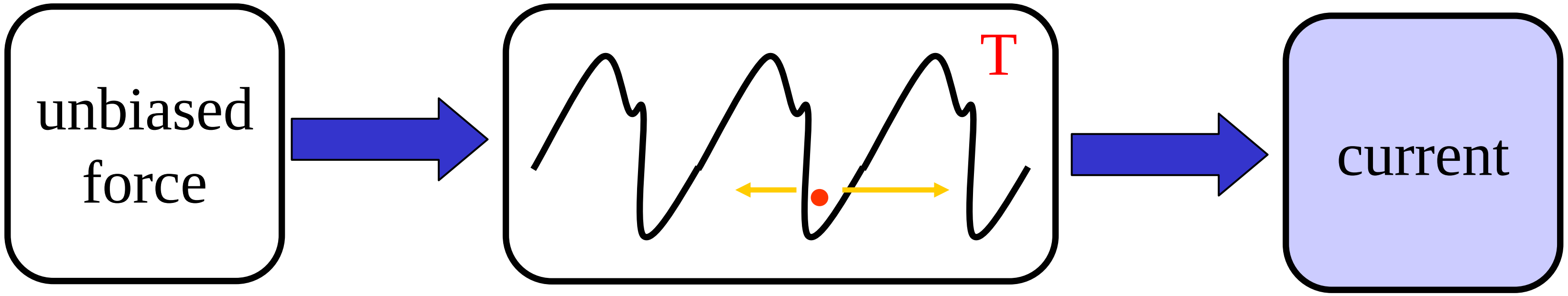
QUANTUM RATCHETS



M. Grifoni, J. Peguiron
J.B. Majer, H. Mooij

- THE RATCHET EFFECT
- EXPERIMENTS
- A MODEL FOR QUANTUM RATCHETS

BASIC CONCEPTS

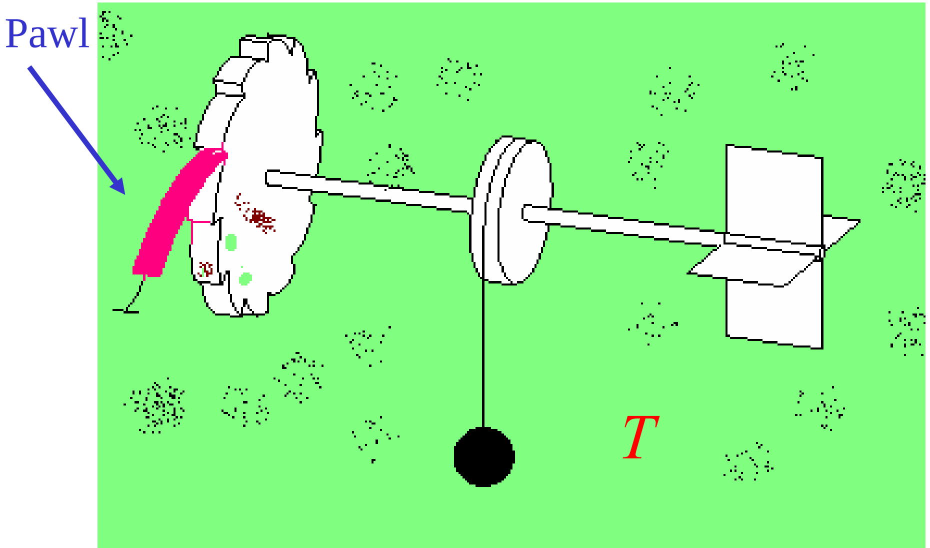
Is it possible to gain useful work out of unbiased random fluctuations?

- Macroscopic fluctuations

- Microscopic fluctuations ?

Gedankenexperiment Smoluchowski 1912, Feynman 1966

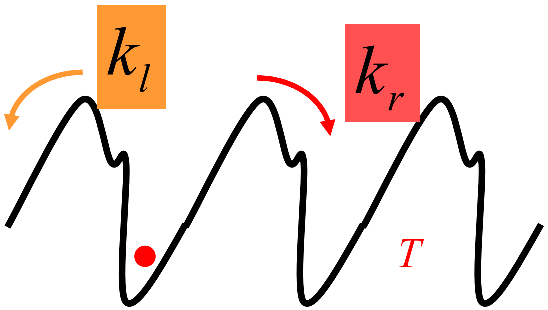
THE FEYNMAN RATCHET



THE FEYNMAN LECTURES ON PHYSICS (1966)

<u>INGREDIENTS</u> :

- periodic asymmetric system
- thermal equilibrium fluctuations

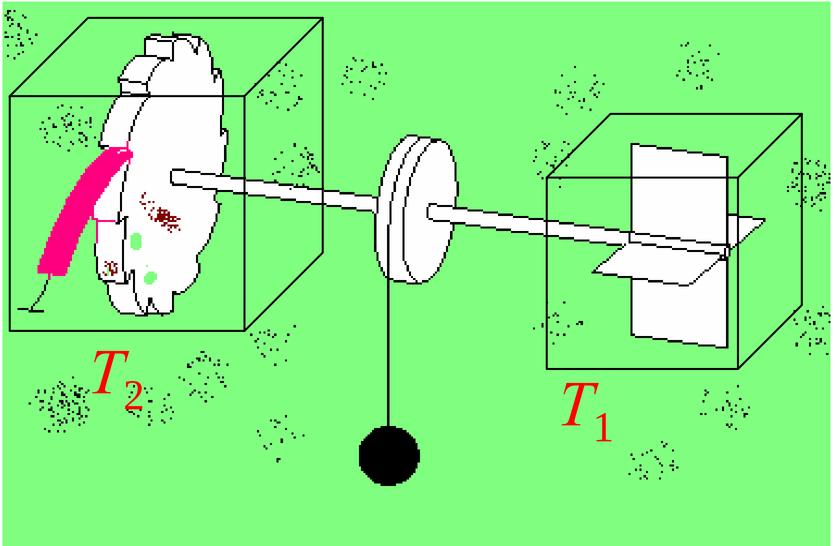


$$V_R(x+L)=V_R(x)$$

$$k_l = k_r$$

$$J = L(k_r - k_l) = 0$$

FEYNMAN RATCHETS II.



$T_1 \neq T_2$

THERMAL RATCHETS

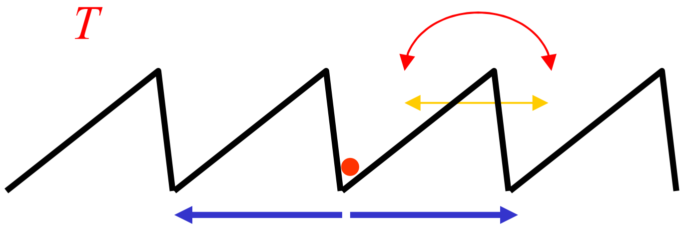
INGREDIENTS :

- periodic asymmetric system
- non-thermal equilibrium

ROCKED RATCHETS

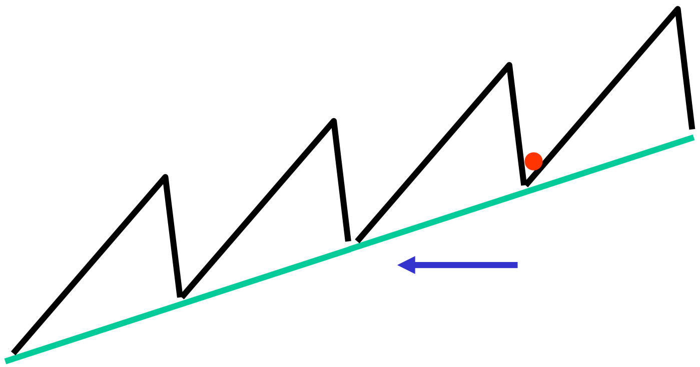
INGREDIENTS :

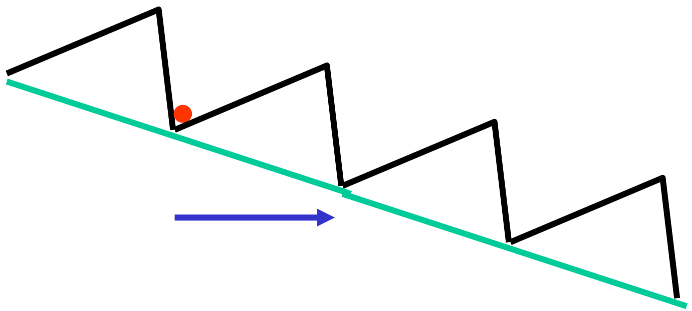
- periodic asymmetric potential
- thermal equilibrium fluctuations



$$V_R(x+L) = V_R(x)$$

- unbiased forcing $V_{eff}(x,t) = V_R(x) - xf(t)$, $\langle f^{2n+1}(t) \rangle \equiv 0$



