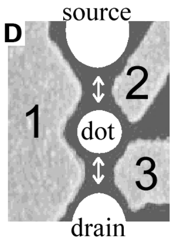
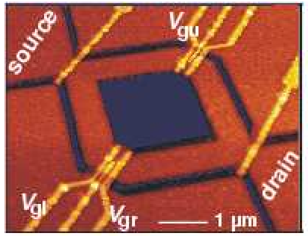
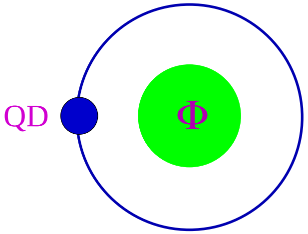
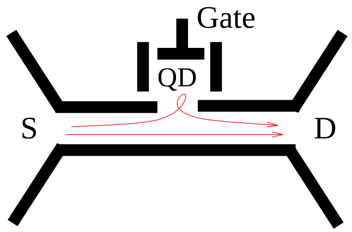
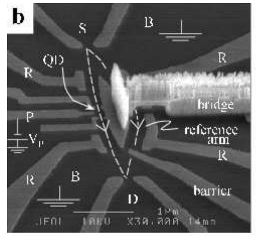
Kondo Effect and Phase-Coherent Transport in Quantum Dots











Kicheon Kang

Department of Physics

Chonnam National University

Gwangju, Korea

전남대학교 CHONNAM NATIONAL UNIVERSITY

Collaborators

Theory : Quantum Dots, Rings, Wires

G. L. Kim *(Chonnam National University)*

S. Y. Cho, R. McKenzie *(Univ. of Queensland, Australia)*

M.-S. Choi *(Korea University)*

L. Craco *(Köln, Germany)*

Experiment : Semiconductor Quantum Devices, CNT Devices

M. Jang, S. Lee *(ETRI)*, K.W. Park *(University of Seoul)*

J. Kim, N. Kim *(KRISS)*

J.-J. Kim *(Chonbuk National University)*

Outline

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- **Coherent transport in quantum dots**

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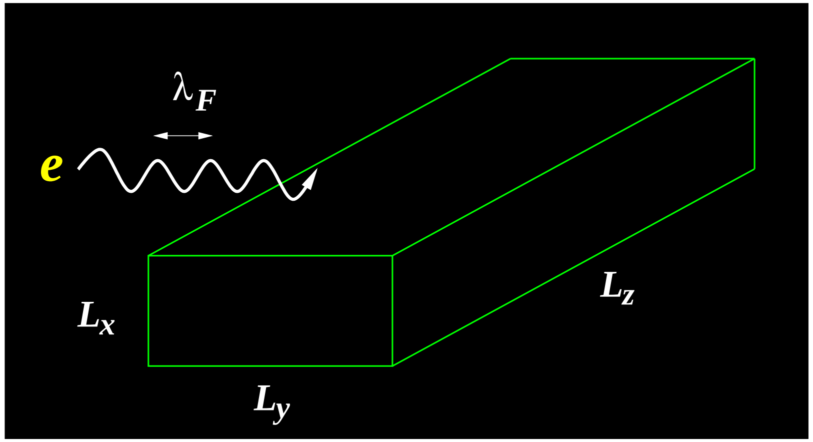
Fermi Wavelength (λ_F) and Dimensionality



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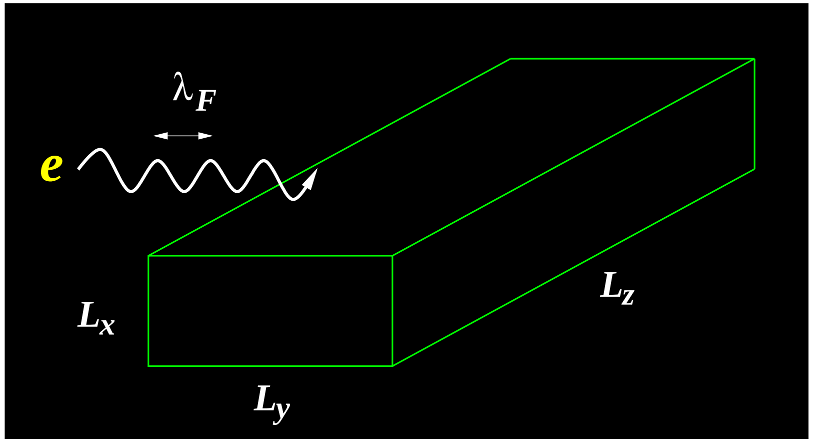

