Single electron spin decoherence in a quantum qot.

Outline of the talk:

1. Introduction, mechanisms of spin relaxation and decoherence in GaAs quantum dots.
2. Spin-orbit interaction. Suppression of the spin relaxation for the localized electron state.
3. Absence of the spin decoherence due to spin-orbit interaction in GaAs quantum dots.
4. Spin relaxation for the interacting problem (at the singlet-triplet transition).
5. Decoherence due to inhomogeneous hyperfine coupling to the nuclei .
6. Conclusions.

A. Khaetskii, V. Golovach , D. Loss, L. Glazman, J. Schliemann

The spin dynamics of electrons in semiconductors is of central interest now (particularly, the controlled manipulation of spin).

S.A. Wolf et al., Science **294**, 1488 (2001).

J.M. Kikkawa, D.D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).

Salis et al., PRL **86**, 2677 (2001); PRB **64**, 195304 (2001).

Which mechanisms of the spin phase decoherence are important for electrons confined to a closed system like a quantum dot with a spin ½ ground state?

These systems are promising candidates for scalable spin qubits-

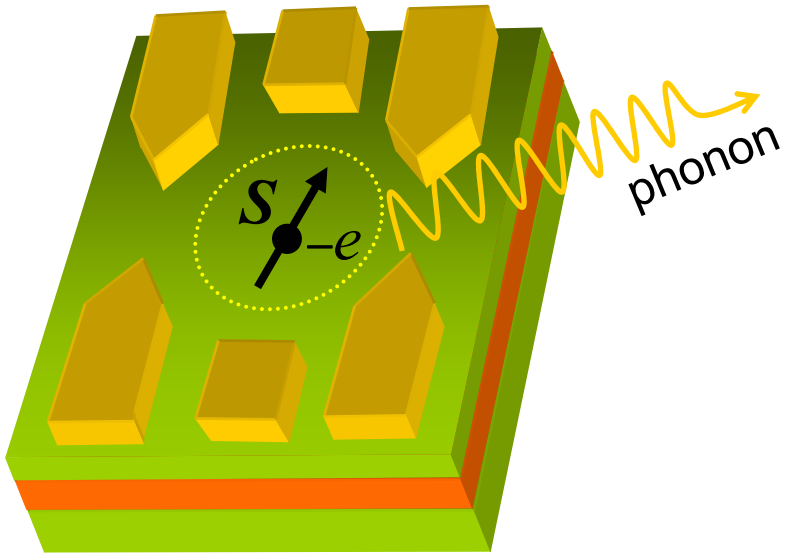
D. Loss, D.P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

Recent measurements of T_1 time in a single electron and 2 electron quantum dots:
T. Fujisawa et al., Nature **419**, 278 (2002); Phys. Rev. B **63**, R081304 (2001).

R. Hanson et al., PRL **91**, 196802 (2003).

Extremely long T_1 time was observed ! 50-200 μs

Relaxation and Decoherence of the Electron Spin in Quantum Dots



Sources:

- spin-orbit (S.-O.) interaction
- hyperfine interaction with nuclei
- Zeeman interaction with external magnetic fields
- exchange interaction with neighboring quantum dots or magnetic impurities

The main sources of spin relaxation and decoherence in **GaAs QD:**
1) Spin-orbit interaction 2) Hyperfine interaction with nuclei

Hyperfine interaction:

A. Khaetskii, D. Loss, L. Glazman, PRL, 2002;
J. Schliemann, A. Khaetskii, D. Loss, J.Phys.: Cond. Matt., 2003.-Topical Review
I. Merkulov, Al. Efros, M. Rosen, Phys. Rev. B, 2002.
R. De Sousa, Das Sarma, Phys. Rev. B, 2003.
S.I. Erlingsson, Yu.V. Nazarov, Phys. Rev. B, 2002.

Some properties of the GaAs like crystals

There is no inversion symmetry → a) spin-orbit splitting in the electron spectrum
b) strong coupling to the piezo-phonons

→ For delocalized (3D, 2D) states spin-orbit interaction is often the main source of spin-flips (D`yakonov-Perel). However, it is not the case for the localized electron states!

$$H = \frac{p^2}{2m^*} + U(r) + U_{e-ph}(r) + H_{SO} + H_{ph} + \frac{1}{2} g\mu_B \boldsymbol{\sigma}\mathbf{B}$$

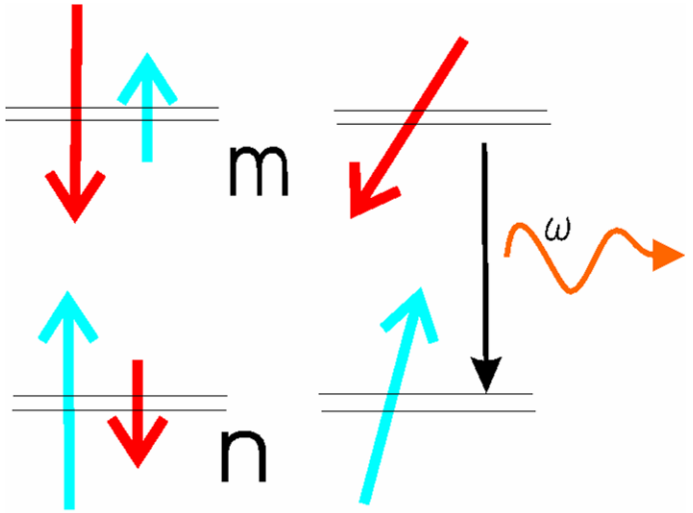
$$H_{SO} = \beta(-p_x\sigma_x + p_y\sigma_y) + \alpha(p_x\sigma_y - p_y\sigma_x)$$

$$\mathbf{p} = -i\hbar\nabla + \frac{e}{c}\mathbf{A}(\mathbf{r})$$ - 2D momentum operator

$$U_{e-ph}(r) = \sum_{j\mathbf{q}} \frac{F(q_z)e^{i\mathbf{q}_{//}\mathbf{r}}}{\sqrt{2\rho\omega_{qj}}}(e\beta_{\mathbf{q}j} - iq\Sigma_{\mathbf{q}j})(b^{+}_{-\mathbf{q}j} + b_{\mathbf{q}j})$$

Mechanisms of the spin-flip

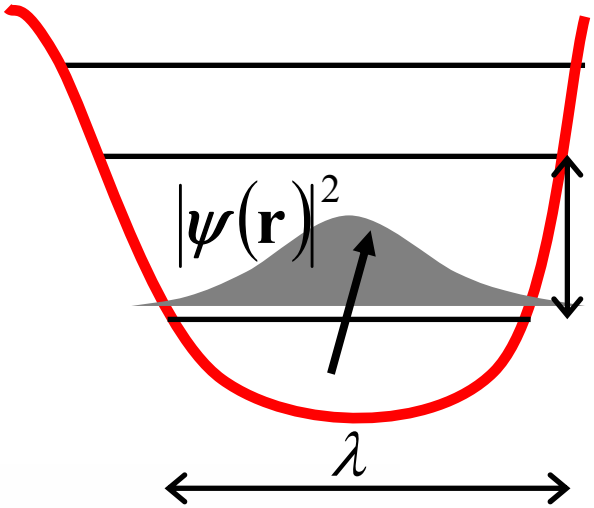
1) The admixture mechanism.



➔

2) Direct spin-phonon coupling.