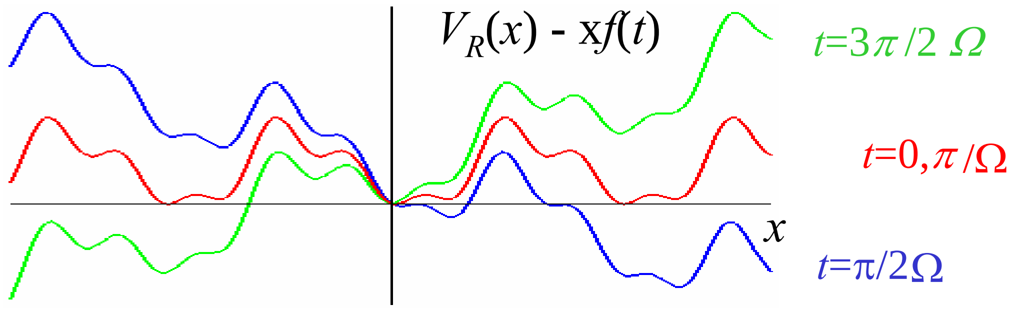


$$f(t) = \pm F \equiv F^{\pm} \quad \Rightarrow \mathrm{v}_{\mathrm{RAT}} = [\mathrm{v}(F^{+}) + \mathrm{v}(F^{-})]/2 \neq 0$$