- $\lambda_F \ll L_x, L_y, L_z$: 3-dimension

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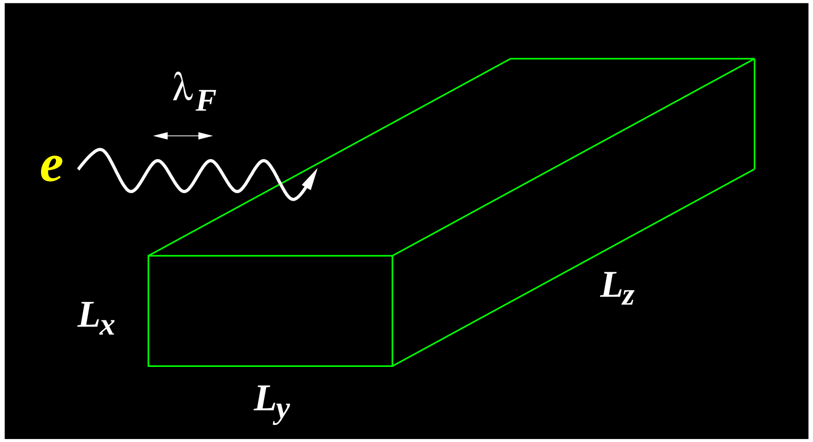

- $\lambda_F \ll L_x, L_y, L_z$: 3-dimension
- $L_x < \lambda_F \ll L_y, L_z$: 2-dimension (2-D electron gas)

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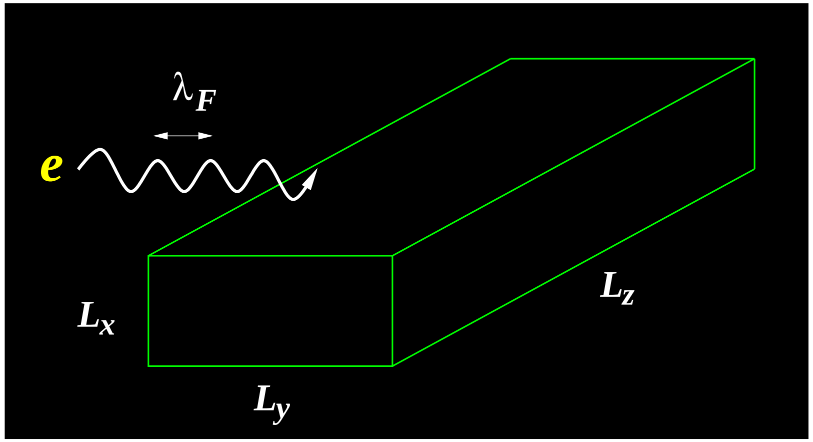

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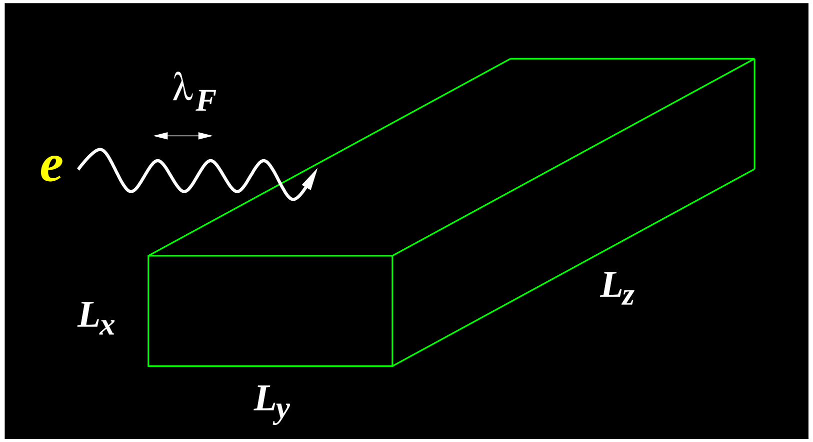

