Half-Integer Shapiro Steps at the 0-π Crossover of a Ferromagnetic Josephson Junction

- Investigation of current phase relation (CPR) of SFS junctions near the crossover between the 0 and the π ground state (driven both by thickness and temperature)
- At a certain thickness (17 nm) a nonzero critical current is observed which is analyzed by a high frequency excitation
  ⇨ observation of half-integer Shapiro steps
- These half-integer steps are attributed to the doubling of the Josephson frequency due to a sin(2γ) component of the CPR explained by level splitting of the energy levels in the ferromagnetic exchange field

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Outline of the talk

- Introduction to the RSJ-model
  - resistively (and capacitively) shunted junction (RCSJ)
  - mechanical analog
  - explanation of Shapiro steps
- Introduction to ferromagnetic Josephson junctions
  - quick review of energy level distribution
  - peculiarity of ferromagnetic junctions
  - π state
- Half-integer Shapiro steps at the 0-π crossover of a ferromagnetic Josephson junction
  - setup
  - experiment
  - interpretation

Introduction to the RSJ-model

- fundamental Josephson relations:
  (1) $$I_s = I_c \sin(\gamma)$$
  $$\frac{d\gamma}{dt} = \frac{2eV}{\hbar} = \frac{2\pi V}{\Phi_0}$$
  (2)

  with γ being the gauge invariant phase

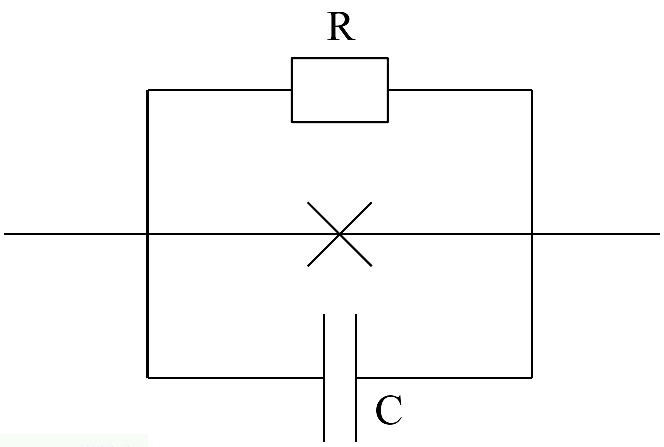
- (1) is sufficient for dc phenomena but for situation involving the ac Josephson effect (2) a more complete description is required

- model the physical Josephson junction by an ideal one described by (1), shunt⇨by a resistance R and a capacitance C

⇨ R builds in dissipation in finite voltage regime

C reflects the capacitance between the electrodes

The RSJ-model



- equivalent circuit of the junction:
- the bias current is then given by:

$$I = I_c \sin(\gamma) + \frac{V}{R} + \frac{dV}{dt}$$

(3)

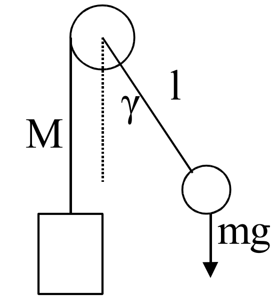
- replacing V in favor of γ using (2) one gets

$$I = I_c \sin(\gamma) + \frac{\Phi_0}{2\pi R}\dot{\gamma} + C\frac{\Phi_0}{2\pi}\ddot{\gamma}$$

(4)

- there exists only an analytic solution to this in the limit of C=0 but one can understand the behavior qualitatively by considering a mechanical analog

