

# Fabry-Perot interference in a nanotube electron waveguide

*W. Liang et al.\**

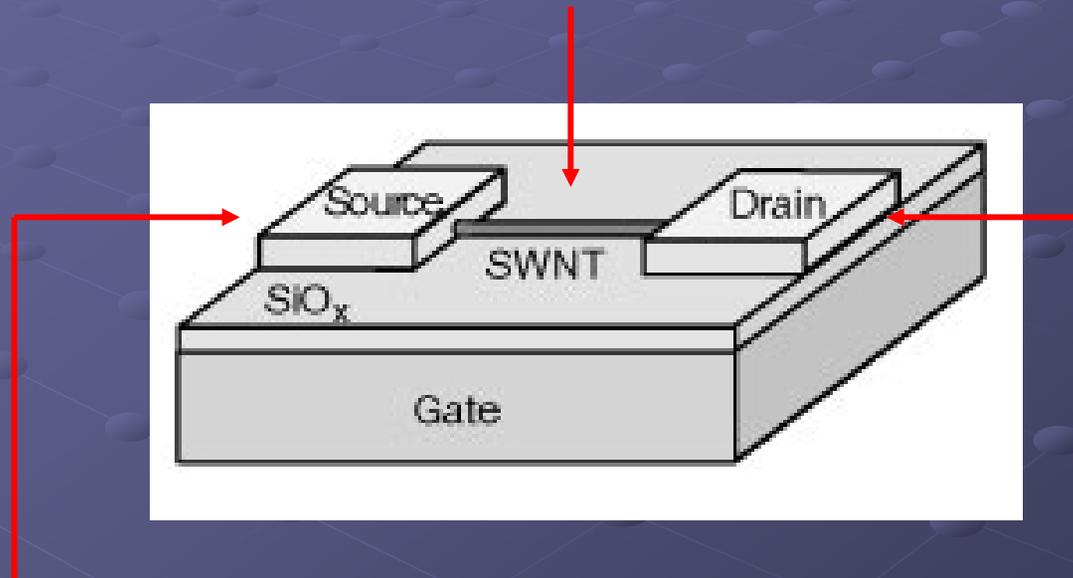
*(Nature, Vol 411, June 2001)*

presented by  
Nils Meyer

\*Department of Chemistry and Chemical Biology and Department of Physics, Harvard University, Cambridge, Massachusetts

# Experimental Setup

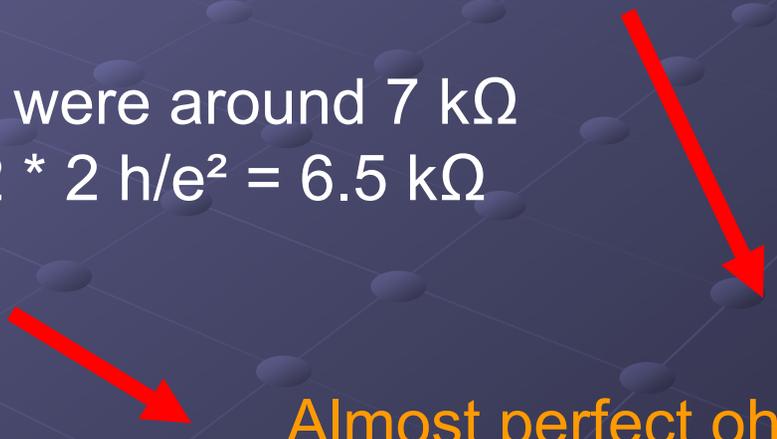
SWCNT synthesized on **degenerately doped silicon wafer with 1- $\mu\text{m}$  oxide layer** by chemical vapour deposition



Nanotube devices were fabricated by defining two **Au/Cr electrodes on top of SWCNT** by electron beam lithography

# Carbon nanotube classification

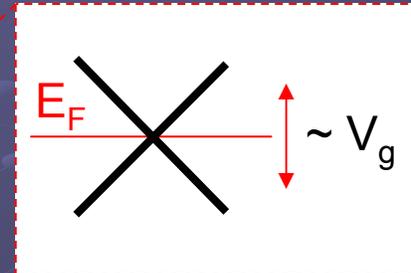
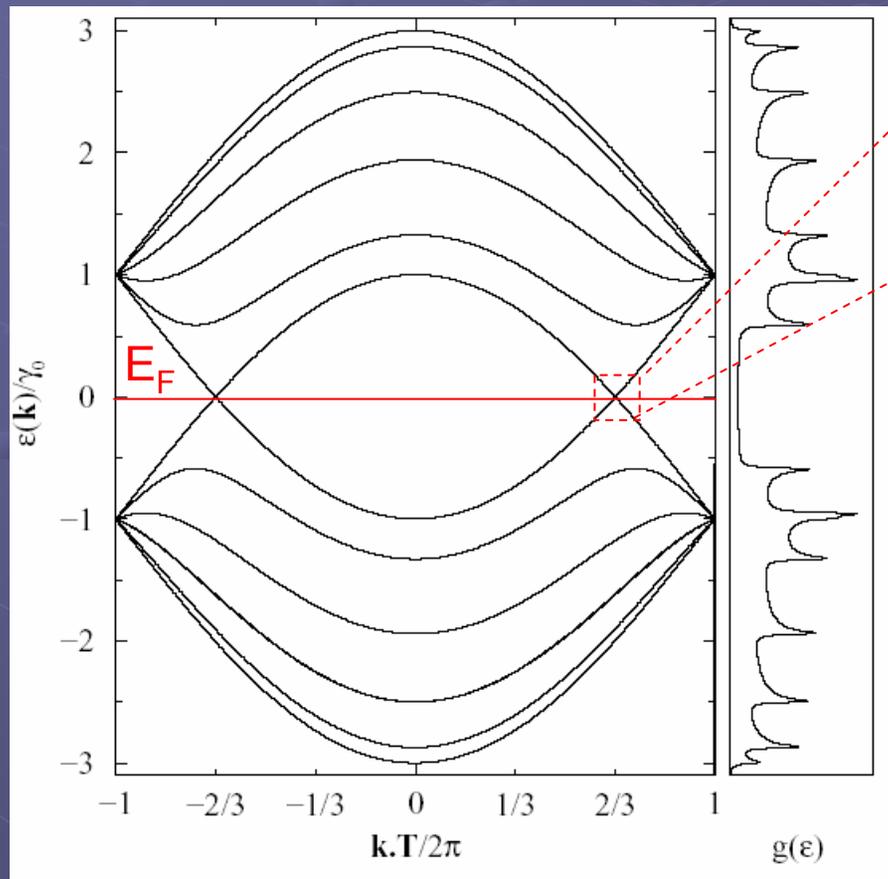
- SWCNTs were classified by their resistance versus gate voltage ( $V_g$ ) behaviour
- Only **metallic tubes with room-temperature resistances  $\leq 15 \text{ k}\Omega$**  have been used
- Lowest resistances were around  $7 \text{ k}\Omega$   
Theoretical value:  $2 * 2 \text{ h}/e^2 = 6.5 \text{ k}\Omega$