DRIVING

$$V_{eff}(x,t) = V_R(x) - xf(t)$$

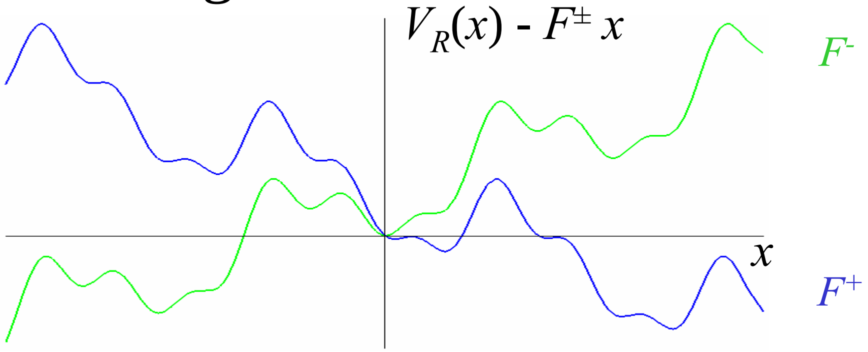
- AC-driving



$$f(t) = F\sin(\Omega t)$$

ratchet current: $\langle \mathrm{v}(f(t)) \rangle_\Omega$

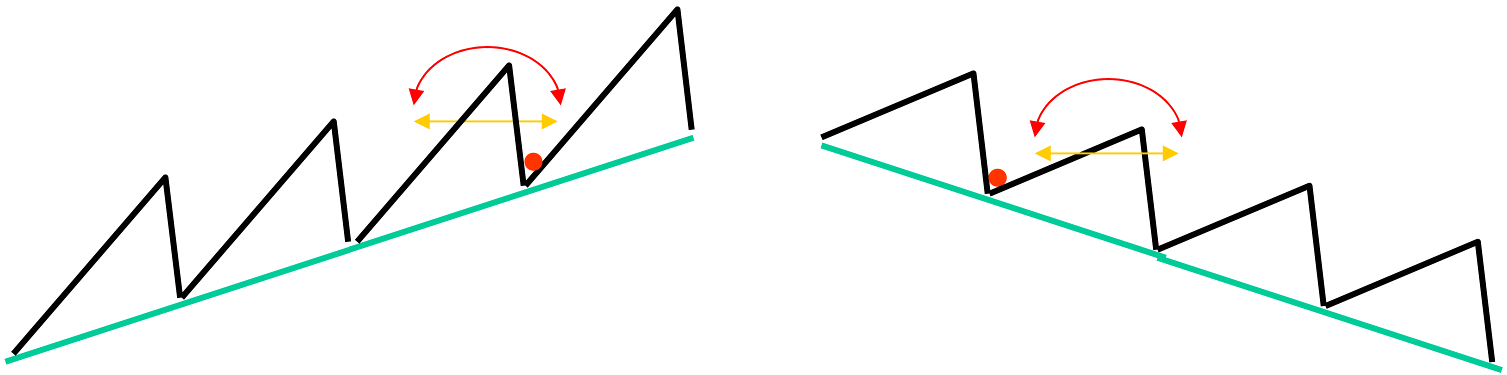
- DC-driving

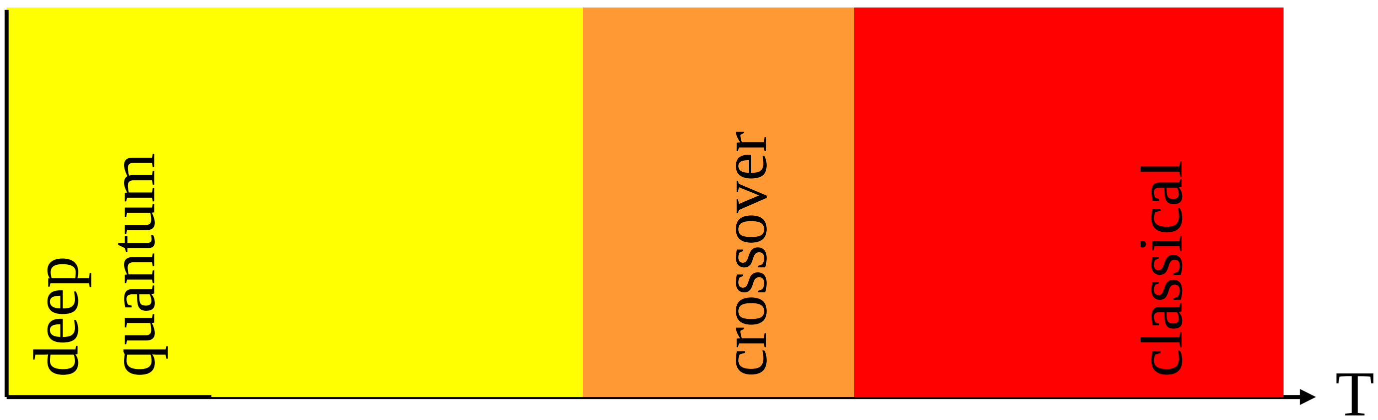


$$f(t) = \begin{cases} F^+ = +|F| \\ F^- = -|F| \end{cases}$$

ratchet current: $[\mathrm{v}(F^+) + \mathrm{v}(F^-)]/2$

ROCKED RATCHETS II

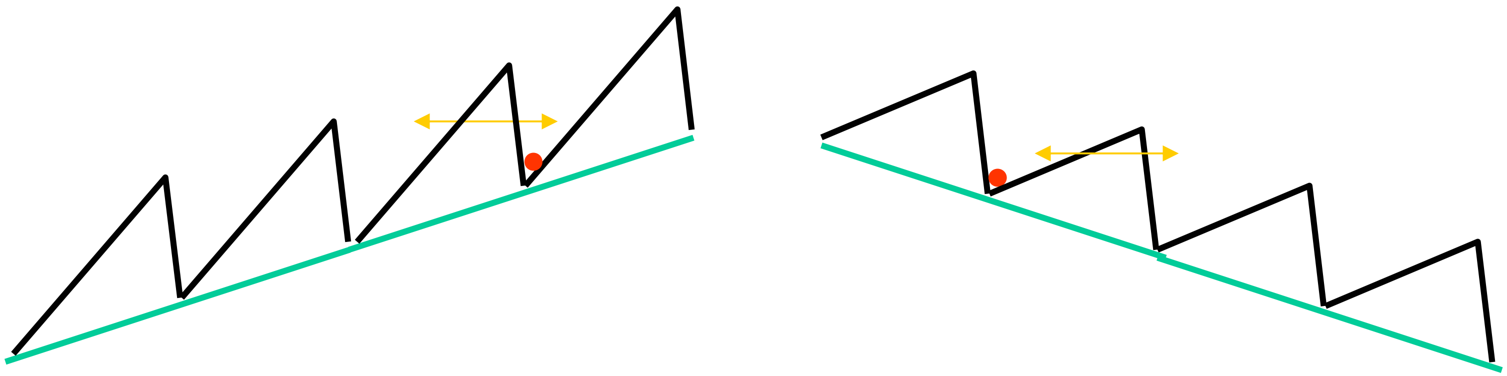


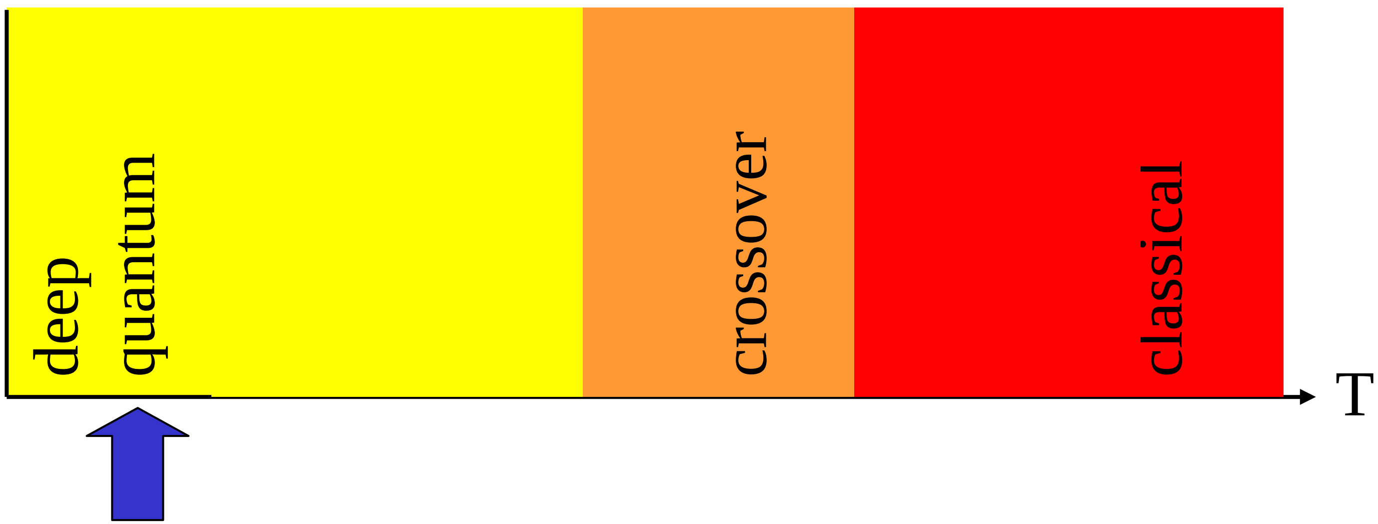


P. Reimann, Phys. Rep. **361**, 57 (2002)

Appl. Phys. A **75**, 167 (2002), special issue

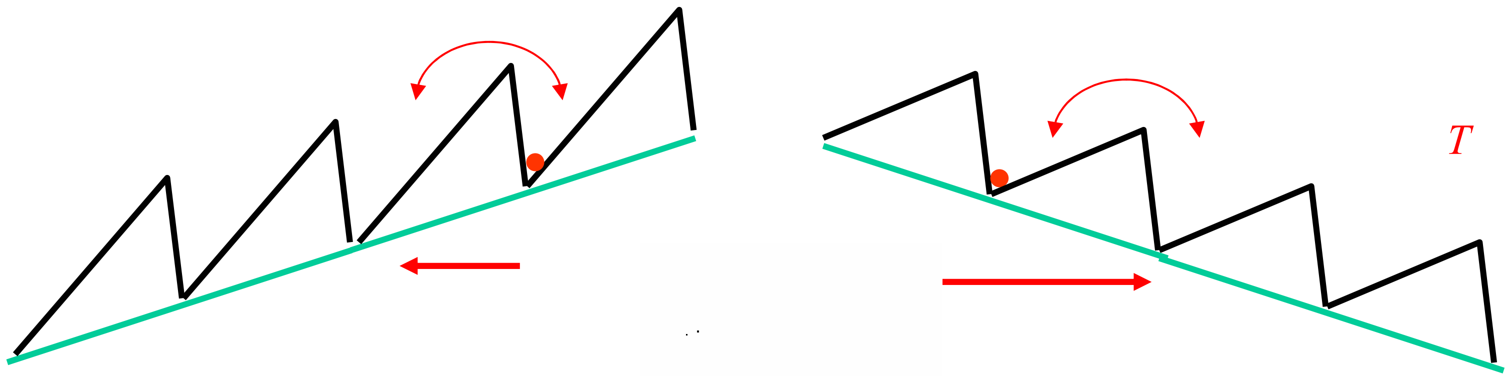
ROCKED RATCHETS III



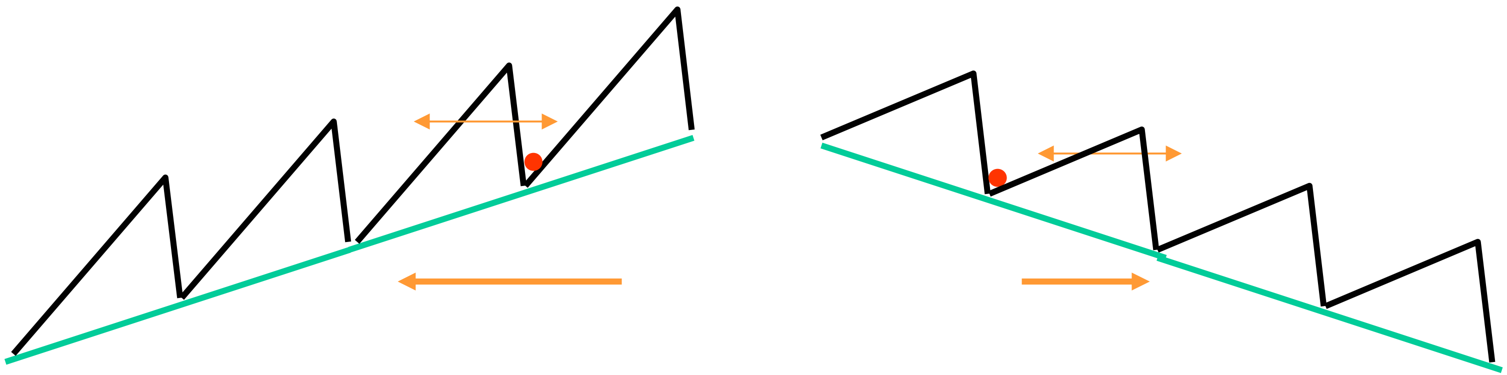


What is the current direction ?

CURRENT INVERSION



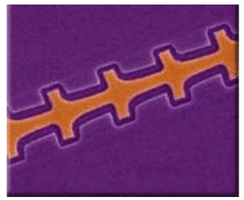
What is the current direction ?



Current inversion

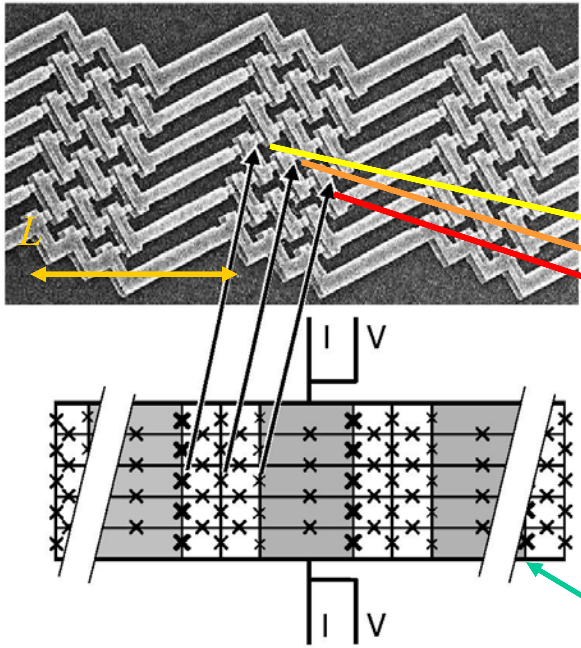
Theory (semiclassical bounce approach): P. Reimann, M. Grifoni, P. Hänggi, PRL **79**, 10 (1997)

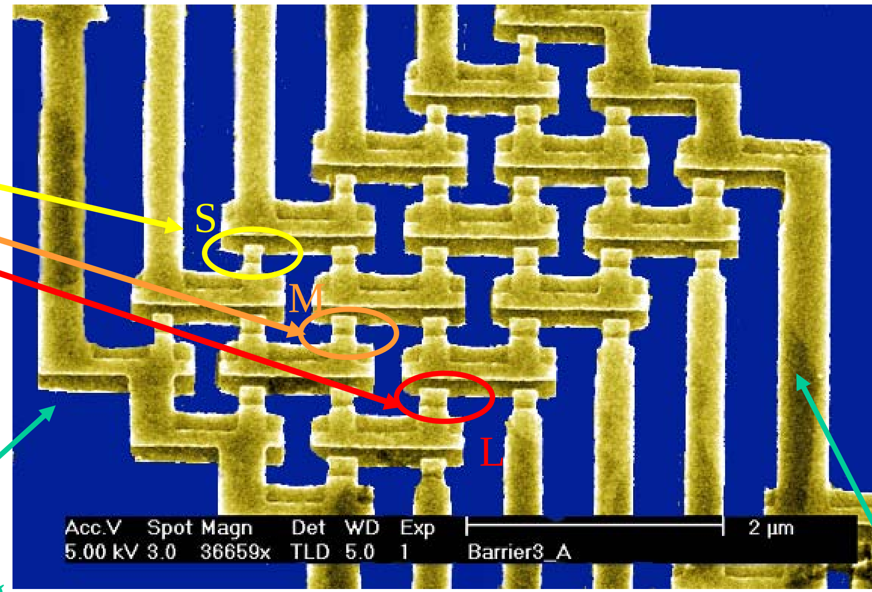
Experiment (semiconductor heterostructures): H. Linke, *et al.*, Science **286**, 2314 (1999)



VORTEX RATCHETS

J.B. Majer, *et al*. PRL **90**, 056802 (2003)