Mechanical Analog: The Physical Pendulum



- the equation of motion for a physical pendulum is

$$M = mgl\sin\gamma + \Gamma\dot{\gamma} + \Theta\ddot{\gamma} \qquad (5)$$

with
- Θ: momentum of inertia
- Γ: damping constant
- M: external torque

Pendulum	Jos. Junction
M	I
mgl	I_c
Γ	$\Phi_0/(2\pi R)$
Θ	$C\Phi_0/(2\pi)$

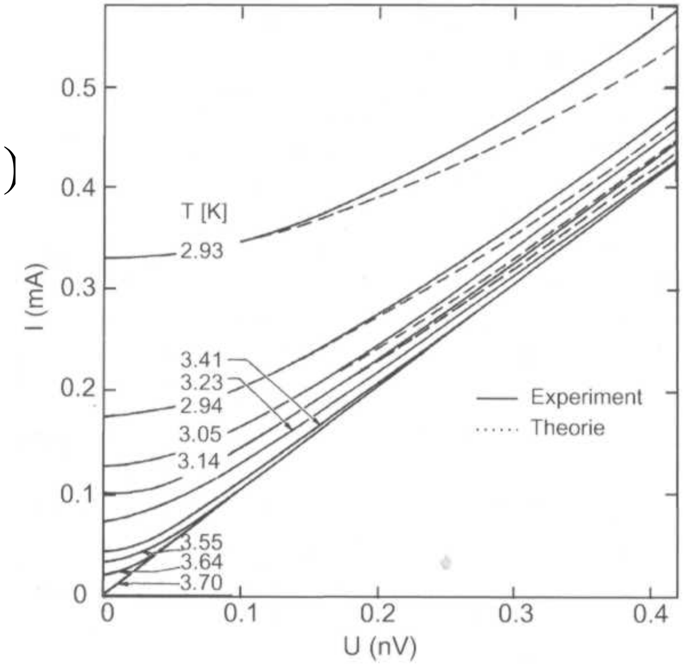
- the analogy is the given by identifying the angle γ with the gauge invariant phase

- the change in γ with time (angular velocity) corresponds to the voltage drop V over the Junction

Approximation: the overdamped case

- for most cases one can use the approximation RC→0 yielding a first order nonlinear differential equation

$$\dot{\gamma} = \frac{mgl}{\Gamma}\left(\frac{M}{mgl} - \sin\gamma\right) \quad (6) \qquad \dot{\gamma} = \frac{2\pi R I_c}{\Phi_0}\left(\frac{I}{I_c} - \sin\gamma\right) \quad (7)$$

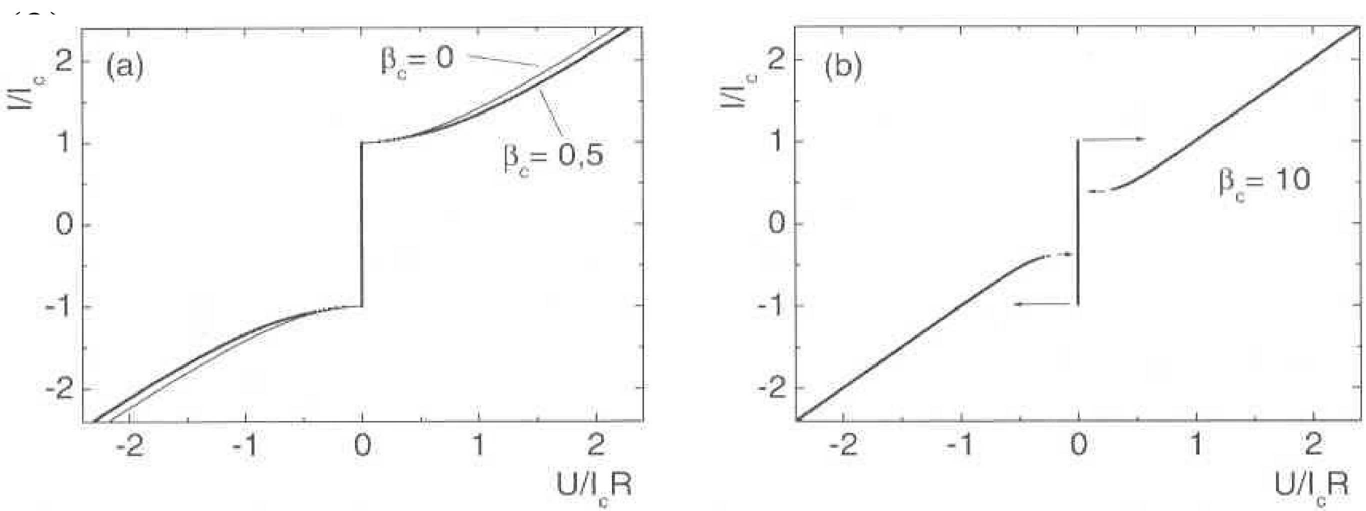


- the time average voltage can be found by integrating over time and using the Josephson frequency relation (2) giving

$$V = R(I^2 - I_c^2)^{1/2} \quad (8)$$

Remarks about numerical solution

- the RSJ-model can of course be treated numerically for which occasion one usually defines one single dimensionless parameter using the characteristic values of the junction: $V_c = I_c R$
  $\tau_c = \Phi_0 / 2\pi I_c R$
  $f_c = 1/2\pi\tau_c$
- with those one can write the differential equation in the compact form $$i = \sin\gamma + \dot{\gamma} + \beta_c \ddot{\gamma}$$