**Almost perfect ohmic contacts**  
**Little reflection**

# Band structure & density of states

Metallic SWCNT (5,5)



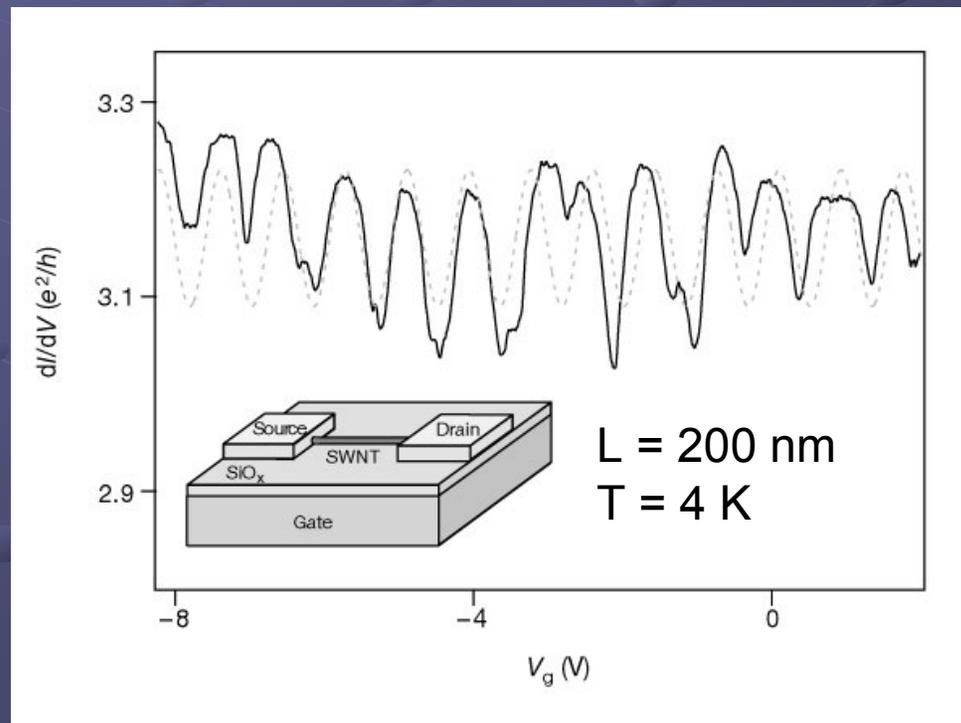
Current is carried by two spin-degenerate one-dimensional transport modes with linear dispersion for small gate voltages  $V_g$



Maximum  $\partial I/\partial V$  of  $4 e^2/h$

# Zero-bias differential conductance

- Below  $T = 10$  K **oscillations in  $\partial I/\partial V$**  which are quasi-periodic in  $V_g$  **due to resonant tunneling**
- **Average differential conductance around  $3.2 e^2/h$**

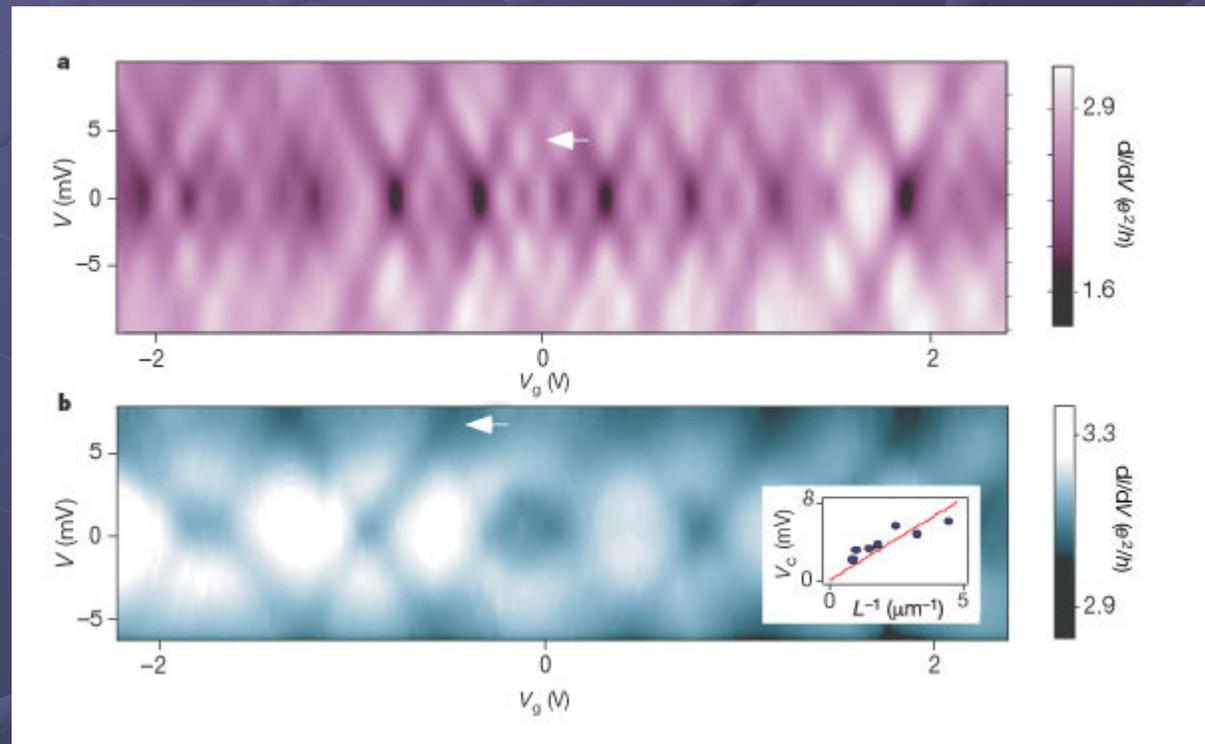


# $\partial I/\partial V - V - V_g$ patterns (I)

- Smooth mesh of crisscrossing dark lines due to **interference**
- **Average differential conductance 2-3  $e^2/h$**  for all devices
- Non-zero  $\partial I/\partial V$  indicates difference to Coulomb-diamonds