In the presence of the spin-orbit term the electron 'spin-up' state contains a small admixture of the 'spin-down' state and vice-versa



$$\hbar\omega_0 \sim \frac{\hbar^2}{m^*\lambda^2}$$

$$\Phi_{n,\uparrow} = \phi_n(\mathbf{r})|\uparrow> + \sum_{k\neq n} \frac{(\hat{H}_{SO})^{\downarrow\uparrow}_{kn}}{(E_n - E_k + g\mu B)}\phi_k(\mathbf{r})|\downarrow>; \quad \frac{\langle n|H_{SO}|m\rangle}{E_n - E_m} \sim \frac{\lambda}{\lambda_{SO}} << 1$$

$$\Phi_{m,\downarrow} = \phi_m(\mathbf{r})|\downarrow> + \sum_{k\neq m} \frac{(\hat{H}_{SO})^{\uparrow\downarrow}_{km}}{(E_m - E_k - g\mu B)}\phi_k(\mathbf{r})|\uparrow>, \quad \lambda_{SO} = \hbar / m^*\beta$$

We expect: spin-flip rate for the transitions between different orbital states

$\Gamma_{\uparrow\downarrow} \propto \Gamma_0 (\frac{m^* \beta^2}{\hbar\omega_0})$,where Γ_0 is the energy relaxation rate.

Sic! The effect of the spin-orbit interaction cancels in the first order in the spin-orbit coupling constant (in strong contrast to the situation in the bulk).

$$\Phi^*_{n\uparrow}\Phi_{m\downarrow} = 0$$ **, i.e. proportional at least to** β^2

➔ Then the spin-flip rate $\Gamma_{\uparrow\downarrow} \propto \beta^4$

This can be seen by making spin-dependent transformation which removes the term linear in β from the Hamiltonian

$$\hat{\Phi} = \hat{U}(\hat{\sigma}, x, y)\hat{\Phi}'$$ (valid only for localized state!)

A.V. Khaetskii and Yu.V. Nazarov, PRB **61**, 12639 (2000);

B.I. Halperin et al., Phys.Rev.Lett. **86**, 2106 (2001).

I.L. Aleiner and V.I. Fal'ko, Phys. Rev.Lett. **87**, 256801 (2001).

$$\Phi^{\dagger}_{n,\uparrow}\Phi_{m,\downarrow} = \sum_{k\neq m}\frac{(\hat{H}_{SO})^{\uparrow\downarrow}_{km}}{(E_m - E_k)}\phi_k(\mathbf{r})\phi^{\star}_n(\mathbf{r}) + \sum_{k\neq n}\frac{(\hat{H}_{SO})^{\uparrow\downarrow}_{nk}}{(E_n - E_k)}\phi^{\star}_k(\mathbf{r})\phi_m(\mathbf{r}) =$$

$$= (im_0/\hbar)\beta_{il}(\sigma_i)^{\uparrow\downarrow}[-\sum_{k\neq m}(x_l)_{km}\phi_k(\mathbf{r})\phi^{\star}_n(\mathbf{r}) + \sum_{k\neq n}(x_l)_{nk}\phi^{\star}_k(\mathbf{r})\phi_m(\mathbf{r})] =$$

$$= (im_0/\hbar)\beta_{il}(\sigma_i)^{\uparrow\downarrow}[-x_l\phi_m(\mathbf{r})\phi^{\star}_n(\mathbf{r}) + x_l\phi^{\star}_n(\mathbf{r})\phi_m(\mathbf{r})] = 0,$$

where

$$(\hat{H}_{SO})^{\uparrow\downarrow}_{km} = \beta_{il}(\hat{\sigma}_i)^{\uparrow\downarrow}(\hat{p}_l)_{km} = (im_0/\hbar)\beta_{il}(\hat{\sigma}_i)^{\uparrow\downarrow}(E_k - E_m)(x_l)_{km}.$$

$$< n\uparrow|\, U^{q\alpha}_{e-ph}\,|\, n\downarrow> = \sum_{k\neq n}[\frac{(U^{q\alpha}_{e-ph})_{nk}(H_{SO})^{\uparrow\downarrow}_{kn}}{E_n - E_k - g\mu_B B} + \frac{(H_{SO})^{\uparrow\downarrow}_{nk}(U^{q\alpha}_{e-ph})_{kn}}{E_n - E_k + g\mu_B B}]$$

For localized states in a quantum dot the spin-flip processes due to S-O interaction are strongly suppressed. ➔ unusually long T_1 time.

For transitions between different orbital levels $T_1 \sim 10^{-5} s$

! Contribution to the spin-flip rate $\propto \beta^2$ appears if one takes into account the finite Zeeman splitting ➔ additional smallness $(g\mu_B B/\hbar\omega_0)^2$.

For transition between Zeeman sublevels $T_1 \sim 1$ms at B=1T, $\hbar\omega_0$ =10K.

$$\frac{\hbar}{T_1} = \Lambda \frac{(g\mu_B B)^5}{(\hbar\omega_0)^4}(1+\cos^2\theta), \quad \Lambda = \frac{4}{35\pi}\frac{(eh_{14})^2\beta^2}{\rho\hbar}\left(\frac{1}{s_l^5}+\frac{4}{3s_t^5}\right)$$

This formula is valid in the limit:

$$\lambda << \lambda_{ph}, \hbar s/\lambda_{ph} = g\mu_B B$$

A.V. Khaetskii and Yu.V. Nazarov, PRB **64**, 125316 (2001).

What about decoherence, i.e. T_2 time? For the parameter range $T \ll \hbar\omega_0$ and $m^*\beta^2 \ll g\mu_B B \ll \hbar\omega_0$ we obtain the effective Hamiltonian:

$$H_{\text{eff}} = g\mu_B \hat{\mathbf{S}}\cdot\mathbf{B} + g\mu_B \hat{\mathbf{S}}\cdot\mathbf{h}(t),$$

$$\mathbf{h}(t) = 2\mathbf{B}\times\boldsymbol{\Omega}(t),$$

$$\boldsymbol{\Omega}(t) = <\psi\,|\,[(\hat{L}_d^{-1}\,\boldsymbol{\xi}), U_{e-ph}(t)]\,|\,\psi>, \psi -$$ is the orbital wave function.

Vector $\boldsymbol{\Omega}(t) \propto \beta, \alpha$ depends on the dot geometry and randomly on time.

In 1st order in spin-orbit interaction, there can be only transverse fluctuations of the effective magnetic field, i.e.

$$\mathbf{h}(t)\cdot\mathbf{B} = 0$$

This statement holds true for spin coupling to any fluctuations (not only to phonons).

V. Golovach, A. Khaetskii, D. Loss, cond-mat/0310655.

The full dynamics of the electron spin is governed by the Bloch equation:

$$\left\langle \dot{\mathbf{S}} \right\rangle = \boldsymbol{\omega} \times \left\langle \mathbf{S} \right\rangle - \Gamma \left\langle \mathbf{S} \right\rangle + \Im \quad \text{, where} \quad \begin{aligned} \boldsymbol{\omega} &= \omega \mathbf{B} / B, \\ \omega &= g\mu_B B / \hbar. \end{aligned}$$

For arbitrary fluctuation field $\mathbf{h}(t)$ the tensor $\Gamma = \Gamma^r(\omega) + \Gamma^d(0)$

where the relaxation and decoherence parts are expressed through the spectral function of the fluctuations:

$$J_{ij}(\varpi) = \int_0^{+\infty} \left\langle h_i(0) h_j(t) \right\rangle e^{-i\varpi t} dt$$

The tensor $\Gamma^r(\omega)$ is due to processes of energy relaxation.

The dephasing part $\Gamma^d(0)$ (which enters at zero frequency) can be non-zero only due to elastic scattering of spin. This term contributes to the decoherence time T_2

In the bulk, at $\omega \tau_c >> 1 \quad \Gamma^d(0) >> \Gamma^r(\omega)$

due to elastic scattering by impurities. That is why in the bulk usually $T_2 << T_1$

In our case $\Gamma^d(0)$ is identically zero for admixture mechanism due to the transverse nature of the fluctuating field.