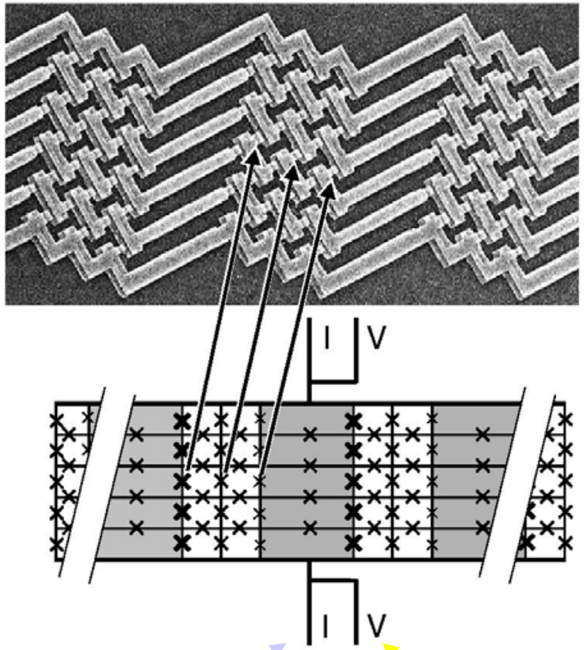


high precision design

Quasi 1D vortex dynamics

303 cells, length 2 µm
Josephson coupling $E_J \sim 5\mathrm{K}k_B \sim 0.5$ µeV
Temperature $T = 12$ mK
1D vortex density $s = 0.61/L$
$E_J/E_C \sim 11$

VORTEX RATCHETS II



Vortices

- Brownian particles of mass $M \sim 1/E_C$
- Height of periodic potential $\Delta U \sim E_J$
- Friction $\eta \sim 1/(RL^2)$

force $F^{\pm} = \dfrac{\Phi_0}{N_c a} I^{\pm}$

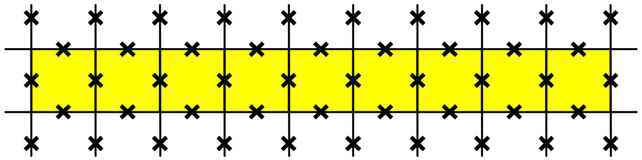
velocity $\mathrm{v} = \dfrac{L}{s\Phi_0} V$

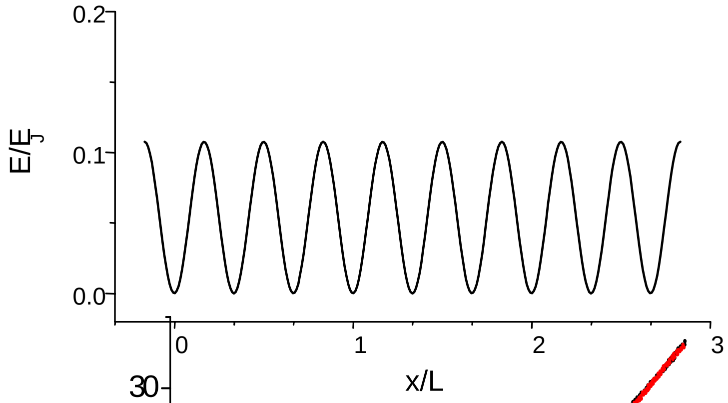
dc-ratchet

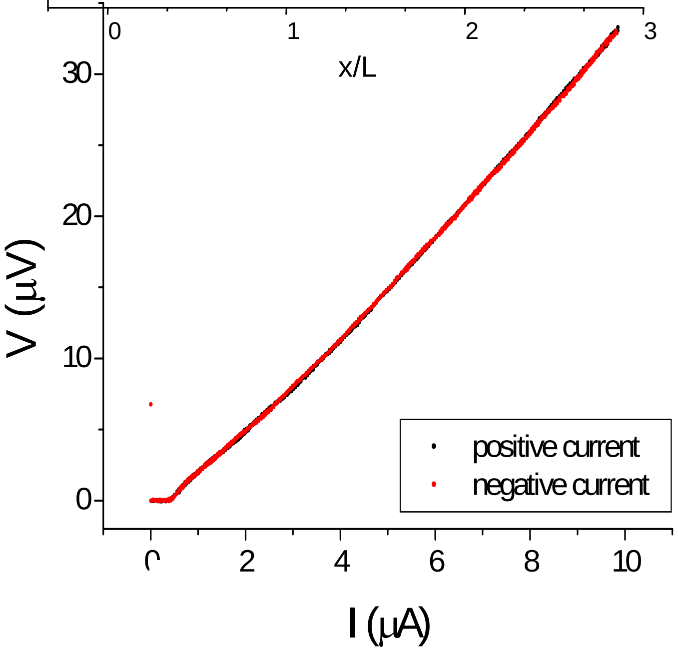
$\Phi_0 = h/2e$

RECTIFICATION ...

Regular Array



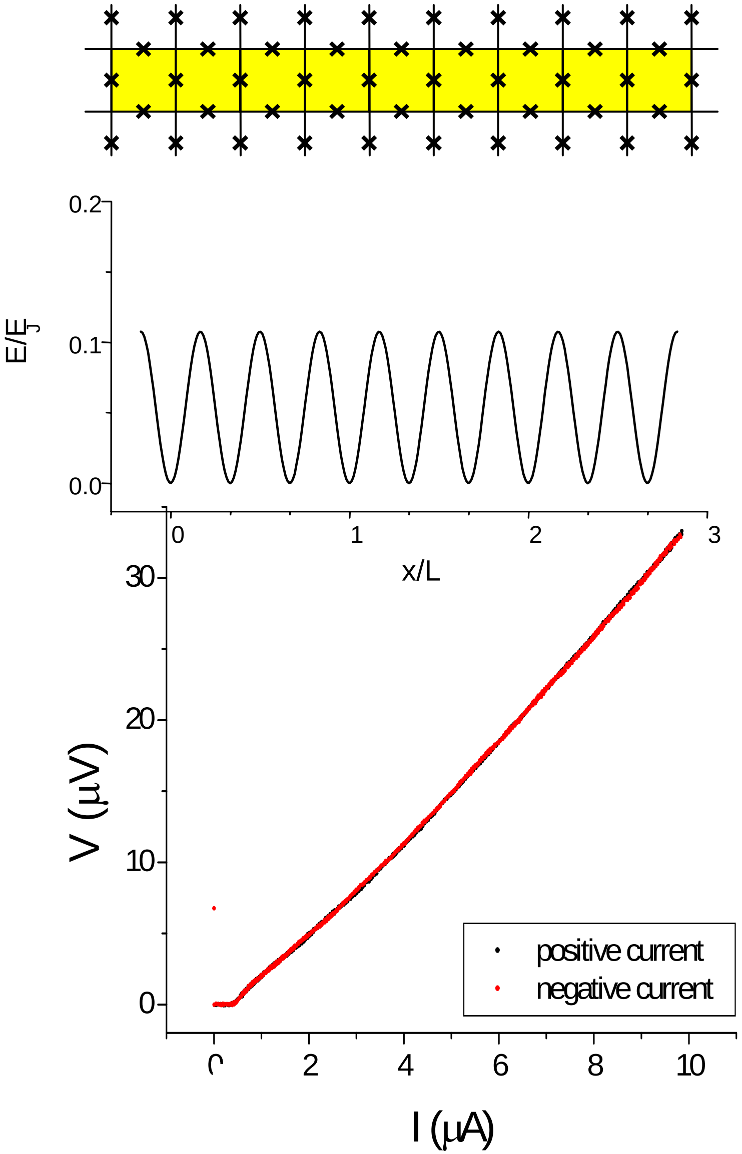




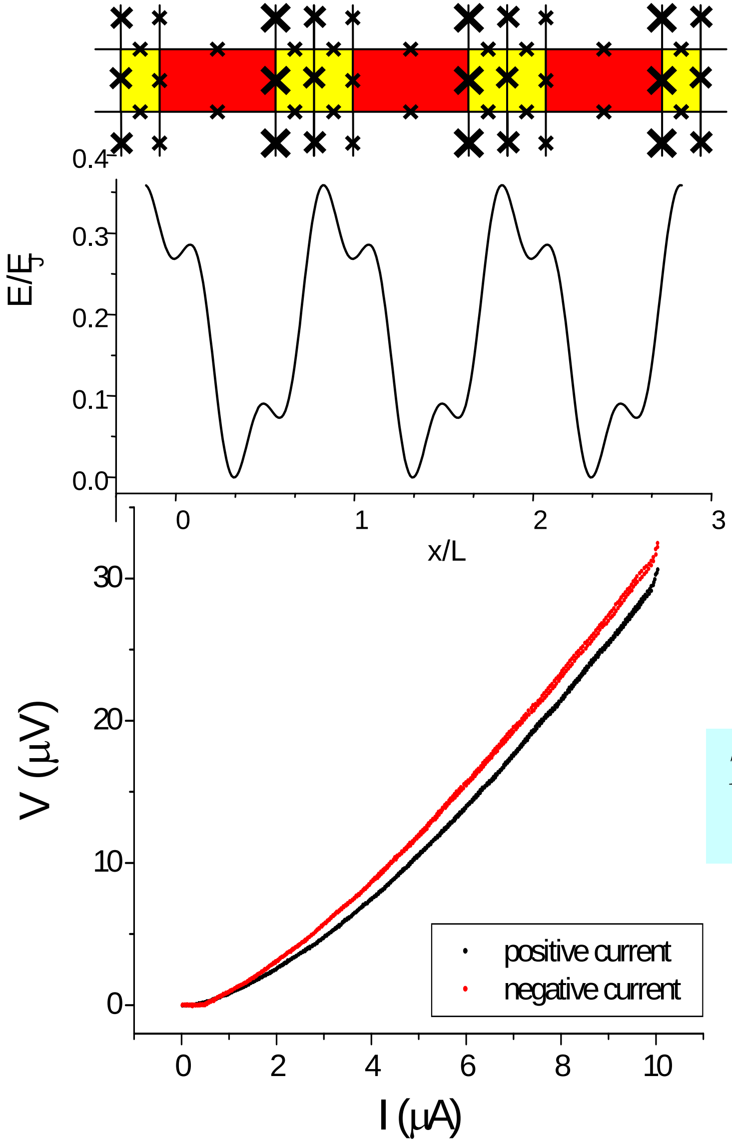
T =12 mK
s = 0.6

RECTIFICATION ...