L = 530 nm

L = 220 nm



# $\partial I/\partial V - V - V_g$ patterns (II)

- First crossing of slopes at voltage  $V_c$  (indicated by  $\leftarrow$ ) is proportional to inverse length  $L$

$$V_c \sim 1/L \text{ (inset)}$$

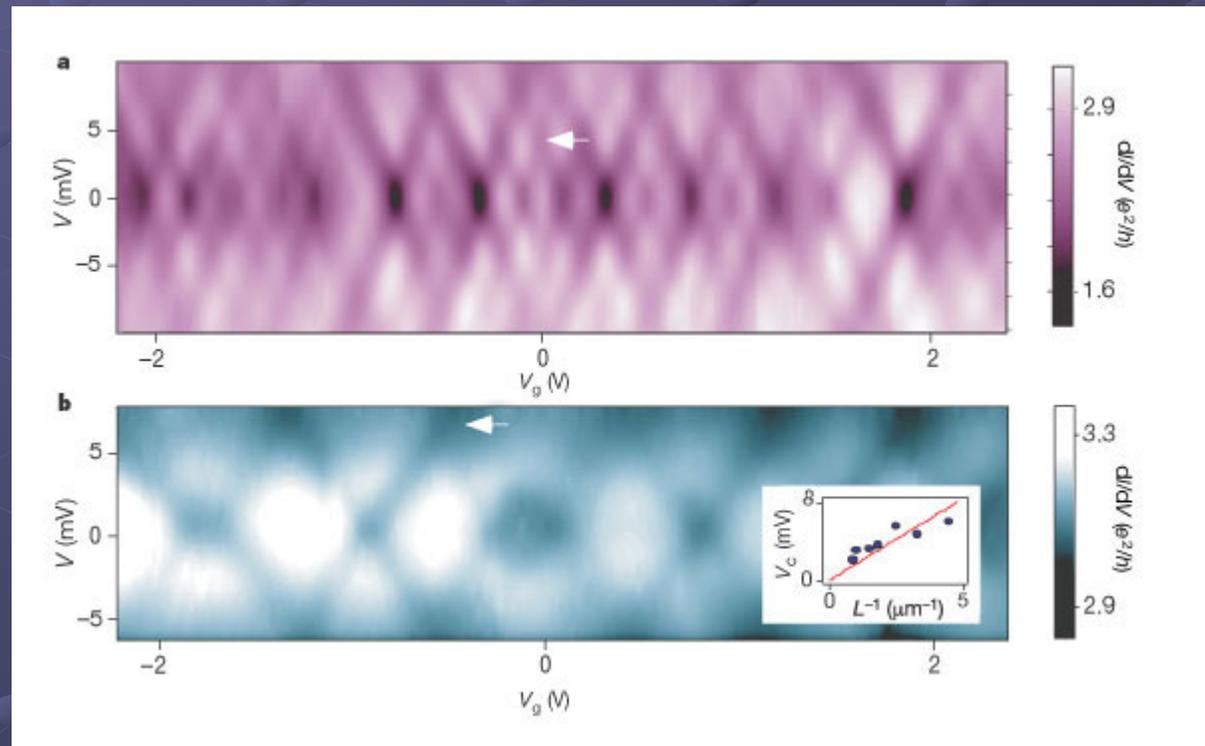
- electron scattering occurs mostly at the nanotube-metal interface (ballistic transport in the tube)

$L = 530 \text{ nm}$

$V_c = 3.5 \text{ meV}$

$L = 220 \text{ nm}$

$V_c = 6.5 \text{ meV}$

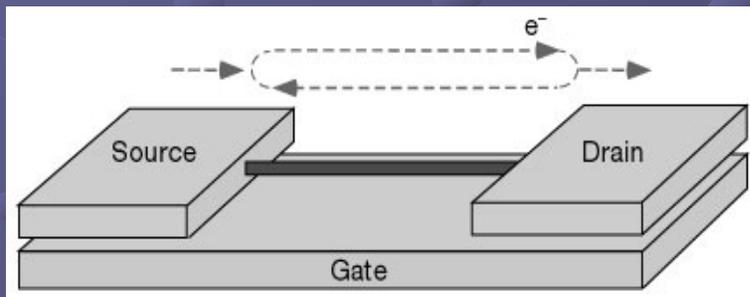


# About the paper title ...

Ballistic transport  
+ Scattering



Quantum interference  
between multiply reflected  
electron waves

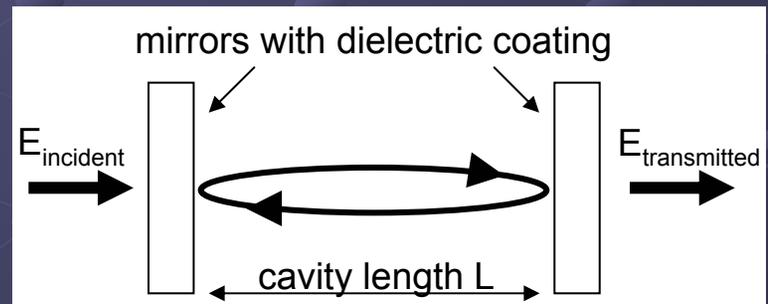


device characteristics

Fabry-Perot-Cavity



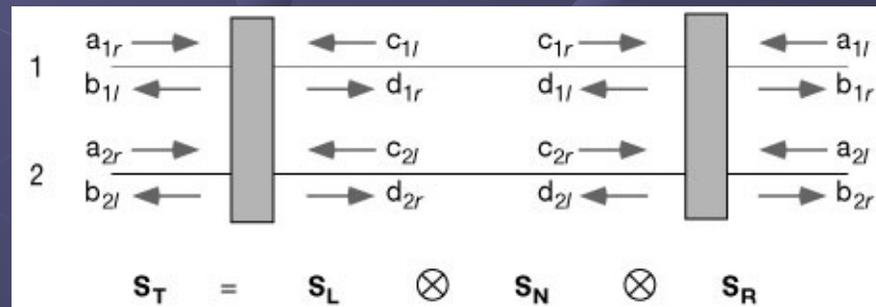
Light interference in cavity  
between multiply reflected  
light waves