For the other mechanisms, like direct spin-phonon coupling, is it zero because the density of acoustic phonons is zero at zero frequency.
Thus, there are no intrinsic dephasing mechanisms related to S-O interaction in our system at T << $\hbar\omega_0$ and, as a result,

$$T_2 = 2T_1$$

--- S-O interaction in GaAs quantum dots causes a spin decay with the largest possible decoherence time.

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{(g\mu_B)^2}{2\hbar^2}\int_{-\infty}^{+\infty}\langle h_Z(0)h_Z(t)\rangle dt$$

$$\frac{1}{T_1} = \mathrm{Re}(J_{XX}(\omega) + J_{XX}(-\omega)) + \mathrm{Re}(J_{YY}(\omega) + J_{YY}(-\omega))$$

The X,Y plane is perpendicular to the direction Z of the external magnetic field **B**

Spectral function:

Bose function

$$\mathrm{Re}\,J_{XX}(z)=\frac{\omega^2 z^3\left(\bar{N}_z+1\right)}{\left(\Lambda_+ m^* \omega_0^2\right)^2}\sum_j \frac{\hbar}{\rho_0 s_j^5 \pi}\int_0^{\pi/2} d\theta \sin^3\theta$$

$$\times e^{-(z\lambda\sin\theta)^2/2s_j^2}\left|F\left(\frac{|z|}{s_j}\cos\theta\right)\right|^2\left(e^2\bar{\beta}_{j\theta}^2+\frac{z^2}{s_j^2}\bar{\Xi}_j^2\right)$$

$z \to \omega = g\mu_B B$ — quantum well — piezo — deformation

$s_1 \approx 4.7\times10^3\, m/s, s_2 = s_3 \approx 3.37\times10^3\, m/s$ speed of sound

$\bar{\Xi}_j = \delta_{j,1}\Xi_0, \Xi_0 \approx 7\,eV$, $\bar{\beta}_{1,\vartheta} = 3\sqrt{2}\pi h_{14}\kappa^{-1}\sin^2\vartheta\cos\vartheta$, $\bar{\beta}_{2,\vartheta} = \sqrt{2}\pi h_{14}\kappa^{-1}\sin 2\vartheta$,

$\bar{\beta}_{3,\vartheta} = 3\sqrt{2}\pi h_{14}\kappa^{-1}(3\cos^2\vartheta-1)\sin\vartheta$, $h_{14} \approx 0.16\,C/m^2$, $\kappa \approx 13$

$$\frac{2}{\Lambda^2_{\pm}} = \frac{1-l_{x'}^{\ 2}}{\lambda_-^{\ 2}} + \frac{1-l_{y'}^{\ 2}}{\lambda_+^{\ 2}} \pm \sqrt{\left(\frac{1-l_{x'}^{\ 2}}{\lambda_-^{\ 2}} + \frac{1-l_{y'}^{\ 2}}{\lambda_+^{\ 2}}\right)^2 - \frac{4l_z^{\ 2}}{\lambda_+^{\ 2}\lambda_-^{\ 2}}}$$

effective SO length

The Bloch Equations (Born approx. in δB):

$$\boxed{\left\langle \dot{\mathbf{S}} \right\rangle = \boldsymbol{\omega} \times \left\langle \mathbf{S} \right\rangle - \Gamma \left\langle \mathbf{S} \right\rangle + \mathbf{Y},}$$

$\boldsymbol{\omega} = \omega \boldsymbol{l}$, $\omega = g\mu_B B/\hbar$, $\boldsymbol{l} = \mathbf{B}/B$; $\tau_c = \lambda / s = 100$ ps $\ll T_{1,2}$ ➔ Markov OK

$$\Gamma = \Gamma^r + \Gamma^d, \quad \text{tensor}$$

relaxation:
$$\Gamma^r_{ij} = \delta_{ij}\left(\delta_{pq} - l_p l_q\right) J^+_{pq}(\omega) - \left(\delta_{ip} - l_i l_p\right) J^+_{pj}(\omega) - \delta_{ij}\varepsilon_{kpq} l_k I^-_{pq}(\omega) + \varepsilon_{ipq} l_p I^-_{qj}(\omega),$$

dephasing:
$$\Gamma^d_{ij} = \delta_{ij} l_p l_q J^+_{pq}(0) - l_i l_p J^+_{pj}(0)$$

spectral function:
$$J_{ij}(w) = \frac{g^2 \mu_B^2}{2\hbar^2} \int_0^\infty \left\langle \delta B_i(0) \delta B_j(t) \right\rangle e^{-iwt} dt$$

$$J^\pm_{ij}(w) = \mathrm{Re}\left[J_{ij}(w) \pm J_{ij}(-w)\right], \qquad I^\pm_{ij}(w) = \mathrm{Im}\left[J_{ij}(w) \pm J_{ij}(-w)\right]$$

inhomogeneous part:

$$2\mathrm{Y}_i = l_j J^-_{ij}(\omega) - l_i J^-_{jj}(0) + \varepsilon_{ipq} I^+_{pq}(\omega) + \varepsilon_{iqk} l_k l_p \left[I^+_{pq}(\omega) - I^+_{pq}(0) \right]$$

In secular approximation ($\Gamma_{ij} << \omega = g\mu_B B$):

$$\frac{1}{T_1} := l_p l_q \Gamma_{pq} = \Gamma_{ZZ}$$

$$\frac{1}{T_2} := \frac{1}{2}\left(\delta_{pq} - l_p l_q\right)\Gamma_{pq} = \frac{1}{2}\left(\Gamma_{XX} + \Gamma_{YY}\right)$$

In absence of dephasing ($\Gamma^d_{ij} = 0$):

$$\frac{1}{T_1} = \frac{2}{T_2} = J^+_{XX}(\omega) + J^+_{YY}(\omega)$$

The solution of the Bloch equation reads:

$$\langle S_X(t)\rangle = S_\perp e^{-t/T_2}\sin(\omega t+\phi),$$
$$\langle S_Y(t)\rangle = S_\perp e^{-t/T_2}\cos(\omega t+\phi),$$
$$\langle S_Z(t)\rangle = S_T + (S_Z^0 - S_T)e^{-t/T_1},$$
$$\langle \mathbf{S}(0)\rangle = (S_\perp \sin\phi, S_\perp \cos\phi, S_Z^0)$$

$$S_T = -\frac{1}{2}\tanh(\hbar\omega/2kT)$$

is the thermodynamic value of spin.