Regular Array



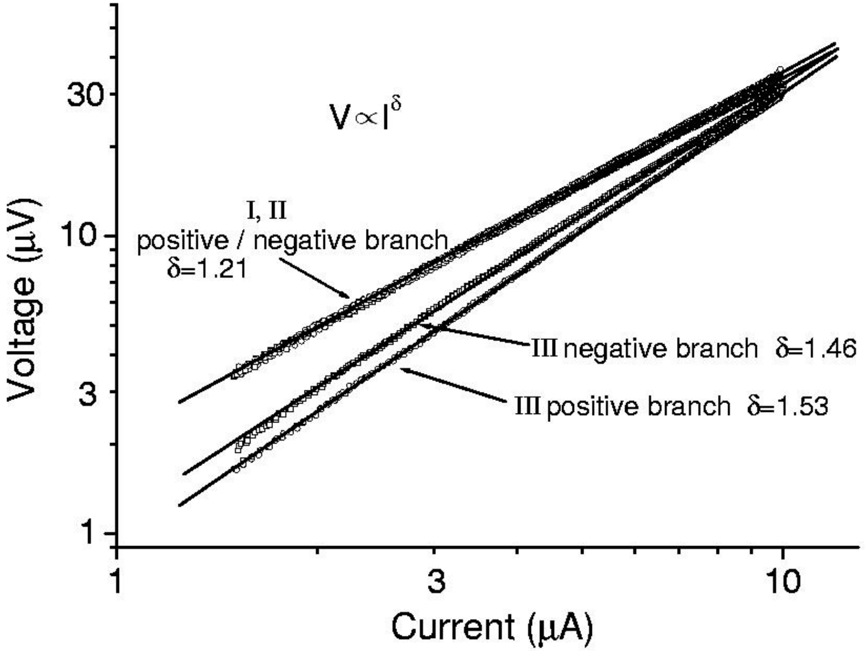
Ratchet



T =12 mK
s = 0.6

… AND POWER LAWS

Power-law dependence of the voltage (velocity)-vs-current (driving)



J.B. Majer et al. PRL **90**, (2003)

T =12 mK
s = 0.6

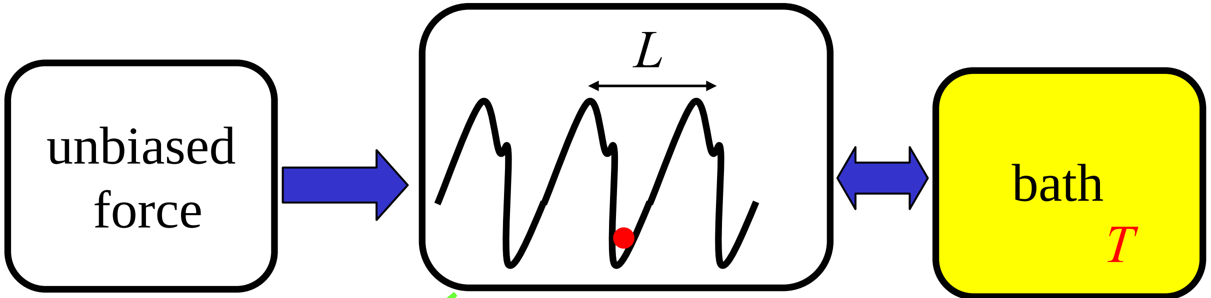
Sample I: regular

Sample II: ratchet but only one relevant band below the barrier

Sample III: ratchet. Different exponents for each branch

NO rectification for one band potentials and universality for sample I, II

A QUANTUM RATCHET MODEL



$$H(t) = H_R(t) + \frac{1}{2}\sum_i \left[\frac{\hat{P}_i^2}{m_i} + m_i\omega_i^2\left(\hat{x}_i - \frac{c_i}{m_i\omega_i^2}x\right)^2\right]$$

with $\hat{\xi} = \sum c_i \hat{x}_i$ stochastic force

$$\left\langle \hat{\xi}(t)\hat{\xi}(0)\right\rangle_\beta = \frac{\hbar}{\pi}\int_0^\infty d\omega\, J(\omega)\frac{\cosh[\tanh\frac{\beta\omega}{2} - i\omega t]}{\sinh(\hbar\beta\omega/2)}, \qquad \left\langle \xi(t)\right\rangle_\beta = 0$$

$$J(\omega) = \frac{\pi}{2}\sum \frac{c_i^2}{m_i\omega_i}\delta(\omega - \omega_i) \equiv \eta\omega$$

$$m\ddot{\hat{x}}(t) = -\eta\dot{\hat{x}} - V_{\mathrm{R}}'(\hat{x}) + f(t) + \hat{\xi}(t)$$ quantum Langevin eq.

SEMICLASSICAL METHOD ?

P. Reimann, M. Grifoni, P. Hänggi, PRL **79**, 10 (1997)

- Local thermal equilibrium
  (adiabatic forcing)

- High potential barriers $\Delta V >> \hbar\omega_0$

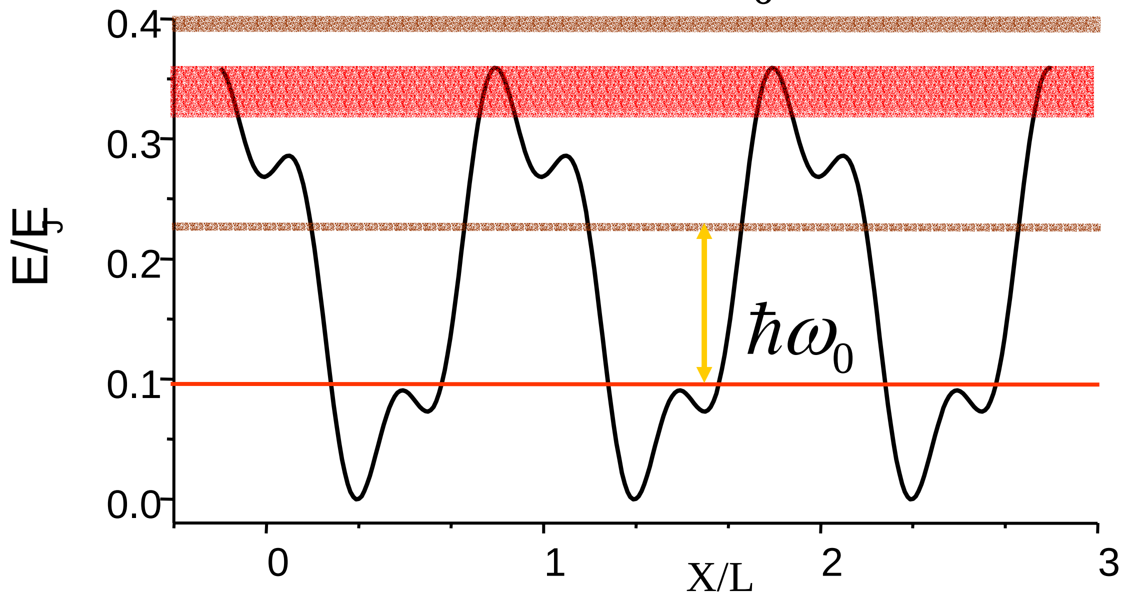
$$v_{RAT} = (v_+ + v_-)/2 = \frac{L}{2}(1 - e^{-\frac{FL}{k_B T}})(\Gamma_r^+ - \Gamma_l^-)$$

adiabatic semiclassical rates

$$\Gamma_{qm} = A e^{-S[q_B]} \quad S[q] = \int_0^{\hbar\beta} d\tau [\frac{m\dot{q}^2}{2} + V_{eff}(q) + \frac{\eta}{4\pi}\int_{-\infty}^{+\infty} d\tau' (\frac{q-q'}{\tau-\tau'})^2]$$