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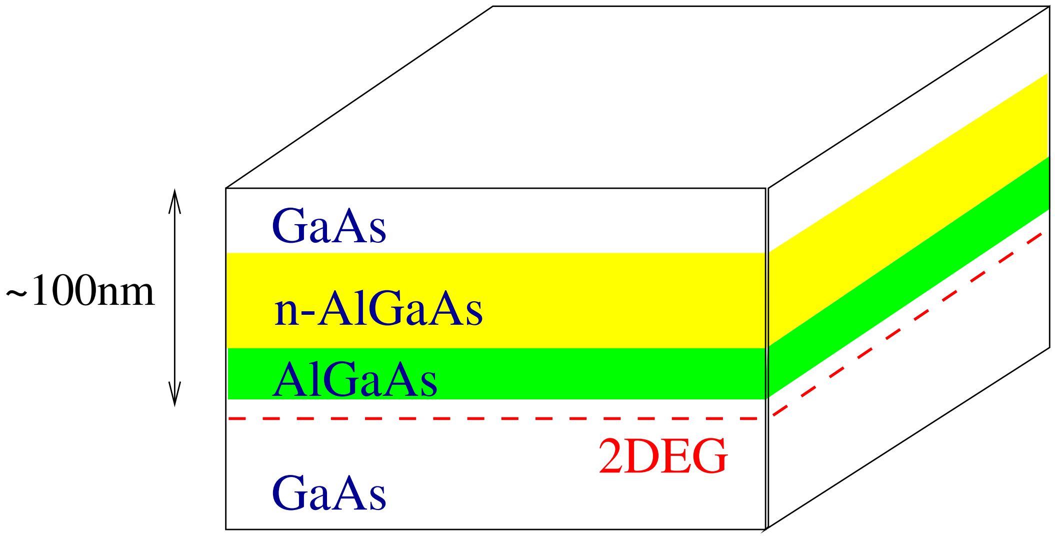
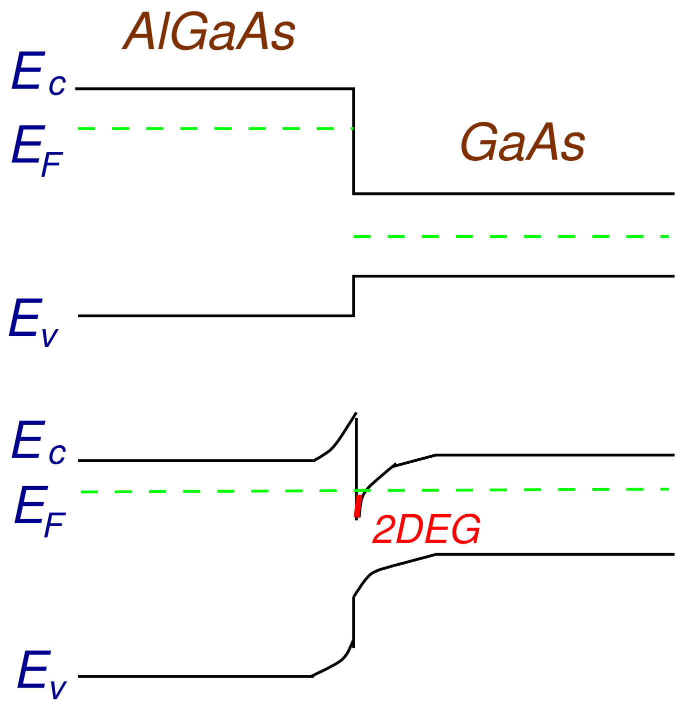
- $L_x, L_y < \lambda_F \ll L_z$: 1-dimension (quantum wire)
- $L_x, L_y, L_z < \lambda_F$: 0-dimension (quantum dot)

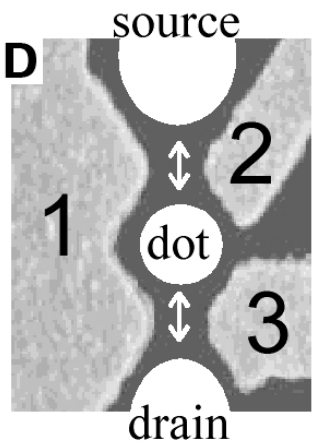
2-Dimensional Electron Gas (2DEG)