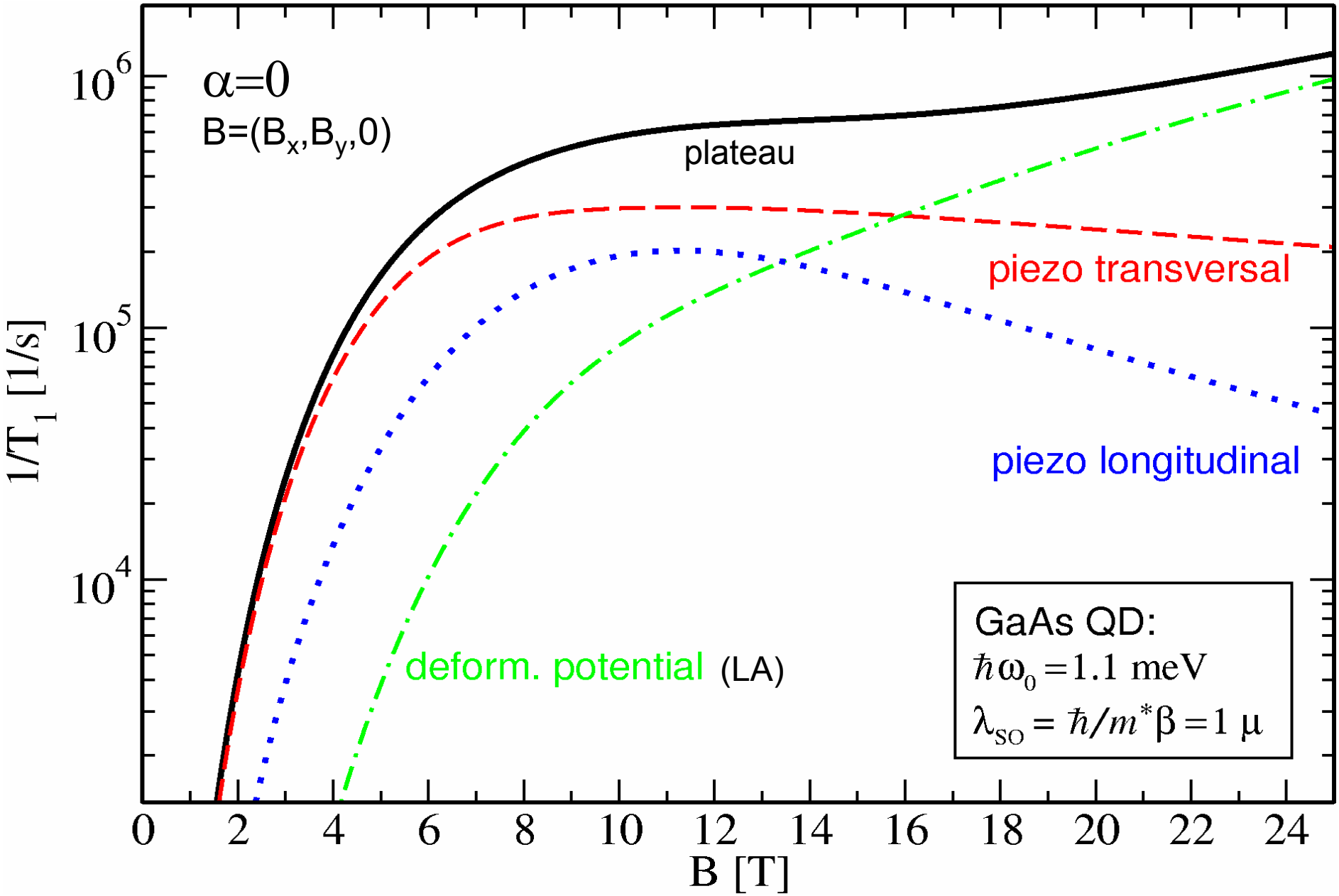
The universal angular dependence of the relaxation rate:

$$\frac{1}{T_1} = \frac{1}{T_1}(\theta=\frac{\pi}{2},\alpha=0)f(\theta,\varphi),$$

$$f(\theta,\varphi) = \frac{1}{\beta^2}[(\alpha^2+\beta^2)(1+\cos^2\theta)+2\alpha\beta\sin^2\theta\sin 2\varphi]$$

depends on the angles of the external magnetic field with respect to the crystallographic axes. Sic! For special orientations of the field within the 2D plane and for $\alpha = \pm\beta$ we obtain $T_1 = \infty$

Result:


α=0
B=(B_x,B_y,0)
plateau
piezo transversal
piezo longitudinal
deform. potential (LA)
GaAs QD:
ħω_0 = 1.1 meV
λ_SO = ħ/m*β = 1 μ
1/T_1 [1/s]
B [T]
10^6
10^5
10^4
0
2
4
6
8
10
12
14
16
18
20
22
24

Other mechanisms based on spin-orbit interaction:

- k^3-terms of Dresselhaus spin-orbit coupling

$$H_{SO} \propto \beta d^2 \left(\sigma_x \left\{ k_x, k_y^2 \right\} - \sigma_y \left\{ k_y, k_x^2 \right\} \right) \quad \text{+phonons;}$$

- direct spin-phonon interaction due to strain field:

$$\Delta H' = \left(V_0 / 4 \right) \varepsilon_{ijk} \sigma_i \left\{ u_{ij}, p_k \right\} \qquad V_0 \approx 8 \times 10^7 \ \text{cm}/s$$

- direct spin-phonon interaction via g-factor modulation:

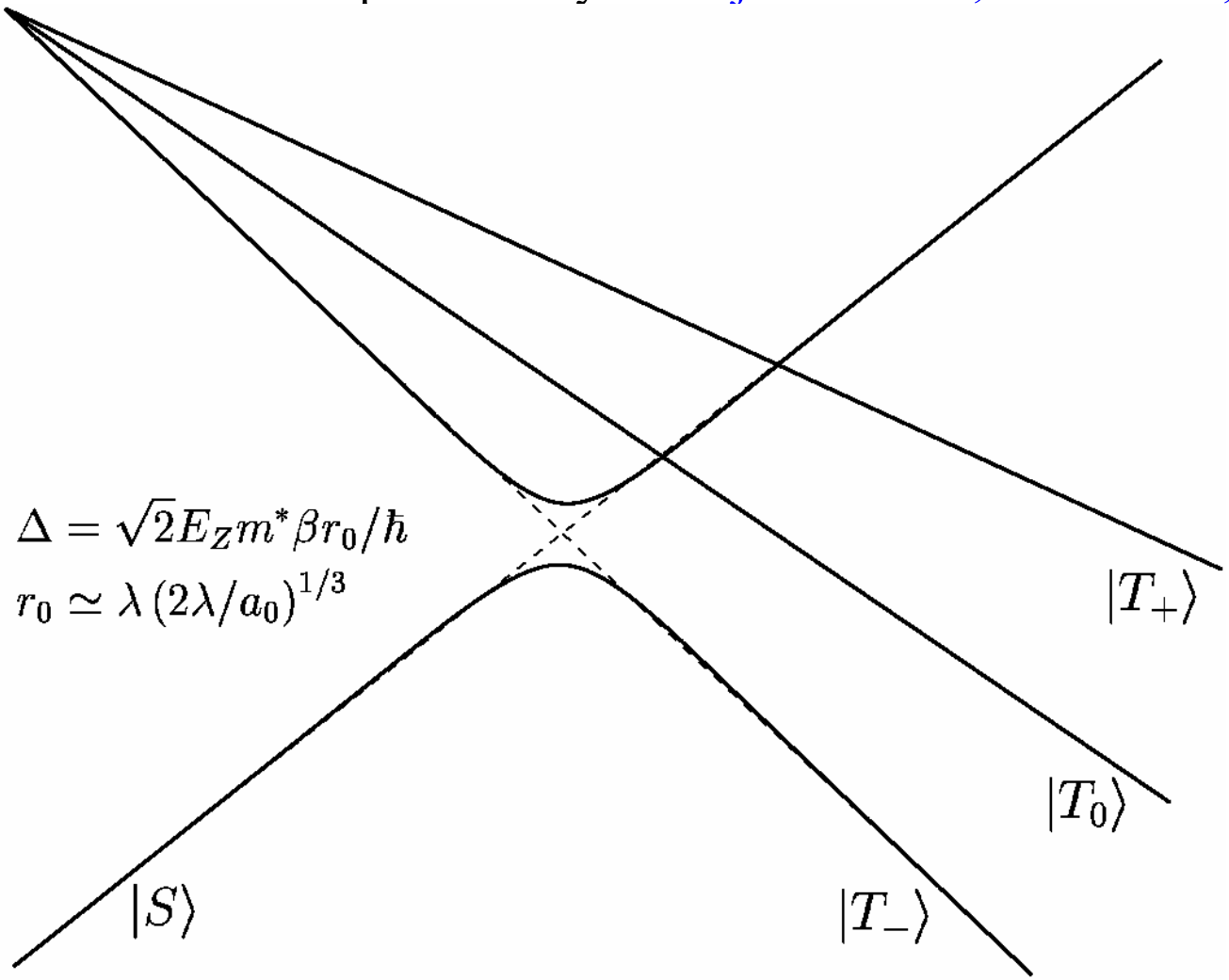
$$\Delta H'' = \tilde{g} \mu_B \sum_{i \neq j} u_{ij} \sigma_i B_j, \qquad \tilde{g} \approx 10$$

For generic $\delta B_i = \sum_{\mathbf{q}} M_i(\mathbf{q}) \left(b_{-\mathbf{q}}^{+} + b_{\mathbf{q}} \right)$, *dephasing* vanishes

if $q \left| M_i(\mathbf{q}) \right|^2 \to 0$, as $q \to 0$.