BEYOND SEMICLASSICS

- shallow potential barriers $\Delta V \geq \hbar\omega_0$

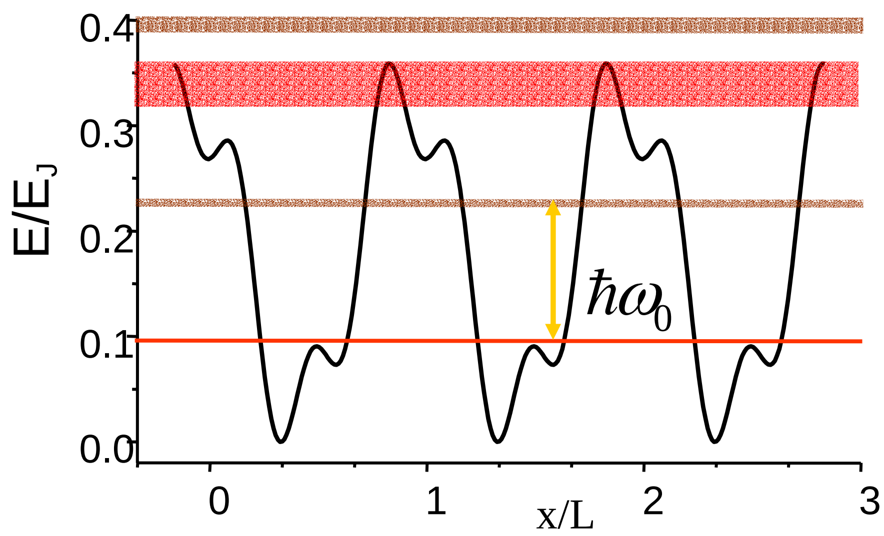


<u>or/and</u>

- nonadiabatic driving

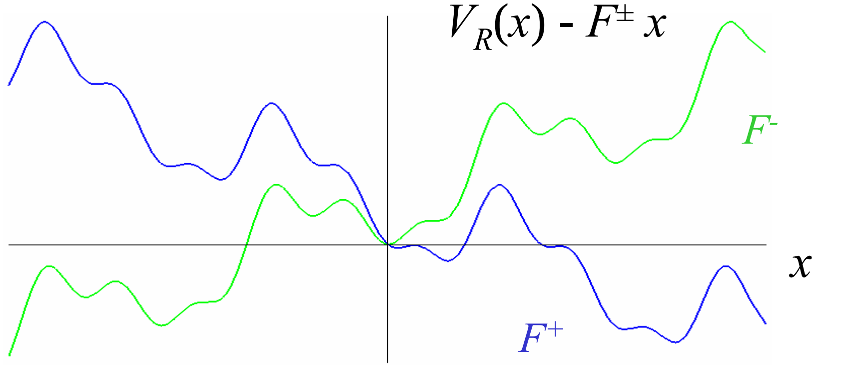
BEYOND SEMICLASSICS II

Two complimentary approaches:



- Few band QR with moderate driving amplitudes & dissipation

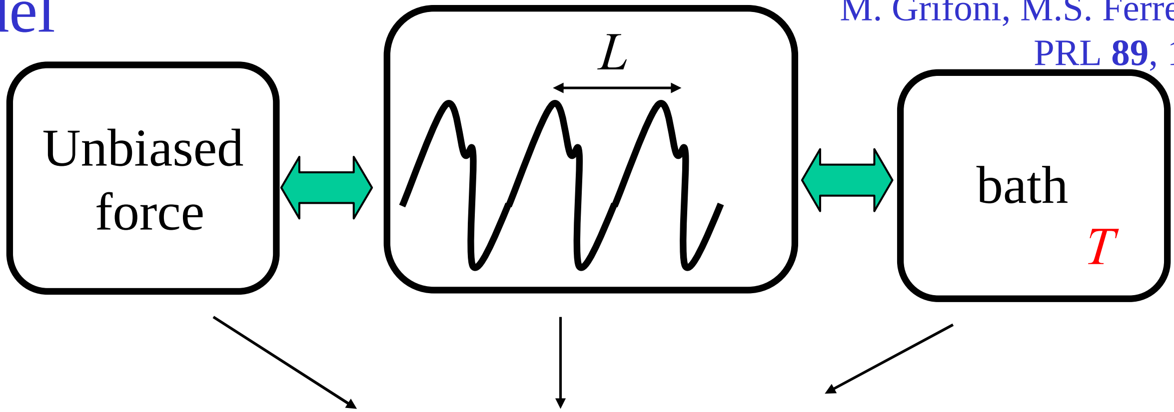
- Duality relation for QR (best for shallow potentials; also strong driving & weak dissipation)



QR WITH FEW BANDS

1) The model

M. Grifoni, M.S. Ferreira, J. Peguiron, J.B. Majer, PRL **89**, 146801 (2002).



$$H_{tot} = xf(t) + H_R + x\xi + H_{BATH}$$ + counterterm

2) Wanted

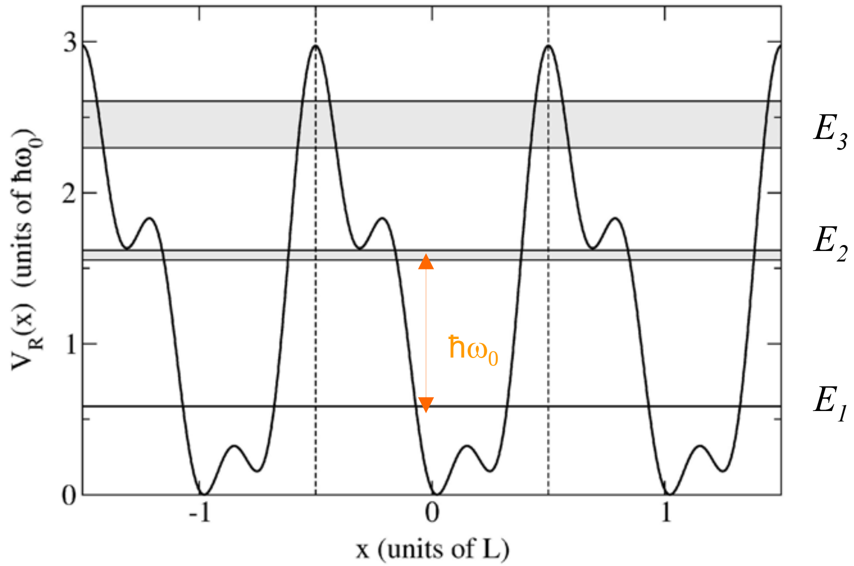
$$\mathrm{v}_\infty = \lim_{t\to\infty} \mathrm{v}(t) = \lim_{t\to\infty} \frac{d}{dt} \mathrm{Tr}_{ratchet}(\rho_{red}(t) x)$$

with $\rho_{red}(t) = \mathrm{Tr}_{bath} W_{tot}(t)$

Conveniently evaluated in the eigenbasis of the operator x coupling to the bath (DVR)

BLOCH BASIS

Periodic potential → Bloch theorem → band structure



$$H_R |n,k\rangle = E_n(k) |n,k\rangle$$

band index

wave vector

$$E_n(k) = E_n + \sum_m \frac{\Delta_n^{(m)}}{2} \cos(mkL)$$

band parameters obtained numerically

DVR BASIS

Bloch basis → **Wannier basis** → **DVR basis**

Bloch basis

- extended states
- x non diagonal

$$|n,k\rangle$$

band — wave vector

Wannier basis

- localized states
- x non diagonal *

$$|n,j\rangle$$

band — cell

DVR basis

- localized states
- x diagonal

$$|\mu,m\rangle$$

pseudo-band — pseudo-cell

$$|n,j\rangle = \frac{1}{\sqrt{N_{\text{cells}}}} \sum_k e^{-ijkL} |n,k\rangle$$

$$|\mu,m\rangle = \sum_{n=1}^{N_{\text{bands}}} \sum_{j \text{ on cells}} U_{n\mu;jm} |n,j\rangle$$

numerical diagonalization

* Wannier and DVR basis differ when more than 1 band is considered

H_R AND x IN DVR BASIS

$$H_R = \sum_{m \text{ on cells}} \left[\sum_{\mu=1}^{N_{\text{bands}}} \varepsilon_\mu |\mu,m\rangle\langle\mu,m| + \sum_{\mu,\mu'=1}^{N_{\text{bands}}} \sum_p \frac{\Delta^{(p)}_{\mu\mu'}}{2} \left(|\mu,m\rangle\langle\mu',m+p| + |\mu',m+p\rangle\langle\mu,m| \right) \right]$$

position

$$x = \sum_{\mu=1}^{N_{\text{bands}}} \sum_{m \text{ on cells}} \left[\left(mL + q_\mu\right) |\mu,m\rangle\langle\mu,m| \right]$$

eigenvalues are periodically repeated

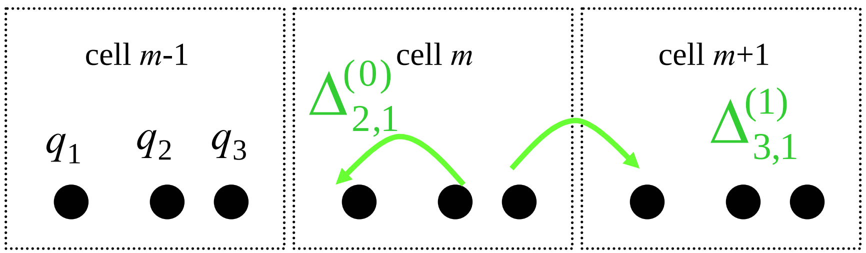
$$\Delta^{(-p)}_{\mu\mu'} = \Delta^{(p)}_{\mu'\mu}$$

no asymmetry in 1 band case

NO current for effective 1 band potentials !