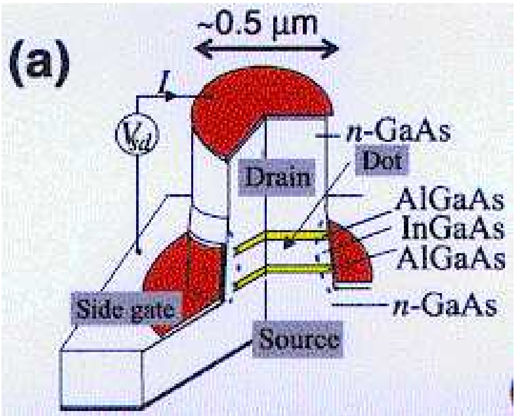


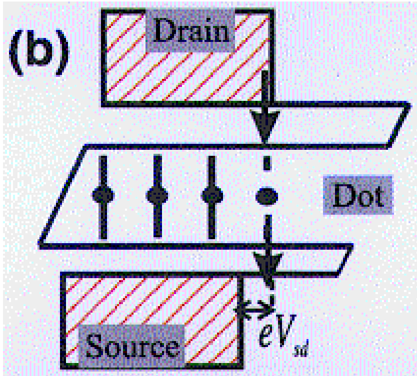


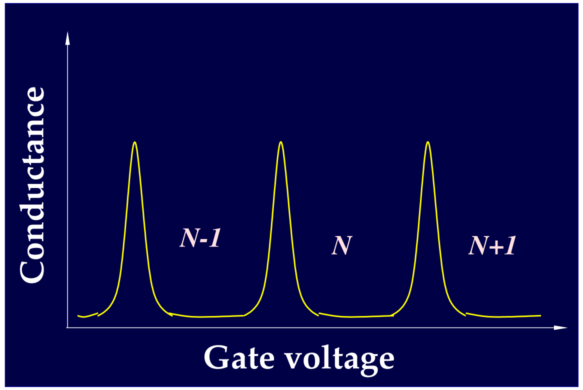
- High mobility/coherence due to separtion of the conduction channel and the doped region
- Gating/etching required to get lower dimension (wire, dot)

Quantum Dot (Artificial Atom)







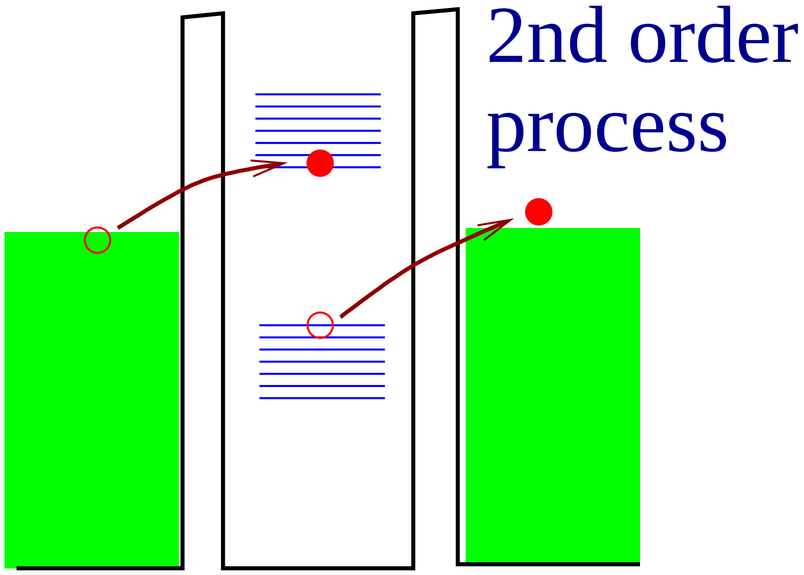


Charge and energy quantization

- E_c: single electron charging energy, Δ: energy level discreteness
- $E_c \equiv e^2/2C \gg k_B T$: Coulomb blockade, single electron tunneling (SET)
- $\Delta \gg k_B T$: Quantum confinement, resonant tunneling $\rightarrow$ phase-coherent process