transmitted intensity

# Theoretical model (I)

- Based on **multichannel Landauer-Büttiker formalism**
- The nanotube is considered as a **coherent waveguide with two propagating modes**
- Electron scattering modelled by **4x4 scattering matrices**  
 $S_L$ ,  $S_R$  (for interfaces) and  
 $S_N$  (energy depending phase accumulation)



# Theoretical model (II)

- The **current** through the nanotube can be calculated by

$$I = \frac{2e}{h} \int_{-eV/2}^{eV/2} \sum_{i,j=1,2} |t_{il,jr} \left( \frac{E}{\hbar v_F} + \frac{\pi}{4} \frac{LC_L V_g}{e} \right)|^2 dE$$

- The **differential conductance** is hence given by

$$\frac{\partial I}{\partial V} = \frac{2e^2}{h} \left[ \sum_{i,j=1,2} |t_{il,jr} \left( \frac{eV}{2\hbar v_F} + \frac{\pi}{4} \frac{C_L V_g}{e} \right)|^2 + \sum_{i,j=1,2} |t_{il,jr} \left( \frac{-eV}{2\hbar v_F} + \frac{\pi}{4} \frac{C_L V_g}{e} \right)|^2 \right]$$

with  $C_L = 20$  electrons / V  $\mu\text{m}$

$v_F = 8 \cdot 10^5$  m / s

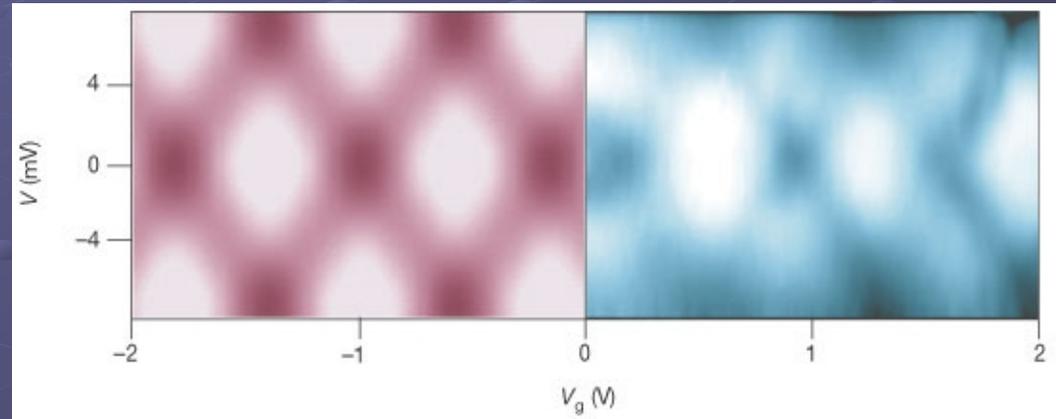
Capacitance per unit length

Fermi velocity

# Theoretical model (III)

- Good agreement between theoretical and experimental data

SWCNT of 220 nm length



- **But: Several features unexplained by theory**
  - magnitudes of  $\partial I/\partial V$  – dips show variations
  - superstructures in 2d-plot which are not periodic in  $V_g$
  - occurrence of heating or dephasing when electron energy deviates from equilibrium