Spin relaxation at the singlet-triplet transition in a quantum dot.

We consider the phonon-assisted transition between singlet and triplet states of two electrons. Experiment by T. Fujisawa et al., Nature **419**, 278 (2002);



The same conclusion holds for an interacting problem,

i.e. the effect of the spin-orbit interaction cancels in the lowest order in S-O coupling constant β

Two particle spin-orbit related problem reduces to a one-particle problem in relative spatial coordinates and relative spin space $\sigma_1 - \sigma_2$

The singlet and triplet states correspond to spin-up and spin-down states in this new space ➔

one to one correspondence with the single-particle problem!

Hamiltonian

We consider two electrons in a lateral quantum dot, described by the Hamiltonian

$$H_0 = H_d + H_{SO} + H_Z, \tag{1}$$

$$H_d = \sum_{i=1,2}\left[\frac{p_i^2}{2m^*} + U(\mathbf{r}_i)\right] + \frac{e^2}{\kappa|\mathbf{r}_1 - \mathbf{r}_2|}, \tag{2}$$

$$H_{SO} = \beta\sum_{i=1,2}(-p_i^x\sigma_i^x + p_i^y\sigma_i^y) + \alpha\sum_{i=1,2}(p_i^x\sigma_i^y - p_i^y\sigma_i^x), \tag{3}$$

$$H_Z = \frac{1}{2}g\mu_B\mathbf{B}\cdot(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2), \tag{4}$$

here $\mathbf{p}_i = -i\hbar\boldsymbol{\nabla}_i + (e/c)\mathbf{A}(\mathbf{r}_i)$ is the i-th electron 2D kinetic momentum, $U(\mathbf{r})$ is the lateral confining potential, and $\boldsymbol{\sigma}$ are the Pauli matrices. The axises x and y point along the main crystallographic directions in the (001) plane of GaAs.

As a result, the energy gap Δ which determines the anticrossing of singlet and some triplet states is very small,

$$\Delta \approx \varepsilon_z \left(\frac{\lambda}{\lambda_{SO}} \right) << \varepsilon_z$$

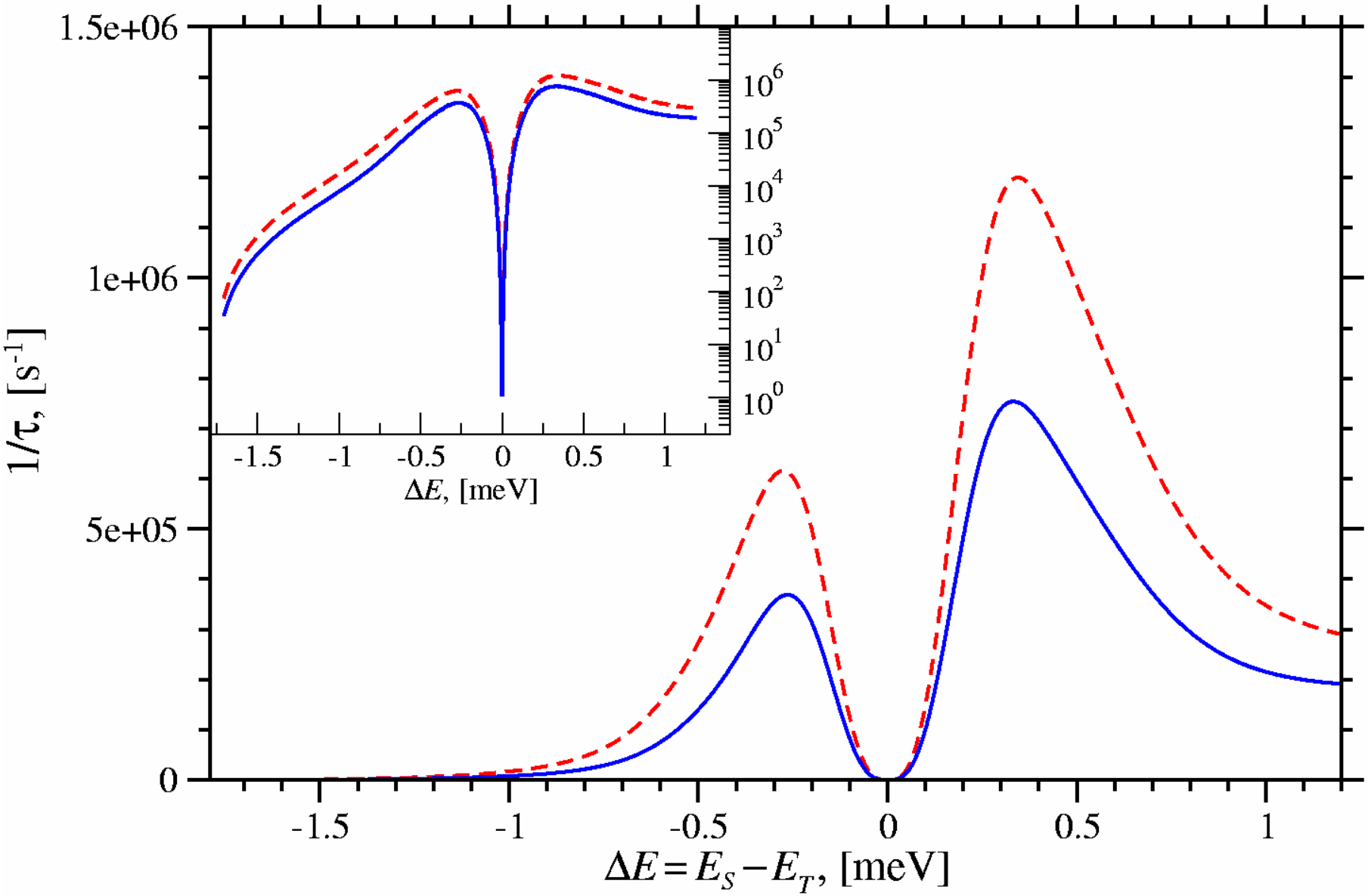
➔ very strong suppression of the spin-flip rate in the neighbourhood of the anticrossing point.

Away from the anticrossing point the wave vectors of the phonons involved are large compared to the dot lateral size and the rate decreases again.

➔ The corresponding curve has a nonmonotonic form with two maxima at the gap values $\hbar s / \lambda$

$$\Gamma_{\uparrow\downarrow} = \Gamma_0(B) \left(\frac{m\beta^2}{\hbar\omega_0} \right) \left(\frac{g\mu_B B}{\varepsilon_{S-T}} \right)^2, \Gamma_0(B) = \frac{(eh_{14})^2 m^2 s_t \omega_0^2}{\rho \varepsilon_{S-T}^3}$$

Here $\varepsilon_{S-T} = \varepsilon_{S-T}(B)$ is the energy distance between singlet and triplet states.