"Cotunneling" in the Coulomb Blockade Region

Averin & Nazarov (1990) - Theory; Eiles et al., PRL (1992) - Experiment



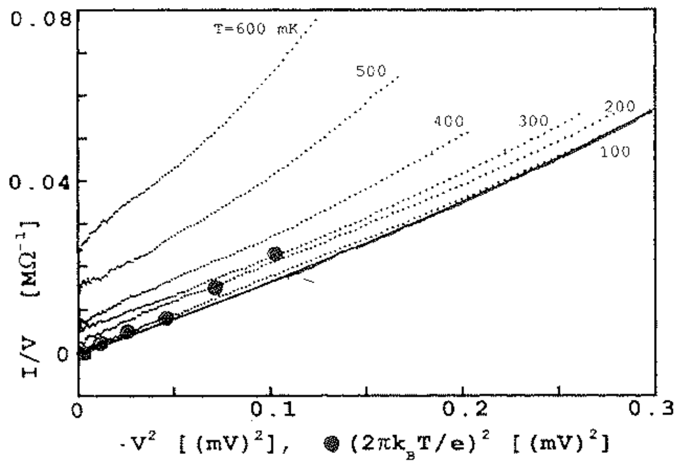


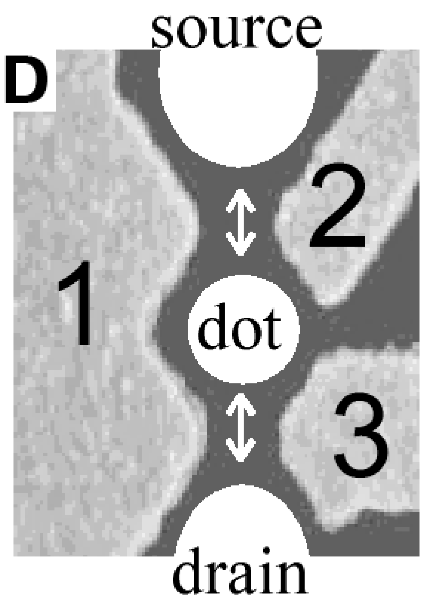
FIG. 3. Plot of I/V vs V^2 for $q = 0.55$. Experimental data (solid points) are plotted for six values of T. Open circles are I/V as $V \to 0$ obtained from the six temperatures plotted vs $(2\pi k_B T/e)^2$. The solid line is the cotunneling prediction of Eq.

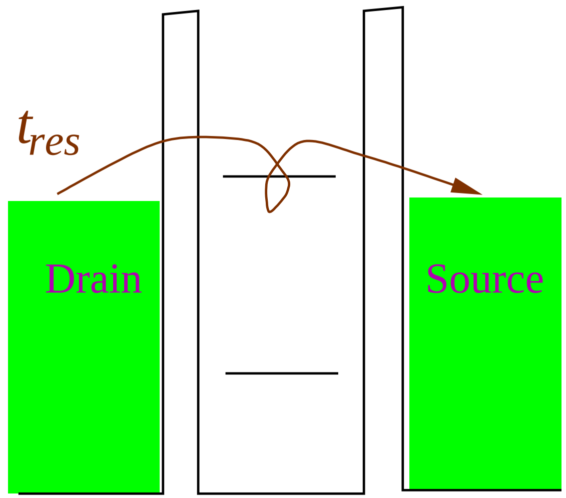
- Cotunneling: 2nd or higher order (virtual) process
  - Macroscopic quantum tunneling of charge

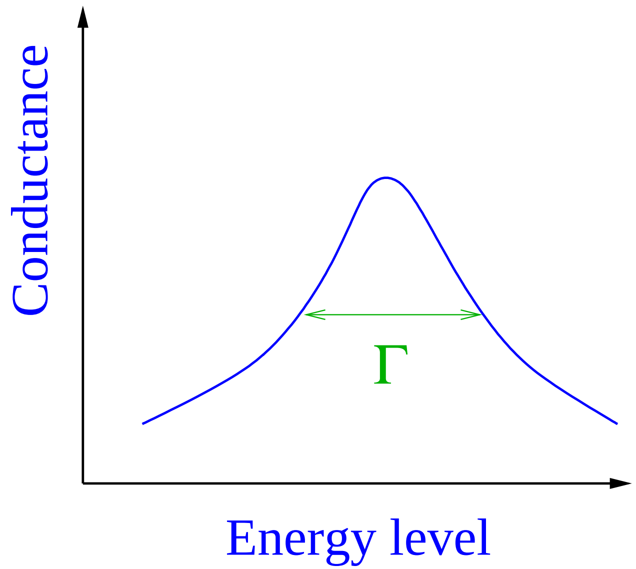
$$G \propto T^2, V^2 \quad \text{for} \quad \Delta \to 0 \quad \text{(Inelastic)}$$