# Interference effects in electronic transport through metallic single walled carbon nanotube

Phys. Rev. B, vol. 66, 073412

S. Krompiewski, J. Martinek and J. Barnas

presented by

Nitesh Ranjan

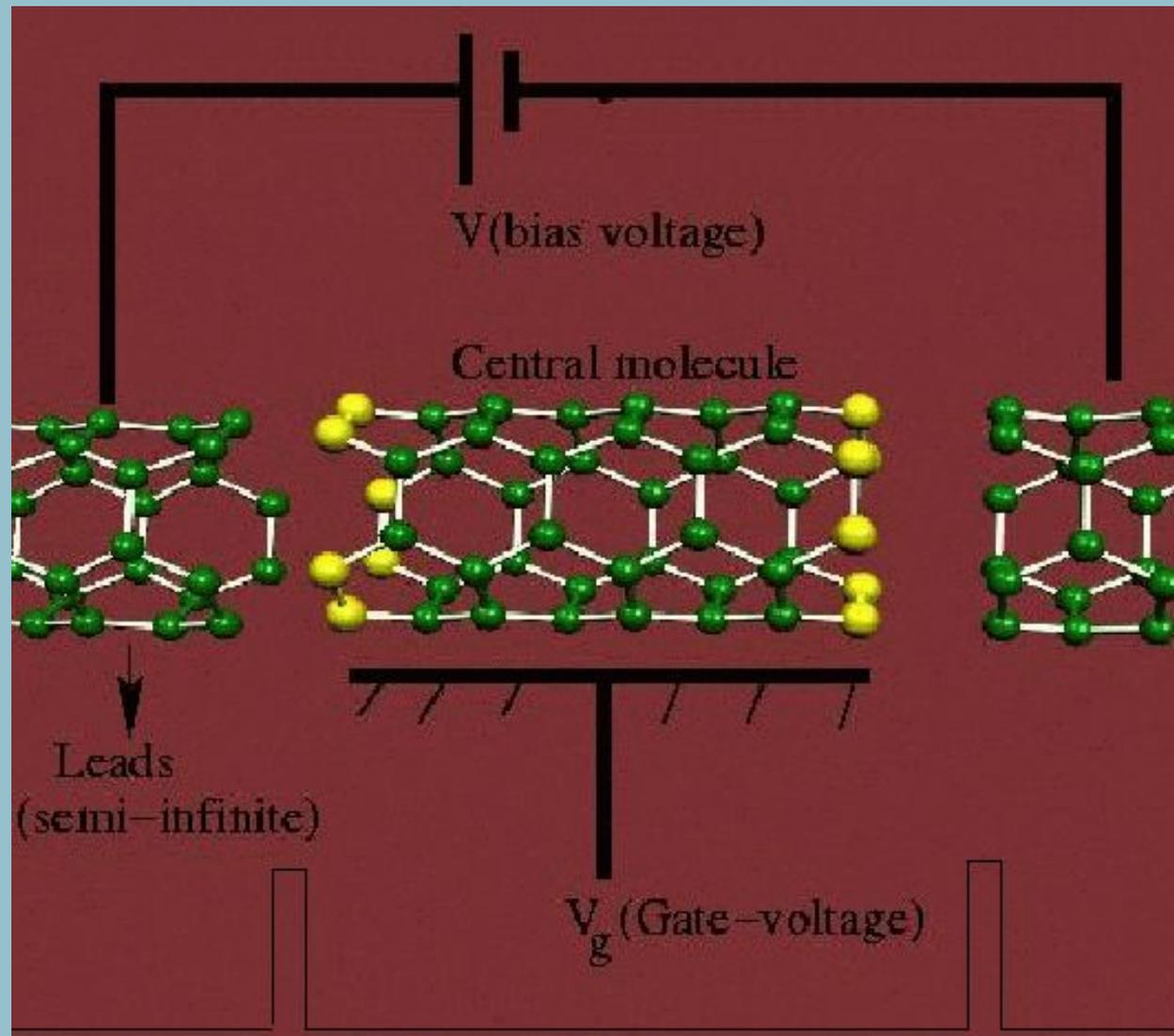
MCG, Uni-Regensburg

# Experimental Details

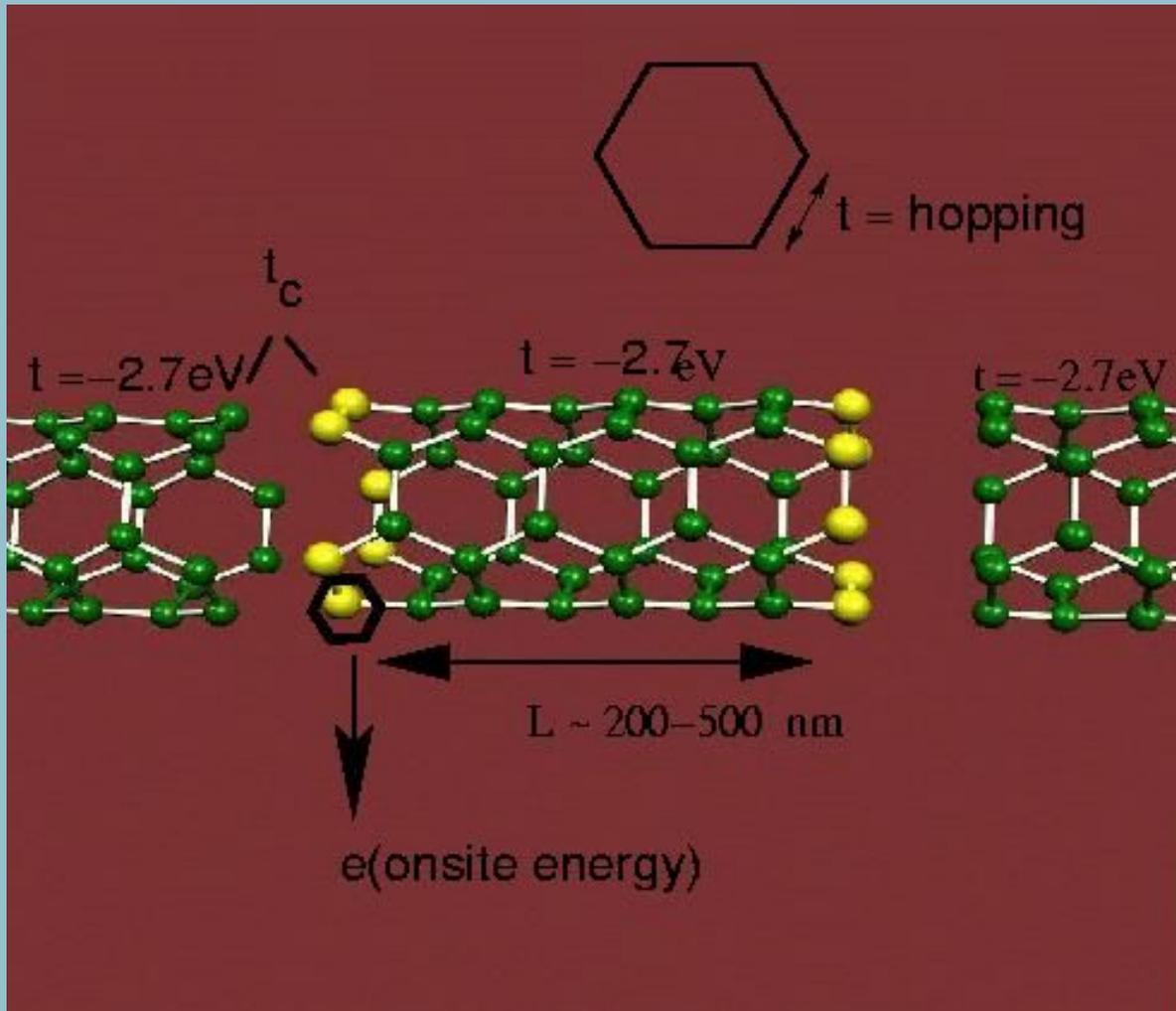
Liang. *et al*, Nature 411, 665 (2001)

- Metallic CNT **strongly coupled** to leads(Au/Cr).
- CNT behave as coherent molecular wave guides.
- Conductance show interference effects.
  - ◆ Multiple reflection from the boundaries.
- Color scale plot revealed characteristic diamond structures