1.5e+06
1e+06
5e+05
0
1/τ, [s-1]
-1.5
-1
-0.5
0
0.5
1
ΔE = E_S − E_T, [meV]
10^6
10^5
10^4
10^3
10^2
10^1
10^0
ΔE, [meV]

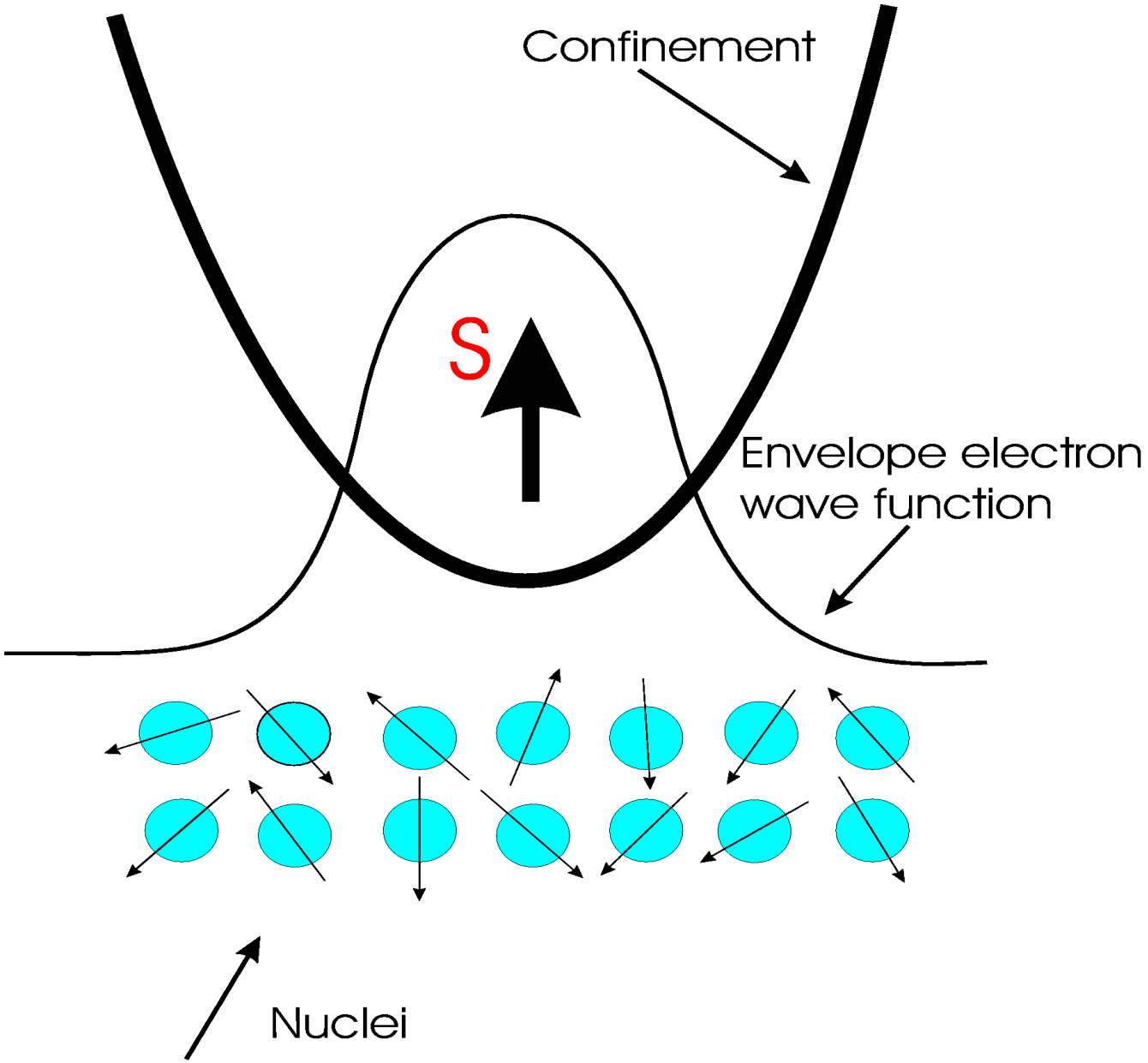
Electron spin decoherence in quantum dots due to hyperfine interaction with nuclei

A. Khaetskii, D. Loss, L. Glazman*, and J. Schliemann,

University of Basel, Switzerland

* Theoretical Physics Institute, University of Minnesota, Minneapolis, USA

System:



$$\hat{H} = g\,\mu_B\,\hat{\mathbf{S}}\cdot\mathbf{B} + \hat{\mathbf{S}}\cdot\hat{\mathbf{h}}_N + \hat{H}_{d-d},$$

$$\hat{\mathbf{h}}_N = \sum_i A_i\hat{\mathbf{I}}_i = g\mu_B\hat{\mathbf{H}}_N$$

$\hat{H}_{d-d}$ - dipole-dipole interaction between nuclei

$$A_i \propto A\cdot|\Psi(\mathbf{r}_i)|^2$$

(hyperfine interaction, A is the hyperfine constant, $\Psi(\mathbf{r})$ the electron envelope wave function)

The typical fluctuating nuclear magnetic field is $H_N \sim A/(\sqrt{N}\, g\, \mu_B)$, where $N \sim 10^5$, $H_N \sim 100$ Gauss in a GaAs quantum dot. The correlation time is $T_{n2} \sim 10^{-4}$ s.

$\omega_N \sim A/\sqrt{N}$ -the electron spin precession frequency in the field of the nuclei.

If $\omega_N \tau_c << 1$ ➔ perturbative regime, dynamical narrowing. In our case $\tau_c = T_{n2}$, ➔

opposite limit $\omega_N T_{n2} >> 1$ ➔ no usual treatment, no Markovian approximation, etc.!

Usual formulas for T_1, T_2 are not applicable here

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{(g\mu_B)^2}{\hbar^2}\frac{1}{2}\int_{-\infty}^{+\infty} d\tau < H_{Nz}(0)H_{Nz}(\tau) >,$$

Method and Equations:

Total angular momentum $\mathbf{S} + \Sigma_i \mathbf{I}_i = \text{const}$.

Each flip-flop process (due to hyperfine interaction) creates a different nuclear configuration. This leads to variation in time of the nuclear field $\mathbf{H}_N = \Sigma_i A_i \mathbf{I}_i$ seen by the electron spin, i.e. to spin decay. The decay is not exponential -- time $(A/N)^{-1}$ (microseconds).
Since $T_{n2} >> (A/N)^{-1}$ -- no averaging over the nuclear configurations !

I=1/2. We consider only a particular (and unpolarized) nuclear configuration $|\{I^i_z\}>$,
with $I^i_z = \pm 1/2$.

We evaluate the correlator:

$$C_n(t) =< n|\delta\hat{S}_z(t)\hat{S}_z|n >, \text{ where } \delta\hat{S}_z(t) = \hat{S}_z(t) - \hat{S}_z, \quad \hat{S}_z(t) = \exp(it\hat{\mathcal{H}})\hat{S}_z \exp(-it\hat{\mathcal{H}})$$

perturbation $\hat{V} = (1/2)(\hat{S}_+\hat{h}_{N-} + \hat{S}_-\hat{h}_{N+})$ (with $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}$). $\hat{\mathcal{H}}_0 = \hat{S}_z\hat{h}_{Nz}$ (eigenenergy ε_n).