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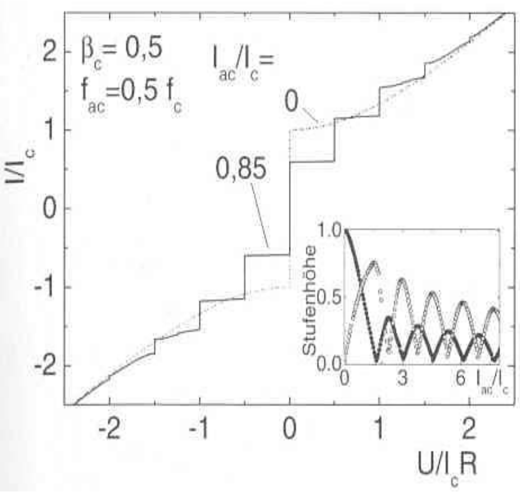
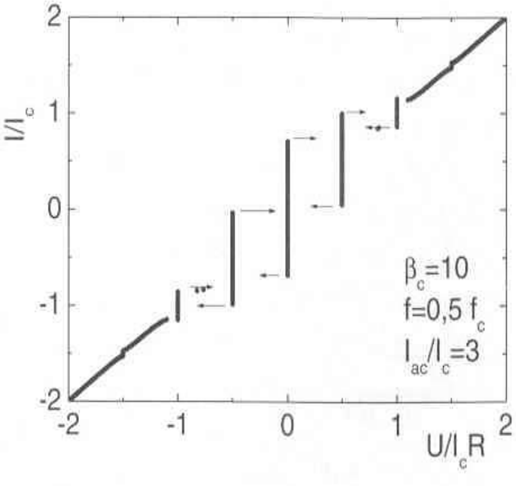
rf-Driven Junctions

- mechanical analog: nonlinear oscillator is tuning its resonance to the driving frequency over a certain frequency interval
- so for one value of applied torque there's is a range of values of angular velocities
- or similar for one value of angular velocities there is a range of torques that give that velocity
- for the Josephson junction this means
  that there will be the same voltage drop over the junction for a range of different bias current values
  namely those that
- this leads to the Shapiro steps





Quick glance at the calculation

- suppose the junction is biased with $V=V_0+V_1\cos(c\omega_1 t)$ then the time dependent phase is given by
  $$\gamma(t) = \gamma_0 + \omega_0 t + \frac{2eV_1}{\hbar\omega_1}\sin\omega_1 t$$
  (10)

  which results in a CPR of $$I_s = I_c \sum (-1)^n J_n\left(\frac{2eV_1}{\hbar\omega_1}\right)\sin\left(\gamma_0 + \omega_0 t - n\omega_1 t\right)$$

  (11)

  where J_n is the nth order Bessel function

- this contribute to a dc component only when ω_0 $V_n = n\hbar\omega_1/2e$ the dc voltage V_0 has one of the Shapiro step values

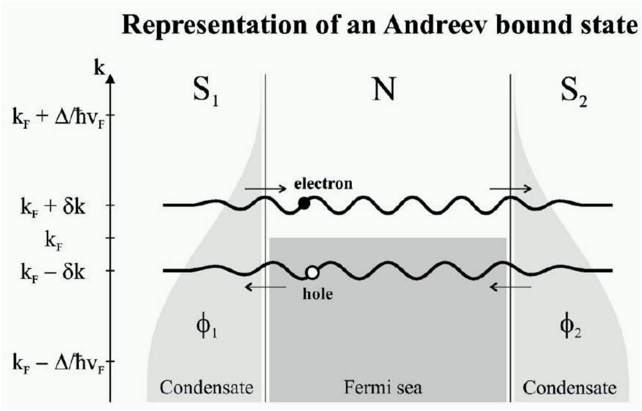
  $$I_n = I_c J_n(2eV_1/\hbar\omega_1)$$

  (12)
- using (11) one can also deduce the half-width of the nth step

(13)

Introduction to Ferromagnetic Josephson Junctions

- formation of bound states when the phase difference of the electron and the hole match the macroscopic phase difference of the to superconducting wave functions