# Visualization of theoretical model



# Our Parameters



\* (3,3) CNT

\*  $884\mu\text{c} \sim 220\text{nm}$

\*  $2129\mu\text{c} \sim 530\text{nm}$

# Theory

## Block Hamiltonian

$$\begin{pmatrix} G_L & G_{LM} & G_{LR} \\ G_{ML} & \boxed{G_M} & G_{MR} \\ G_{RM} & G_{RM} & G_R \end{pmatrix} = \begin{pmatrix} E - H_L & -H_{LM} & 0 \\ -H_{ML} & \boxed{E - H_M} & -H_{MR} \\ 0 & -H_{RM} & E - H_R \end{pmatrix}^{-1}$$

- Solving the equation for the molecule we get  $G_M = [E - H_M - \Sigma_L - \Sigma_R]^{-1}$  **Dyson's**

### Equation

$\Sigma_{L/R}$  are the self energies given by :

$$\Sigma_\alpha = H_{M\alpha} g_\alpha^S H_{\alpha M}, \quad \alpha = L/R$$

- $g_\alpha^S$  is the surface green function of the contact(Right/Left) obtained by iterative calculation of transfer matrices.

## Transmission

$$T_{LR}(E) = \text{Tr}(\Gamma_L G_M \Gamma_R G_M^\dagger) \quad \text{Landauer formula}$$

$$\Gamma_\alpha = i(\Sigma_\alpha - \Sigma_\alpha^\dagger) \quad \alpha = L/R$$

## Differential conductance

Temperature = 0 K

$$I(V) = \frac{2e}{h} \int_{-\infty}^{\infty} [f(E - \frac{eV}{2}) - f(E + \frac{eV}{2})] T(E, \phi) dE$$

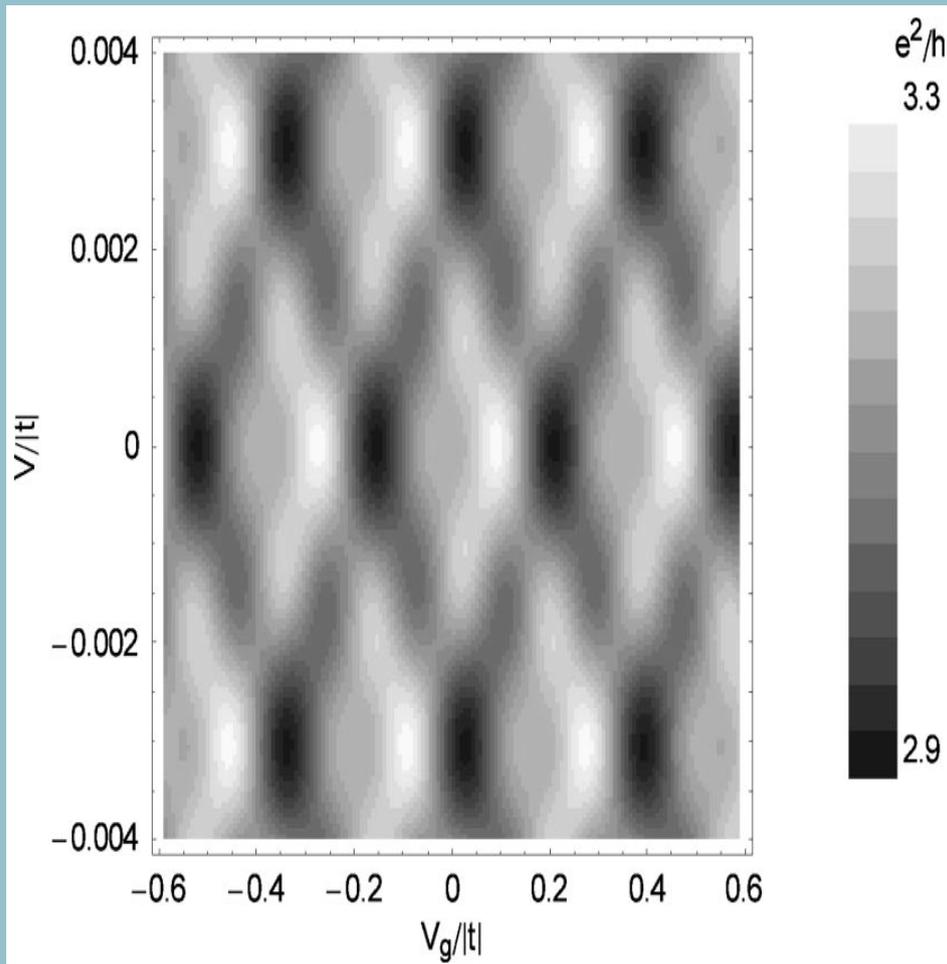
$$\frac{\partial I(V)}{\partial V} = \frac{e^2}{h} [T(E_F + \frac{eV}{2}, \phi) + T(E_F - \frac{eV}{2}, \phi)]$$