Example: $N_{\text{bands}}=3$

DVR basis



$$\Delta^{(0)}_{\mu\mu'} \equiv \Delta^{\text{intra}}_{\mu\mu'} \approx 0.1 - 0.5\,\hbar\omega_0$$

$$\Delta^{(1)}_{\mu\mu'} \equiv \Delta^{\text{inter}}_{\mu\mu'} \approx 10^{-2}\,\hbar\omega_0$$

H_R AND x IN DVR BASIS

$$H_R = \sum_{m \text{ on cells}} \left[\sum_{\mu=1}^{N_{\text{bands}}} \varepsilon_\mu |\mu,m\rangle\langle\mu,m| + \sum_{\mu,\mu'=1}^{N_{\text{bands}}} \sum_p \frac{\Delta_{\mu\mu'}^{(p)}}{2} \left(|\mu,m\rangle\langle\mu',m+p| + |\mu',m+p\rangle\langle\mu,m| \right) \right]$$

position

$$x = \sum_{\mu=1}^{N_{\text{bands}}} \sum_{m \text{ on cells}} \left[(mL + q_\mu) |\mu,m\rangle\langle\mu,m| \right]$$

eigenvalues are periodically repeated

$$\Delta_{\mu\mu'}^{(-p)} = \Delta_{\mu'\mu}^{(p)}$$

no asymmetry in 1 band case

NO current for effective 1 band potentials !

Approximations

- $k_B T$, $\hbar\Omega$, FL $<<$ $\hbar\,\omega_0$ $\rightarrow$ truncation to M bands

- $\Delta_{\mu\mu'}^{(p)} \approx 0$ for $|p|>1$ $\rightarrow$ tight-binding approximation (nearest neighbours)

GENERALIZED MASTER EQ.

$$\mathrm{v}_{as}(t) = \lim_{t\to\infty} \sum_{m=-\infty}^{\infty} \sum_{\mu=1}^{N_{bands}} q_{\mu,m} \dot{P}_{\mu,m}(t)$$

$$P_{\mu,m}(t) = \left\langle \mu, m \right| \mathrm{Tr}_{bath} \left\{ T e^{-\frac{i}{\hbar}\int_{t_0}^{t} d\tau H_{tot}(\tau)} W_{tot}(t_0) e^{\frac{i}{\hbar}\int_{t_0}^{t} d\tau H_{tot}(\tau)} \right\} \left| \mu, m \right\rangle$$

$$W_{tot}(t_0) = \rho_{sys}(t_0) \otimes \frac{1}{Z} \mathrm{Tr}_{bath} \left\{ e^{-\frac{H_{bath}^{(0)}}{k_B T}} \right\}$$

GENERALIZED MASTER EQ. II

$$P_{\mu,m}(t) = \left\langle \mu, m \right| \mathrm{Tr}_{bath} \left\{ T e^{-\frac{i}{\hbar}\int_{t_0}^{t} d\tau H_{tot}(\tau)} W_{tot}(t_0) e^{\frac{i}{\hbar}\int_{t_0}^{t} d\tau H_{tot}(\tau)} \right\} \left| \mu, m \right\rangle$$

perform trace on the bath
(Feynman-Vernon functional)

$$W_{tot}(t_0) = \rho_{sys}(t_0) \otimes \frac{1}{Z} \mathrm{Tr}_{bath} \left\{ e^{-\frac{H_{bath}^{(0)}}{k_B T}} \right\}$$

↓

double path integral
on $q(t)$ and $q'(t)$

↓

$$\dot{P}_{\mu,m}(t) = \sum_{\mu',m'} \int_{t_0}^{t} dt' \, K_{\mu',m';\mu,m}(t,t') \, P_{\mu',m'}(t')$$

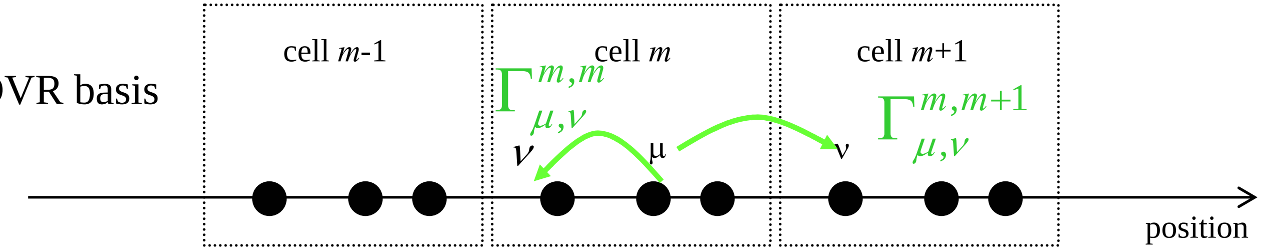
U. Weiss, *Quantum Dissipative Systems*
M.Grifoni, M. Sassetti and U. Weiss, PRE (1996)

Approximations:

- for high dissipation or high temperature: kernel K up to 2nd order in $\Delta_{\mu\mu'}$
- in AC case for large Ω introduce averaged quantities: $P \to \overline{P}, K \to \overline{K}$

ASYMPTOTIC SOLUTION

DVR basis



$$\Gamma^{m,n}_{\mu,\nu} = \int_{t_0}^{\infty} d\tau\, K_{\mu,m;\nu,n}(\tau) \qquad (K \to \overline{K}, \Gamma \to \overline{\Gamma} \quad \text{for AC case})$$

$$\Gamma^{f/b}_{\mu,\nu} = \Gamma^{m,m\pm 1}_{\mu,\nu}$$

$$\Gamma^{intra}_{\mu,\nu} = \Gamma^{m,m}_{\mu,\nu}$$

$$\Gamma_{\mu,\nu} = \Gamma^{intra}_{\mu,\nu} + \Gamma^{f}_{\mu,\nu} + \Gamma^{b}_{\mu,\nu}$$

$$\Gamma_{\mu} = \sum_{\nu \neq \mu} \Gamma_{\mu,\nu}$$

$$p^\infty_\mu := \sum_m \overline{P}^\infty_{\mu,m}$$

$$v_\infty = L \sum_{\mu,\nu} p^\infty_\mu \left(\Gamma^f_{\mu,\nu} - \Gamma^b_{\mu,\nu}\right)$$

$$p^\infty_\alpha = \frac{\Gamma_\beta \Gamma_\gamma - \Gamma_{\beta\gamma}\Gamma_{\gamma\beta}}{\sum_\mu \sum_{\nu>\mu} \Gamma_\mu \Gamma_\nu - \Gamma_{\mu\nu}\Gamma_{\nu\mu}} \quad \circlearrowleft$$

TRANSITION RATES

- DC case

$$\Gamma_{\mu,\nu}^{m,m'} = \left(\frac{\Delta_{\mu,\nu}^{m,m'}}{\hbar}\right)^2 \int_{-\infty}^{\infty} d\tau\, e^{-Q_{\mu,\nu}^{m,m'}(\tau)} \exp\left(\frac{i\tau}{\hbar}\left(\varepsilon_\mu - \varepsilon_\nu - F\left(q_{\mu,m} - q_{\nu,m'}\right)\right)\right)$$

$$Q_{\mu,\nu}^{m,m'}(\tau) = \frac{\left(q_{\mu,m} - q_{\nu,m'}\right)^2}{\pi\hbar} \int_0^{\infty} d\omega\, \frac{J(\omega)}{\omega^2}\left[\coth\left(\frac{\hbar\beta\omega}{2}\right)\left(1-\cos(\omega\tau)\right) + i\sin(\omega\tau)\right]$$

$$q_{\mu,m} = mL + q_\mu$$

bath correlations

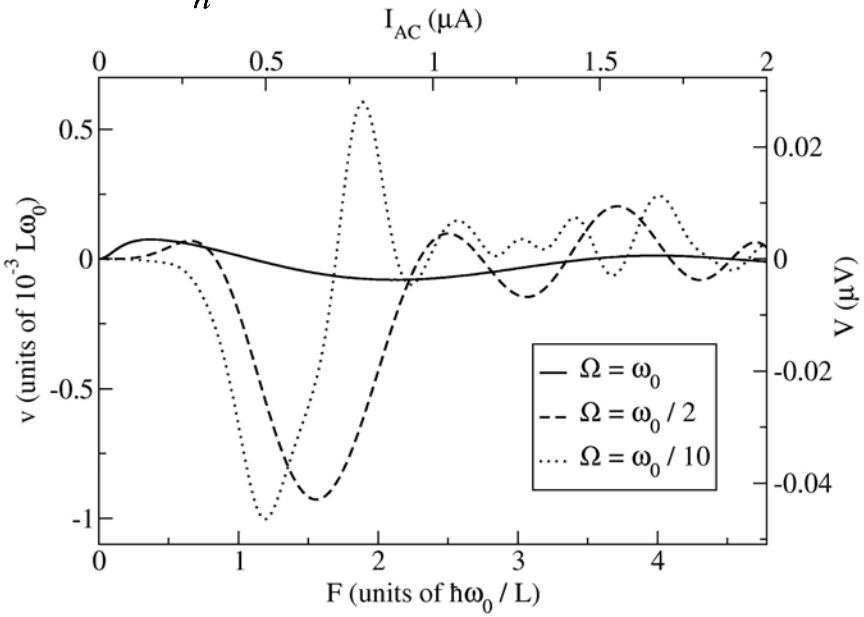
- AC case

$$\overline{\Gamma}_{\mu,\nu}^{m,m'} = \left(\frac{\Delta_{\mu,\nu}^{m,m'}}{\hbar}\right)^2 \int_{-\infty}^{\infty} d\tau\, e^{-Q_{\mu,\nu}^{m,m'}(\tau)} J_0\left(\frac{2F\left(q_{\mu,m} - q_{\nu,m'}\right)}{\hbar\Omega}\sin\left(\frac{\Omega\tau}{2}\right)\right) e^{\frac{i\tau}{\hbar}\left(\varepsilon_\mu - \varepsilon_\nu\right)}$$