In the leading order: $$C_n(t) = \sum_k \frac{|V_{nk}|^2}{\omega_{nk}^2}(\cos(\omega_{nk}t) - 1)\,,$$

Initial state $n = \Uparrow, \{..., I_z^k = -1/2, ...\}$, intermediate state $k = \Downarrow, \{...., I_z^k = +1/2, ...\}$

The energy difference $\omega_{nk} = \varepsilon_n - \varepsilon_k = \varepsilon_z + (h_z)_n + A_k/2$, $(h_z)_n = \langle n | \hat{h}_{Nz} | n \rangle$

For a typical nuclear configuration, $(h_z)_n^2 \simeq \omega_N^2 \gg A_k^2$, at $\tau >> 1$, $\tau = At/2\pi N$,

$N = a_z a^2 / v_0 >> 1$:

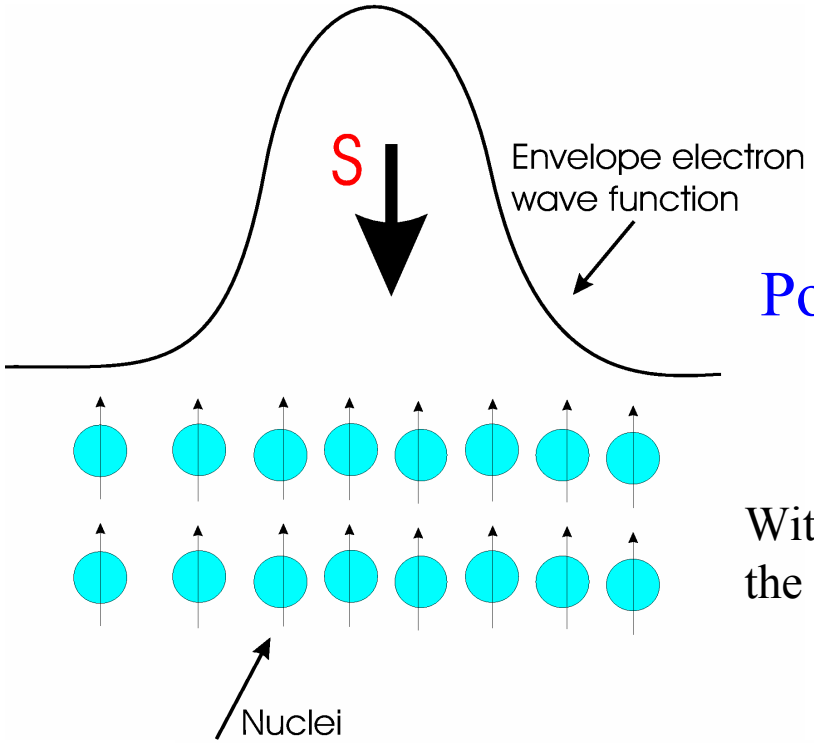
$$C_n(t) \cong -\alpha + \frac{\beta}{\tau^{3/2}} \sin(\tilde{h}_n t - \phi_0),$$

$$\tilde{h}_n = \varepsilon_z + (h_z)_n + A_0/2$$

Power law decay for times $t >> (A/N)^{-1}$. This law $1/\tau^{3/2}$ is the universal one. The amplitude of precession , reached as a result of the decay, is finite.

For a weak Zeeman field $\varepsilon_z = g \mu_B B < \omega_N$ we have $\alpha \sim \beta \sim ½$.

If $g \mu_B B >> \omega_N$, then $\alpha \sim \beta \sim (\omega_N / g \mu_B B)^2 << 1$.



Polarized nuclei. Exactly solvable case.

With the initial wave function Ψ_0 we construct the exact wave function of the system for t >0:

$$\Psi_0 = |\Downarrow;\uparrow,\uparrow,\uparrow \ldots. >, \Psi(t) = \alpha(t)\Psi_0 + \sum_k \beta_k(t)|\Uparrow;\uparrow,\uparrow,\downarrow_k \ldots. >,$$

Normalization condition: $|\alpha(t)|^2 + \sum_k |\beta_k(t)|^2 = 1$,

$$\alpha(t = 0^+) = 1, \alpha(t < 0) = 0.$$

$$
\begin{aligned}
i\frac{d\alpha(t)}{dt} &= -\frac{1}{4}A\alpha(t) + \sum_k \frac{A_k}{2}\beta_k(t) - \epsilon_z\alpha(t)/2, \\
i\frac{d\beta_l(t)}{dt} &= (\frac{A}{4} - \frac{A_l}{2})\beta_l(t) + \frac{A_l}{2}\alpha(t) + \epsilon_z\beta_l(t)/2,
\end{aligned}
\qquad (1)
$$

$A=\Sigma_k A_k$.

The correlation function: $C_0(t) = -\langle\psi_0|\delta\hat{S}_z(t)\hat{S}_z|\psi_0\rangle = (1-|\alpha(t)|^2)/2$

Laplace transform of (1) gives:

$$
\alpha(t) = \frac{\exp(-iA't/4)}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\omega \frac{i\exp(\omega t)}{[i\omega + \epsilon_z + \pi N i\omega \int dz \ln(1 - \frac{iA\chi_0^2(z)}{2\pi N\omega})]},
$$

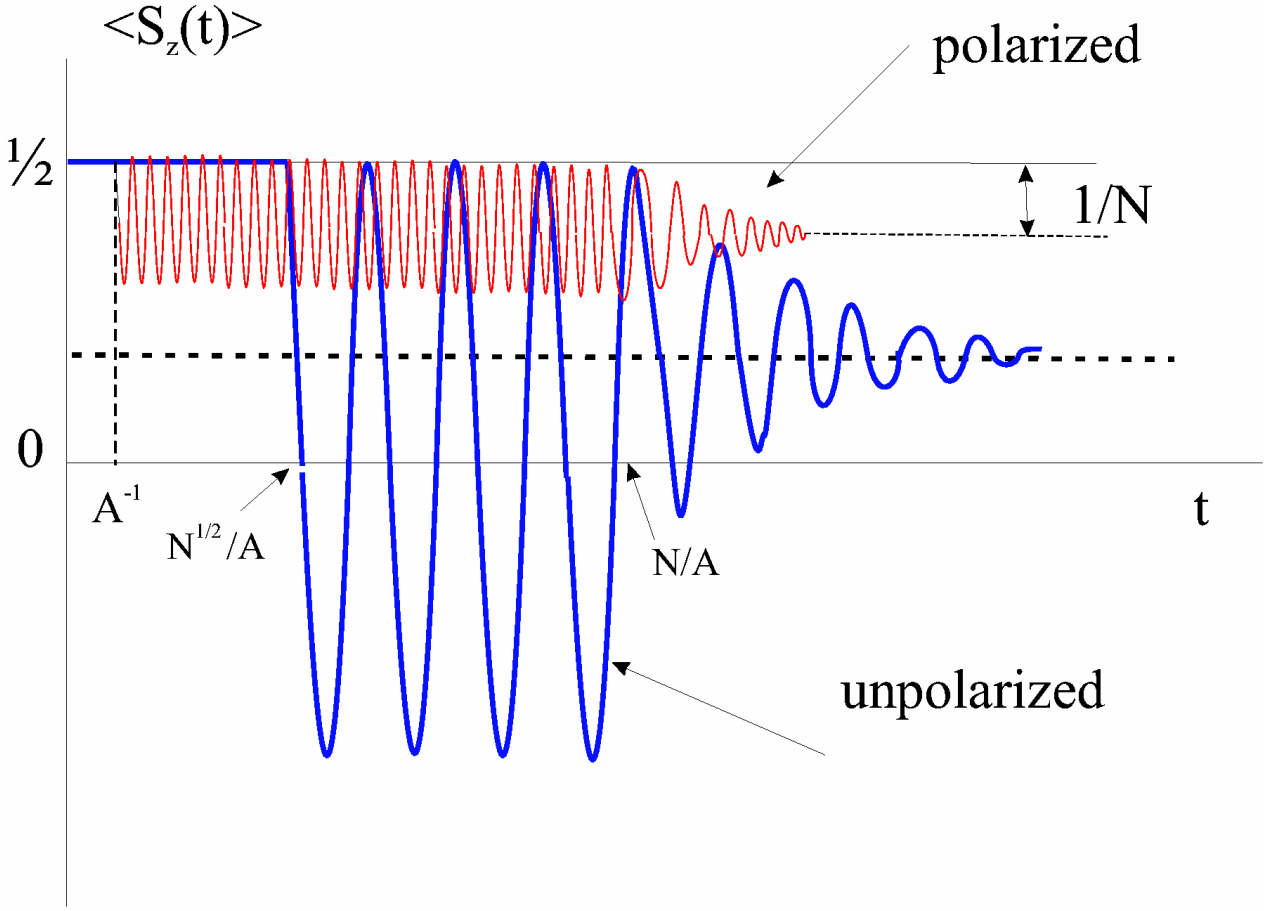
$$
A' = A + 2\epsilon_z.
$$

Some limit cases

1) $\varepsilon_z = 0$. The asymptotics ($\tau >> 1$) is determined by the weakest – coupled nuclear spins (far from the dot). ➔ Diffusion of nuclear spins in the external region induced by the hyperfine interaction!