$V$  = applied bias

$\phi$  = average potential energy due to applied gate voltage

# Results

Length of CNT = 220nm

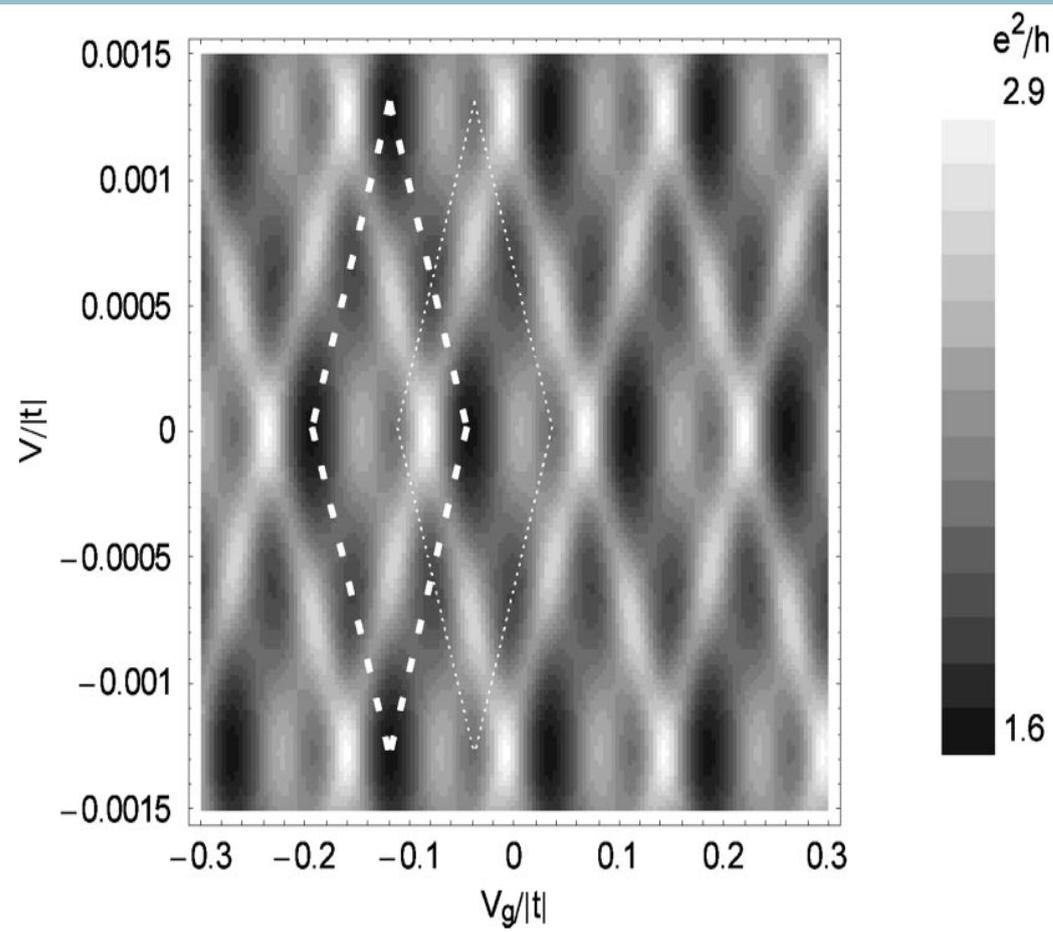


- \* Period in  $V = 16.7\text{mV}$ ; from figure
- \* Estimated value =  $15.2\text{mV}$
- \* From experiment =  $13\text{mV}$

$$**t_c = -1.971\text{eV} ; \epsilon = 0.189\text{eV}$$

# Results(II)

Length = 530nm



\* Period in  $V = 6.78\text{mV}$  (from Figure)

\* Estimated value =  $6.8\text{mV}$

\* From experiments =  $7\text{mV}$

\*\*  $t_c = -1.62$ ,  $\epsilon = 0.945$

\*\* double-diamond structures; observed when difference between  $t_c$  &  $t$  increases.

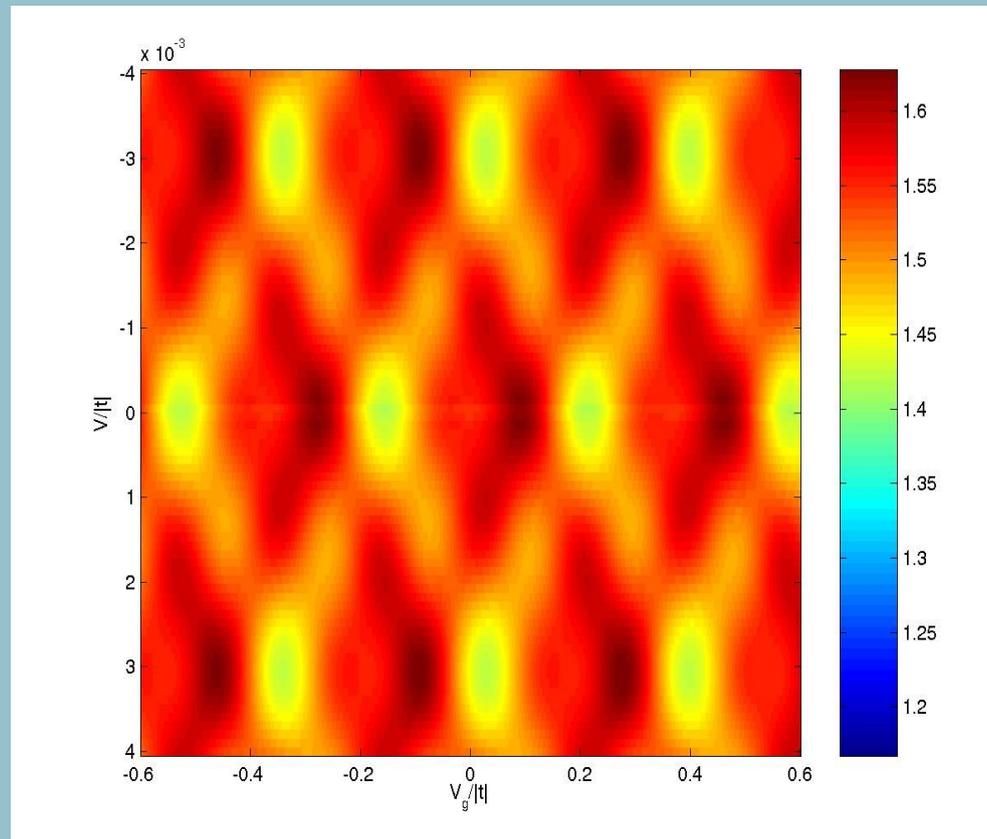
# Results(III)

- Periods in  $V$  and  $V_g \propto 1/L$
- $t_c$  determine coupling of the leads (strong coupling or weak confinement)
- Onsite energy acts as additional reflecting factor.
- These two defects acting together gives satisfactory results.