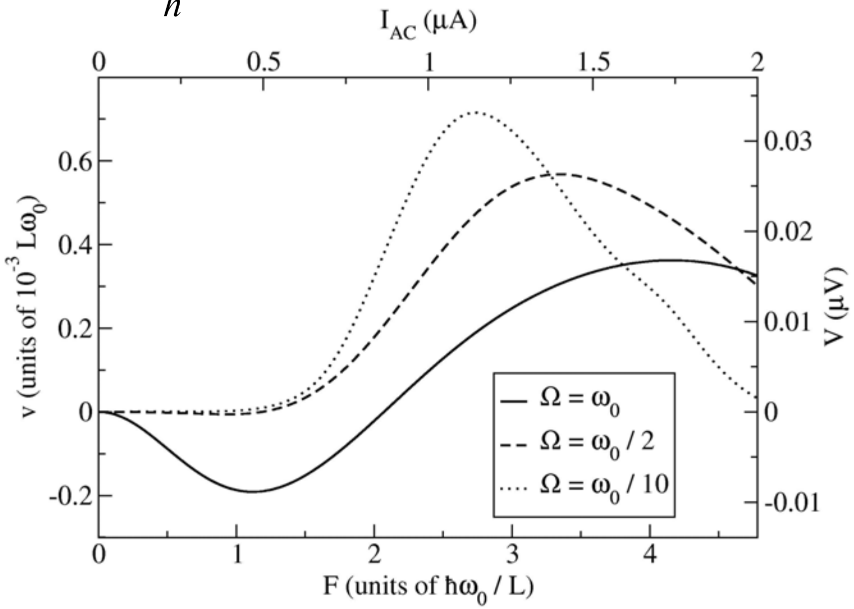
AC DRIVING

Current inversion depending on the parameters

$\alpha = \frac{\eta L^2}{h} = 0.5$ (moderate damping)



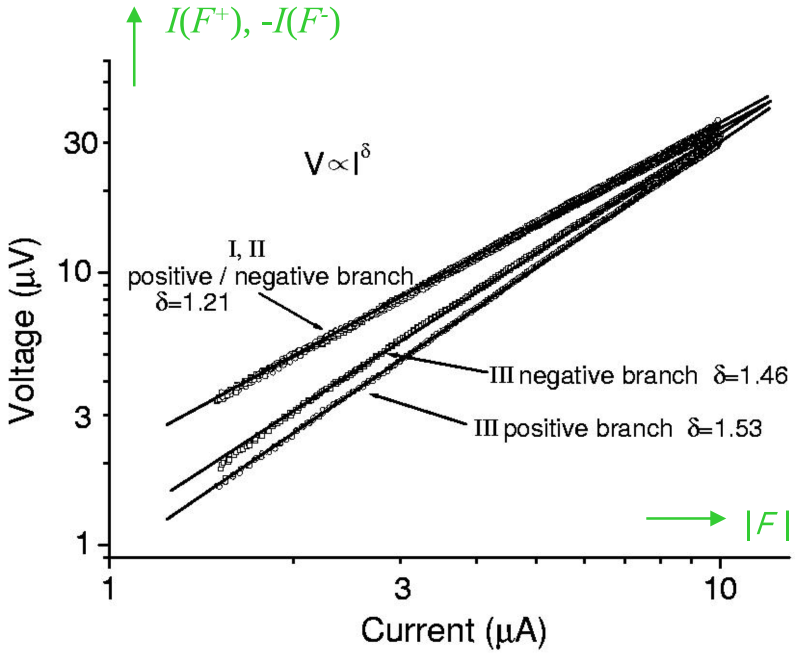
$\alpha = \frac{\eta L^2}{h} = 2.0$ (strong damping)



M. Grifoni, M. S. Ferreira, J.Peguiron and J. B. Majer, PRL **89**, 146801 (2002)

DC-CASE

- Experiment:



Problem

theory valid only

$FL \lesssim \hbar\omega_0 \rightarrow I \lesssim 0.4\mu\text{A}$

- Theory: DC rate for $k_B T << FL << \hbar\omega_c$:

$$\Gamma_{\mu,\nu}^{m,n} = \frac{\pi}{2} \frac{\left(\Delta_{\mu,\nu}^{m,n}\right)^2}{\omega_c \Gamma(2\alpha_{\mu,\nu}^{m,n})} \left(\frac{\varepsilon_{\mu,\nu}^{m,n}}{\omega_c}\right)^{2\alpha_{\mu,\nu}^{m,n}-1}$$

$$\varepsilon_{\mu,\nu}^{m,n} = \varepsilon_\mu - \varepsilon_\nu - F\left(q_{\mu,m} - q_{\nu,n}\right)$$

$$\alpha_{\mu,\nu}^{m,n} = \frac{\eta\left(q_{\mu,m} - q_{\nu,n}\right)^2}{2\pi\hbar}$$

U. Weiss and H. Grabert, Phys. Lett. **A108** (1985)

DUALITY RELATION FOR QR

No quantitative explanation for experiments due to $FL \lesssim \hbar\omega_0 \rightarrow I \lesssim 0.4\mu\text{A}$

OUTLOOK

- ´Exact´ duality transformation mapping continuous problem into a TB

$$\mu_{\mathrm{WB}}(F;L,\alpha) = \mu_0 - \mu_{\mathrm{TB}}(F;\frac{L}{\alpha},\frac{1}{\alpha})$$

Peguiron, Grifoni (2004)

$$\mu_0 = \frac{1}{\eta} \qquad \alpha \equiv \frac{\eta L^2}{2\pi\hbar}$$

Generalization of: M.P.A. Fischer and W. Zwerger, PRB **32**, 6190 (1985)

- Diagonalize $H_R + Fx$ rather than H_R

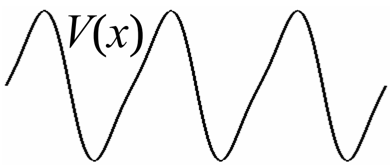
⟶ Truncation for Wannier states rather than Bloch bands

DUALITY RELATION FOR QR

$$\mu_{\mathrm{WB}}(F;L,\alpha) = \mu_0 - \mu_{\mathrm{TB}}\left(F;\frac{L}{\alpha},\frac{1}{\alpha}\right) \quad * \qquad \mu_0 = \frac{1}{\eta}$$

Model 1 (continuous, weak-binding):

$$V_{\mathrm{WB}}(x) = \sum_{l=1}^{\infty} V_l \cos\left(l\frac{2\pi}{L}x - \varphi_l\right)$$



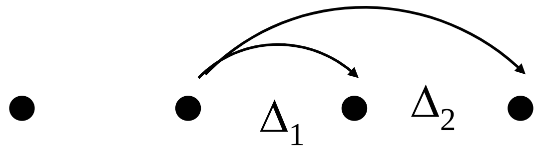
Model 2 (discrete, tight-binding):

$$H_{\mathrm{TB}} = \sum_{n=-\infty}^{\infty}\sum_{l=1}^{\infty}\left(\Delta_l |n+l\rangle\langle n| + \Delta_l^* |n\rangle\langle n+l|\right)$$

$$\tilde{x} = \sum_{n=-\infty}^{\infty} n\tilde{L}|n\rangle\langle n|$$

$$\Delta_l = \frac{V_l}{2}e^{i\varphi_l}$$

harmonics ↔ neighbors coupling



Change of periodicity and dissipation strength:

$$L \quad \rightarrow \quad \tilde{L} = \frac{L}{\alpha} = \frac{2\pi\hbar}{\eta L}$$

$$\alpha \equiv \frac{\eta L^2}{2\pi\hbar} \quad \rightarrow \quad \tilde{\alpha} = \frac{1}{\alpha}$$

Change of bath spectral density:

$$J_{\mathrm{WB}}(\omega) = M\gamma\omega \quad \rightarrow \quad J_{\mathrm{TB}}(\omega) = \frac{M\gamma\omega}{1+(\omega/\gamma)^2}$$

Validity:
TB transition rates Γ << dissipation rate γ

* Generalization of: M.P.A. Fischer and W. Zwerger, PRB **32**, 6190 (1985)

CONCLUSIONS

directed current

$\Updownarrow$

breaking of detailed balance symmetry

- Ratchet effect in few-bands asymmetric potentials
- Current inversion depending on all parameters in the AC case
- Power-law behavior in the DC case (for large ω_c)
- No ratchet effect with only 1 band within this model