- in the ferromagnet an additional phase shift due to the exchange energy

$$q = E_{ex} / \hbar v_F$$

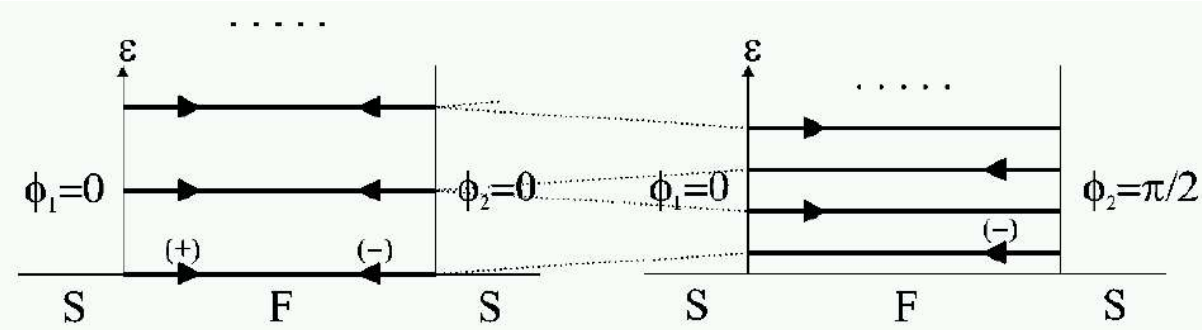
arises

$$\begin{cases} k_e^{\downarrow}(\epsilon) = k_F - q + \delta k \\ k_h^{\uparrow}(\epsilon) = k_F + q - \delta k \end{cases}$$

$$\begin{cases} k_e^{\uparrow}(\epsilon) = k_F + q + \delta k \\ k_h^{\downarrow}(\epsilon) = k_F - q - \delta k \end{cases}$$

Andreev bound states in SFS junctions

- In the ferromagnet the two spin directions of each bound state are indeed shifted by the exchange energy
- In the case that the exchange energy shifts the first level from finite energy to zero energy the direction of the lowest level is negative
- In this case the ground state is at $\Phi=\pi$



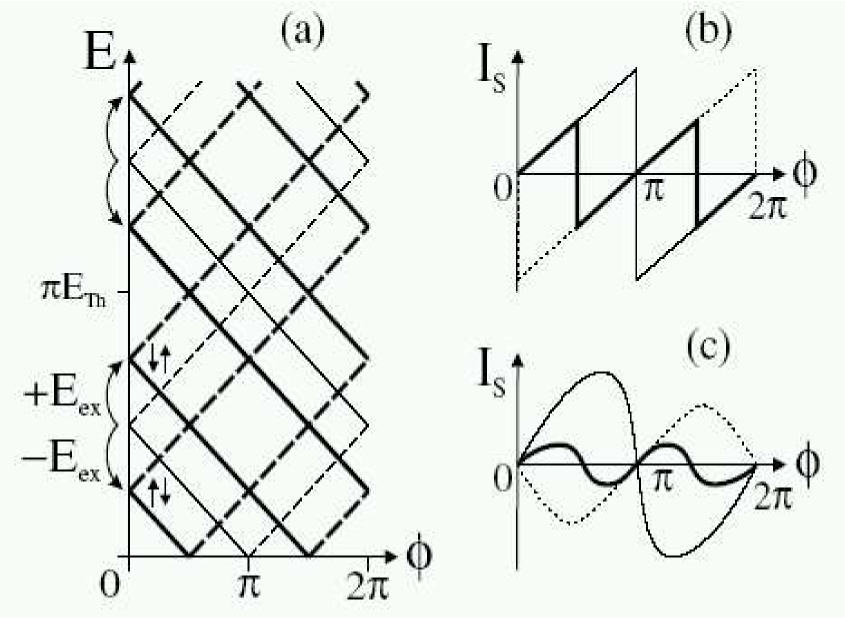
Crossover from 0 to π

- consider the situation where the exchange energy doesn't shift the first energy level completely to zero energy but only half-way

  ⇨ the spectrum contains equidistant levels twice closer than usual

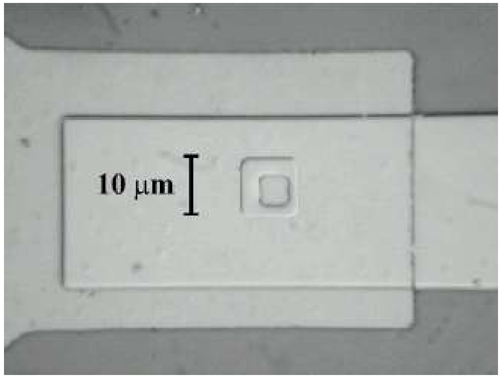
  ⇨ the supercurrent is π periodic in phase