# Our Calculations, To motivate some discussion !!

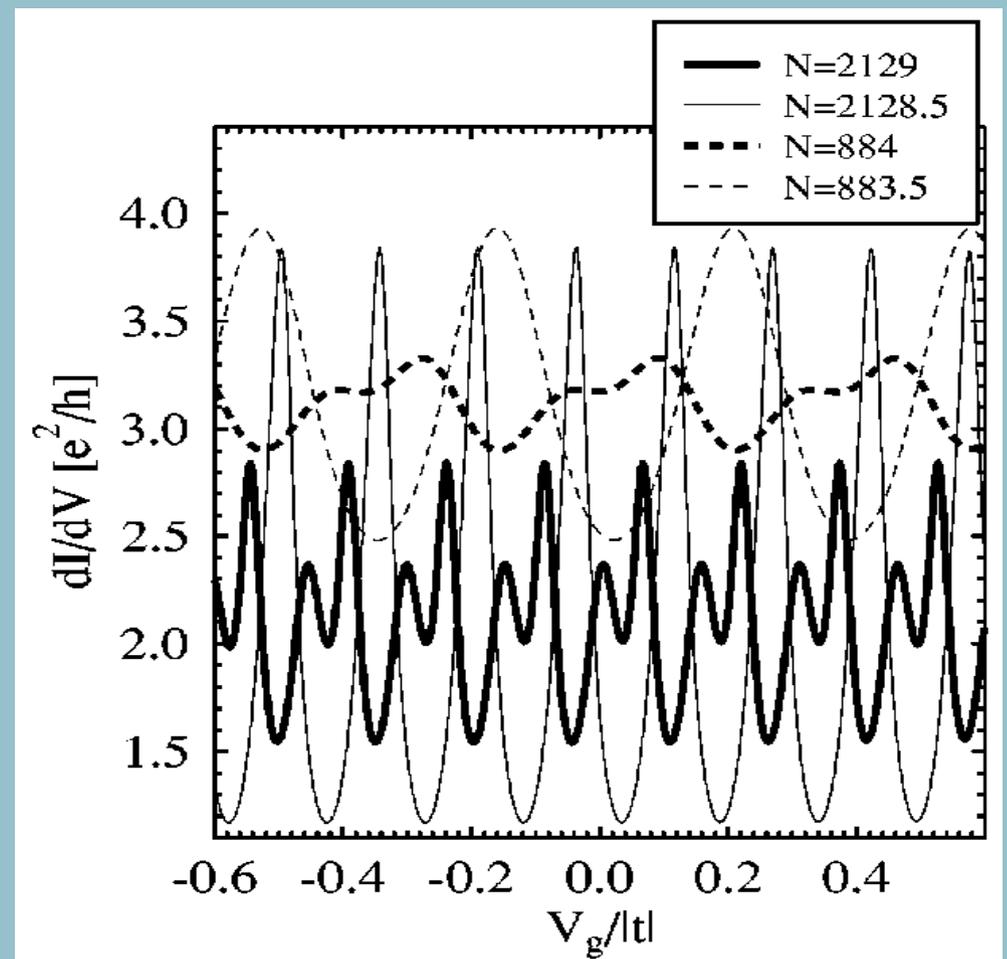
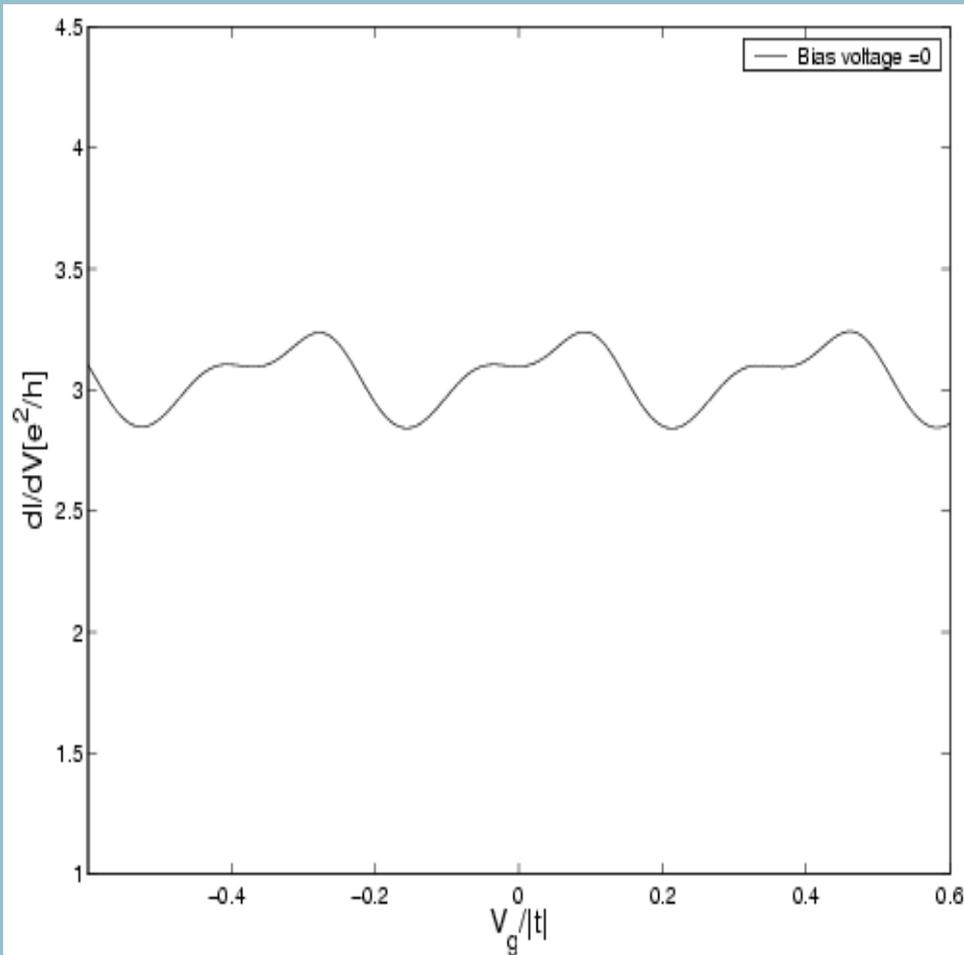
(3,3)CNT, 884  $\mu\text{c}$   $\sim$  220nm

$$t_c = -1.971 \quad \varepsilon = 0.189$$



# Our Calculations(II)

## Differential Conductance



# **Discussion !!**

**Some ideas to incorporate in the model !!**