For $\chi_0^2(z)/\chi_0^2(0) = \exp(-z^2)$ we find $\tilde{\alpha} \propto 1/\ln^{3/2}\tau$. It is not a universal behavior.

Thus, the decay of $|\alpha(t)|$ starts at $t > A^{-1} N$, as in the unpolarized case.
The decaying part of the initial state ~1/N.



Averaging over nuclear configurations (an ensemble of the dots)

Since $\omega_N^{-1} << N/A$, ➔ many precessions in a given nuclear field configuration before decoherence sets in due to the non-uniform hyperfine couplings A_k.

This behavior changes dramatically when we average over nuclear configurations.

If the nuclear field is treated classically, then the exact calculation of the correlator gives:

$$C_n(t) = -\frac{h_{N\perp}^2}{4h_N^2}(1 - \cos h_N t),$$

here $h_N = \sqrt{h_{Nz}^2 + h_{N\perp}^2}$ is the nuclear field, $h_{N\perp}^2 = h_{Nx}^2 + h_{Ny}^2$.

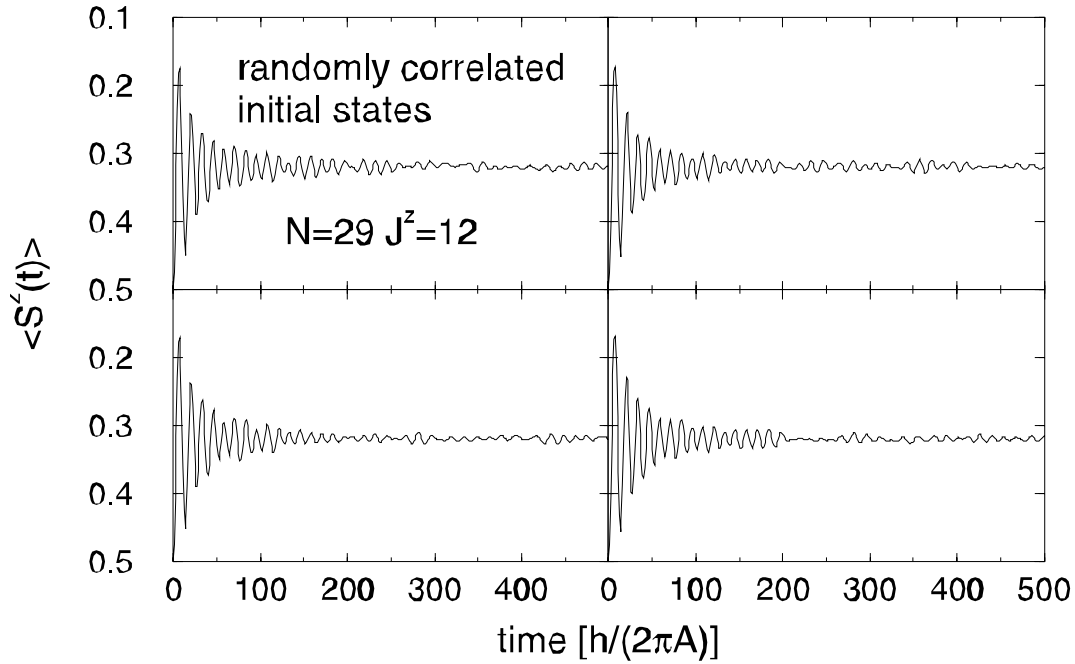
We average this correlator over a Gaussian distribution for h_N, i.e. over

$$P(h_N) \propto \exp(-3h_N^2/2\omega_N^2).$$

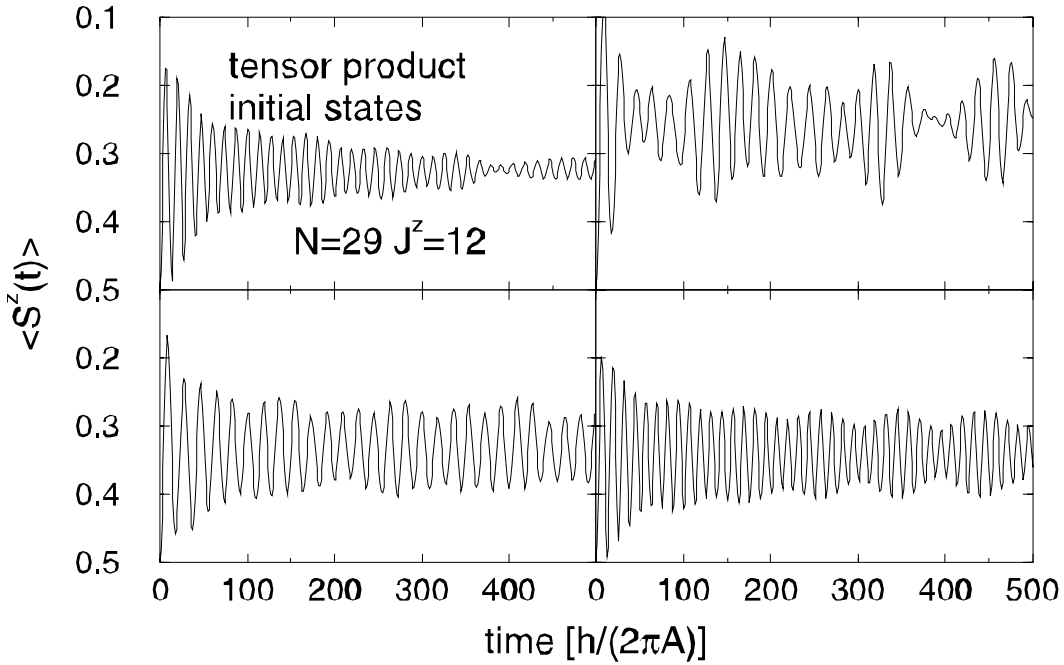
With the definition $C_{cl}(t) = \int dh_N P(h_N) C_n(t)$, we obtain

$$C_{cl}(t) = -\frac{1}{6}[1 + (\frac{\omega_N^2 t^2}{3} - 1)\, e^{-\omega_N^2 t^2/6}].$$

Rapid decay of the correlator for $t >> \omega_N^{-1}$, the dephasing time $T_2^* \sim \omega_N^{-1} = \sqrt{N}/A$.



Electron spin dynamics for an initially randomly correlated nuclear spin system. The electron spin dynamics is practically independent of the realization of the random initial nuclear state.



Here the initial state of the nuclear spins is given by individual tensor product states. Different initial tensor product states clearly lead to a significantly different time evolution of the electron spin. This is strikingly different from randomly correlated initial conditions.

Conclusions

•We have studied spin relaxation and decoherence in GaAs quantum dots due to S-O interaction.

•The T_1 relaxation time is very large for both single-electron and two-electron (interacting) problems. This is due to the localized character of the electron wave functions which kills the effect of S-O interaction in the leading order.

•The decoherence time T_2 is as large as the relaxation time T_1 for the spin decay based on the spin-orbit mechanisms.

•We have studied the spin decoherence of an electron confined to a single quantum dot in the presence of hyperfine interaction with nuclear spins.

•The decoherence time is given by N/A (several μs.)

•The decay of the electron spin correlator does not have an exponential character, it is either a power or inverse logarithm law.