- at this 0-π crossover the CPR contains a dominant sin(2γ) component and the critical current has a nonzero minimum with respect to thickness or exchange energy
- so far the critical current of SFS junctions at the crossover was assumed to vanish completely

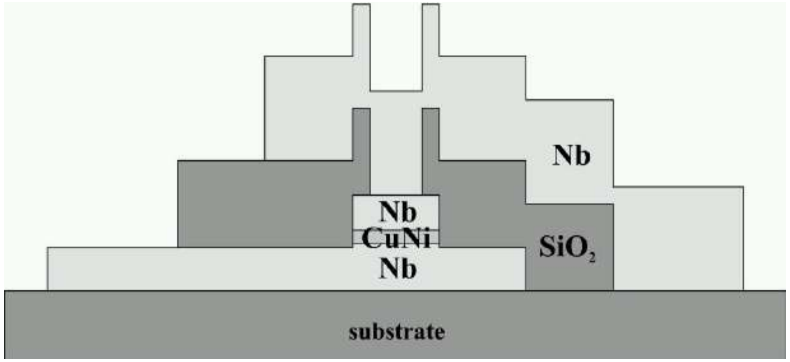


Experimental Setup

- $Nb/Cu_{52}Ni_{48}/Nb$ trilayers deposited in situ and patterned by photolithography



- $T_{curie} = 20K$
- the current through the junction is measured by a SQUID feedback method
- thicknesses of 17 and 19nm

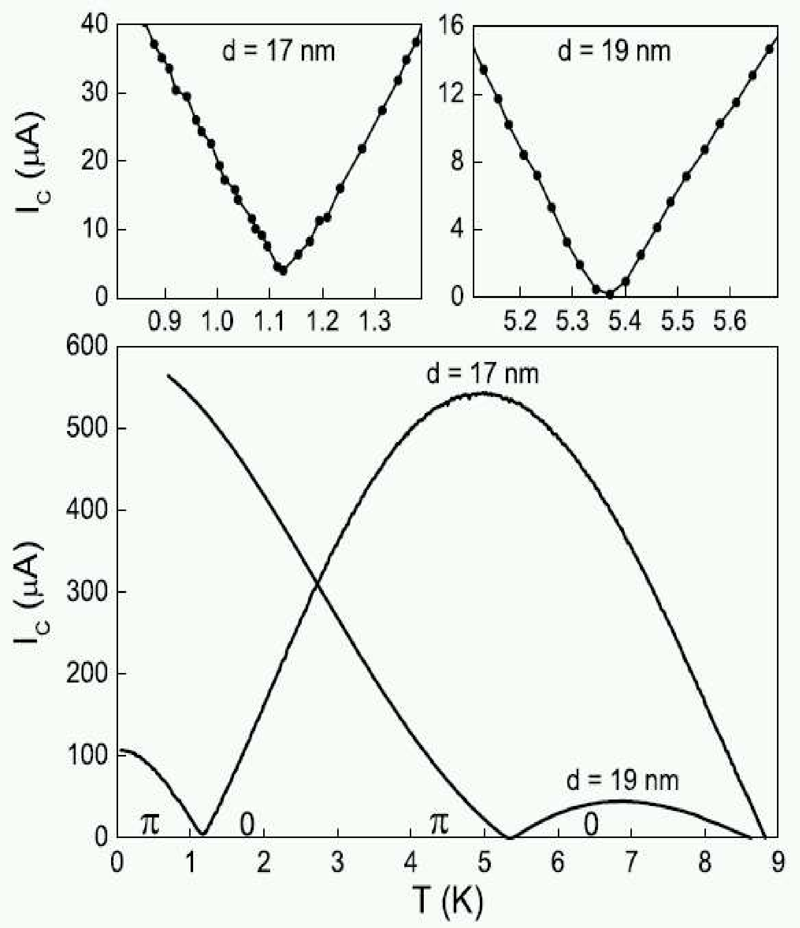


Temperature dependence of critical current

- the critical current presents a deep minimum
  at T^*, the crossover temperature