- For $\Delta \gg k_B T, eV$ inelastic cotunneling current is strongly suppressed (See e.g., Kang & Min, PRB (1997))

Resonant Tunneling through a Quantum Dot







- "Coherent" resonant tunneling through a single impurity level (ε_0) for $\Gamma \gg k_B T$

$$G = \frac{2e^2}{h}|t_{res}(E_F)|^2 = G_{max}\frac{1}{e_0^2 + 1} \qquad \left(e_0 \equiv \frac{2}{\Gamma}(\varepsilon_0 - E_F)\right)$$

- Phase coherence of the transmission amplitude $\boxed{t_{res} = |t_{res}|e^{i\gamma}}$ cannot be directly addressed (Conductance measures $|t_{res}|$ only)

Detecting the Phase Coherence I

- 2-Terminal Aharonov-Bohm (AB) Interferometer (Yacoby *et al.*, PRL (1995))

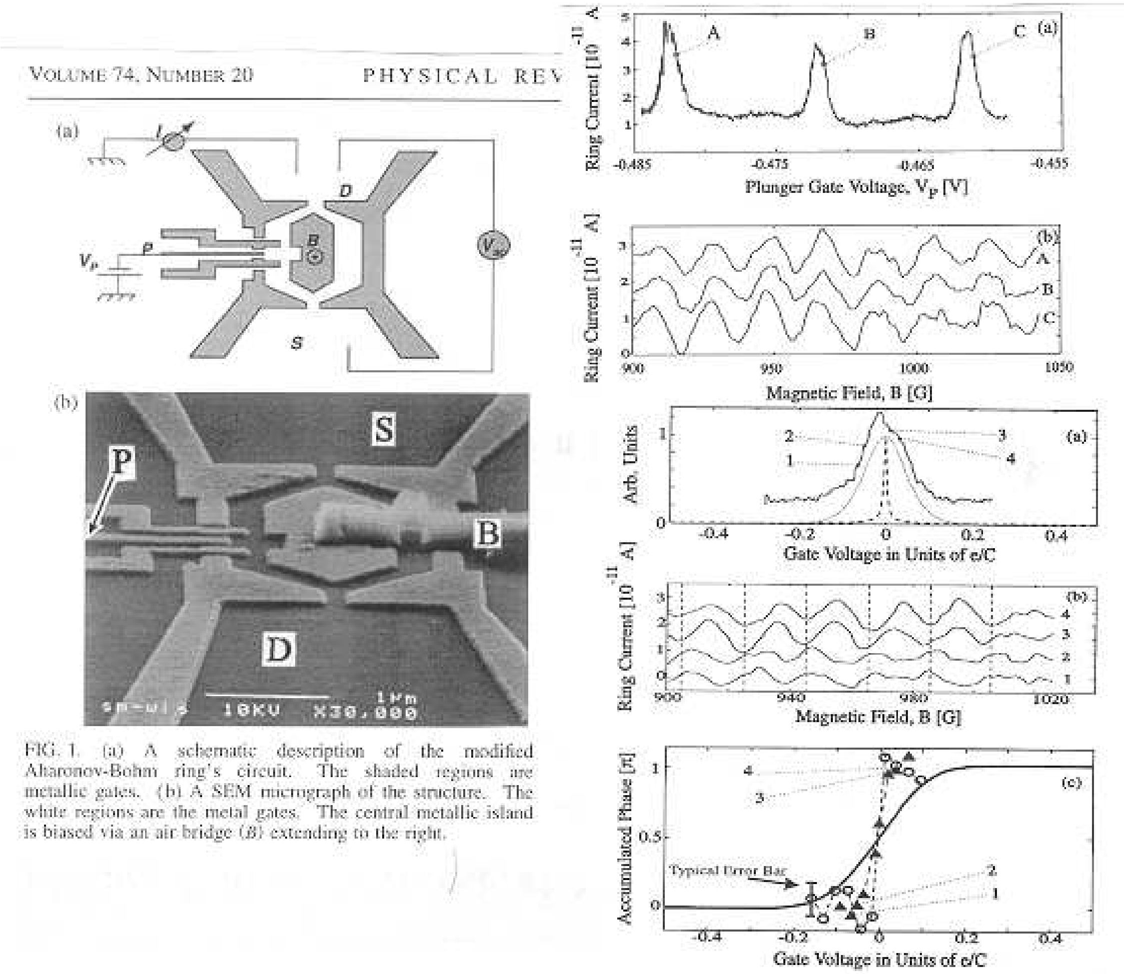
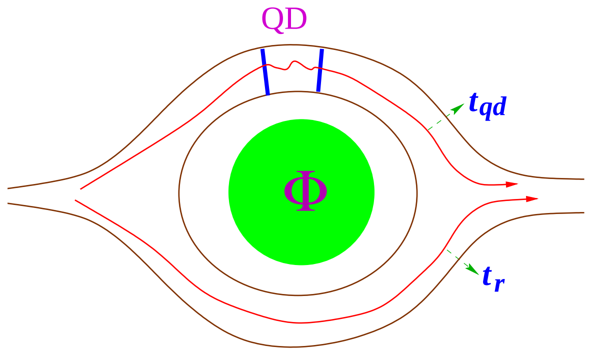


