |p_{\perp}^j\rangle \langle p_{\perp}^i| - \beta \sum_{\langle\langle i,j \rangle\rangle} \cos(\theta_{ij}) \exp\left(\frac{d_{ij} - a}{\delta}\right) |p_{\perp}^j\rangle \langle p_{\perp}^i|$$

Is (and how) intershell coupling acting
on transport mechanism ????

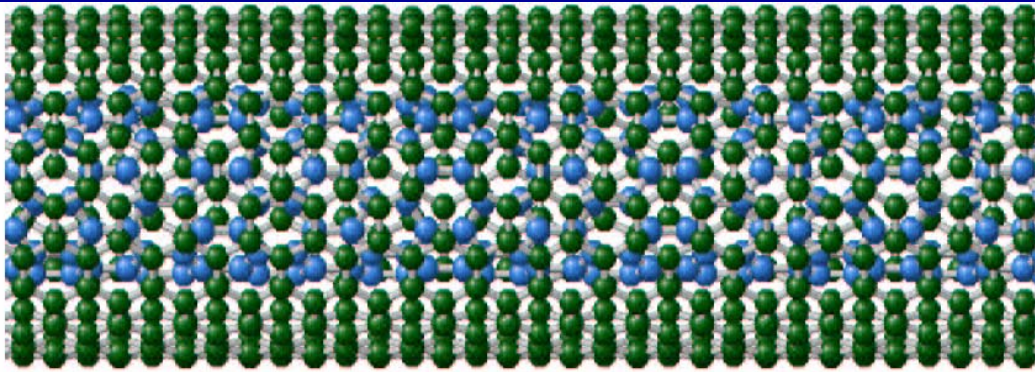


(5,5)@(10,10)

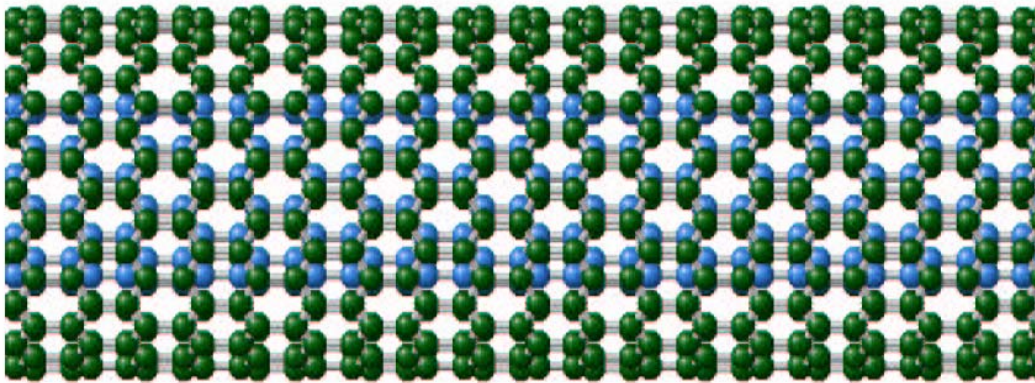
With Anderson-like
Disorder
(CNP: mfp $\sim 75 \text{ nm}$)

Inner shell :
Disorder-free

Intershell coupling in this situation favors delocalization and results in mean free path increase !!!



$(9,0)@(10,10)$



$(9,0)@(18,0)$

**Incommensurate Syst.
Aperiodic**

-No Bloch theorem-
??

$$\left| \vec{T}_{(10,10)} \right| = \sqrt{3} \left| \vec{T}_{(9,0)} \right|$$

**Commensurate Syst.
Periodic**

-Bloch theorem-
Bandstructures/mfp,...

$$\left| \vec{T}_{(18,0)} \right| = \left| \vec{T}_{(9,0)} \right|$$

Non-ballistic propagation modes

Scaling law for conductance (power-law exponents)

commensurate

$$D_{\Psi}(t) \approx t \quad G(L) \approx \frac{e^2}{h} N_{\perp}$$

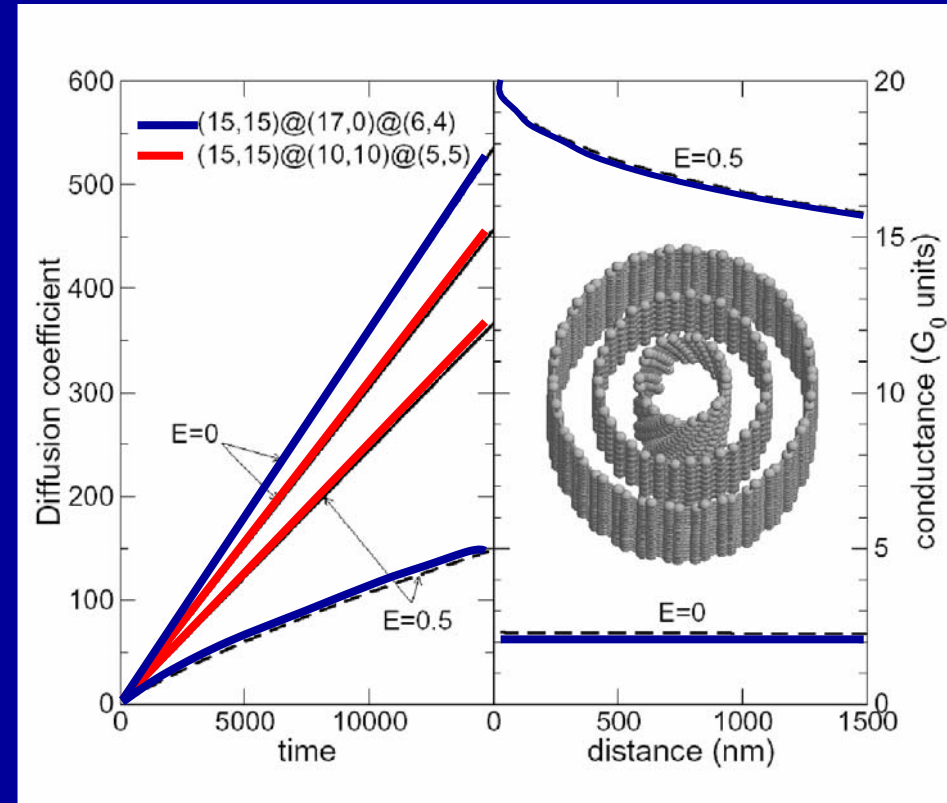
incommensurate

$$D_{\Psi}(t) \approx t^{2\eta-1} \quad G(L) \approx \frac{e^2}{h} \left(\frac{L}{L_0} \right)^{\frac{\eta-1}{\eta}}$$

Experiment

V. Krstic et al.,

Phys. Rev. B 67, 041401 (2003)



S. R, F. Triozon, A. Rubio, D. Mayou, Phys. Rev. B (RC) 64, 121401 (2001)

F. Triozon, S. R, A. Rubio, D. Mayou, Phys. Rev. B (RC) 69, 121410 (2004)

Regensburg Univ. Feb. 05