- there exists a finite current of 4μA at T^* for the 17nm thick junction

- with its crossover temperature so close to zero this junction is almost the optimum case where E_{ex}/E_{th} gives a doubled periodicity of Andreev levels



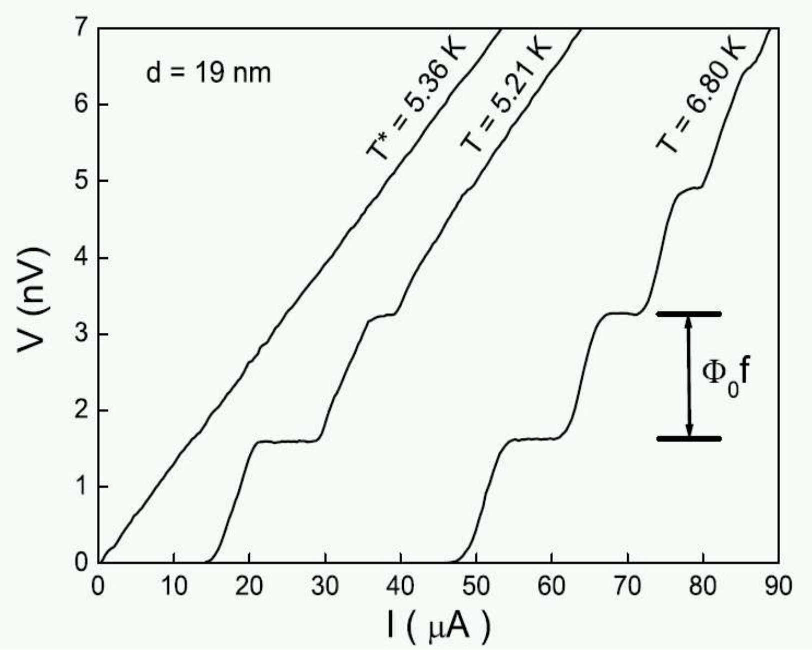
Experiment for 19 nm

- the CPR is analyzed by sending an alternative current at f=800 kHz through the junction when measuring the direct current-voltage curve

- constant voltage Shapiro steps appear due to the synchronization of the Josephson oscillations with the applied excitation

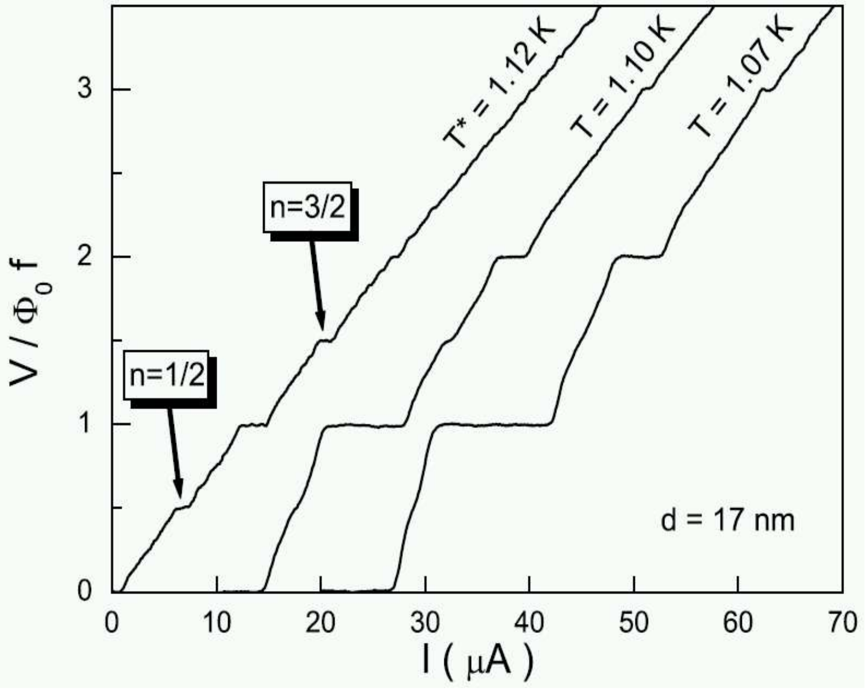
- as expected (by RSJ) the steps appear at voltages equal to $V_n = n\Phi_0 f$

- no steps are present at the crossover temperature T*(19nm)=5.36K



Experiment for 17 nm

- in this case the steps are still present at $T^*(17nm)=1.12K$
- additional steps appear at n=1/2 and n=3/2
- the half-integer steps reveal the existance of supercurrent oscillations at a frequency of $2(V/\Phi_0)$ which synchronize to the excitation at frequency f producing steps at voltage multiples of $\frac{1}{2}(\Phi_0 f)$



Discussion of results 1

- ideally the sin(2γ) component should dominate for this thickness of junction at all temperatures T<<Tc
- the fact that they are visible only close to the crossover temperature T* indicates that the phase dependence departs only slightly from sin (γ)
- the sin(2γ) component dominates only when the large sin(γ) cancels to change its sign
- to first order near T* the supercurrent can be described by

$$I_s(\gamma) = \frac{T - T^*}{T^*} I_1 \sin\gamma + I_2 \sin 2\gamma$$

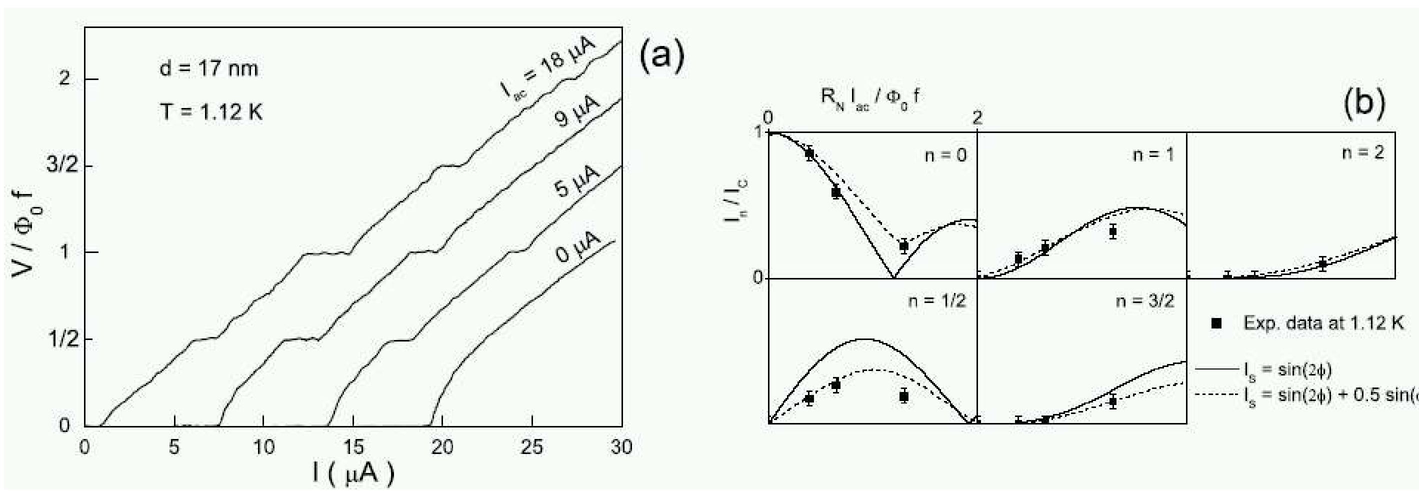
(16)

- in contrast to other situations with half-integer steps they only appear here because of

  the crossover
  the low temperature
  the finite critical current

Discussion of results 2

- investigation of spectral composition in greater detail by applying different excitation amplitudes (18, 9, 5, and 0 μA) at d = 17 nm and $T = T^{*} = 1.12$ K
- comparison of the Shapiro step widths to different RSJ-model predictions



- the best fit of the experimental data is achieved with a spectral composition of $Is = \sin(2\gamma) + 1/2\sin(\gamma)$

Summary

- the study of the finite voltage behavior of SFS junctions under high frequency excitation revealed half-integer Shapiro steps at the crossover temperature where the critical current is nonzero

- the steps show the $\sin(2\gamma)$ dependence of the current phase relation which can be explained by specific level splitting realized at the crossover

- CPR of tunnel junctions:

$$I_s = I_c \sin(\gamma)$$

(14)

- CPR of SNS or SFS junctions are predicted to be nonsinusoidal:

$$I_s(\gamma) = \sum_{n=1}^{\infty} I_c^{(n)} \sin n\gamma$$

(15)

due to a different conduction mechanism explained by Andreev bound states

- additionally the CPR in SFS junctions is strongly distorted by the exchange field of the ferromagnet and can even be reversed ⇨ π state