FIG. 1. (a) A schematic description of the modified Aharonov-Bohm ring's circuit. The shaded regions are metallic gates. (b) A SEM micrograph of the structure. The white regions are the metal gates. The central metallic island is biased via an air bridge (*B*) extending to the right.



$$G \sim |t_{qd} + t_r|^2$$

$$= T_0 + 2|t_{qd}||t_r| \cos\left(\varphi_{qd} - \varphi_{AB}\right)$$

Transmission Amplitude:

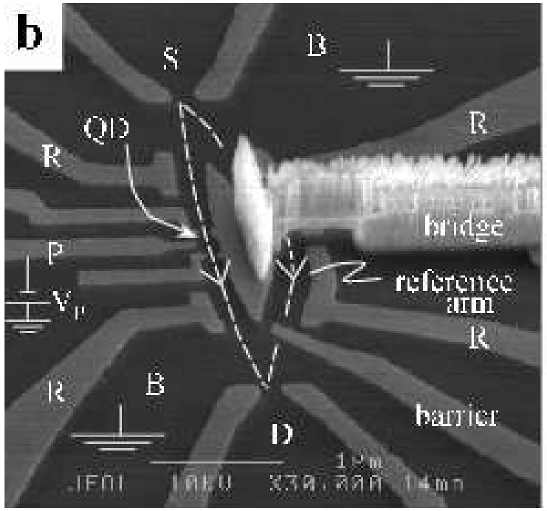
$$t_{qd} = |t_{qd}| \exp\left(i\varphi_{qd}\right)$$

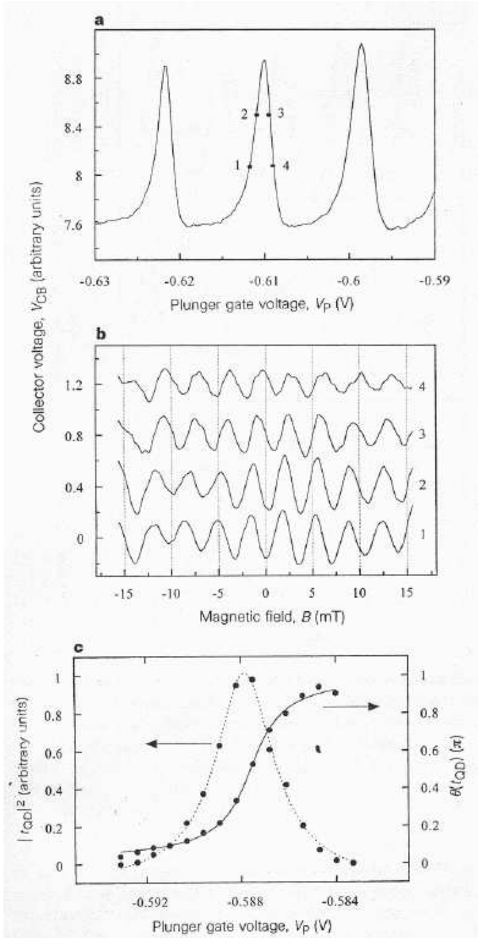
(Conventional SET experiment measures only $|t_{qd}|^2$)

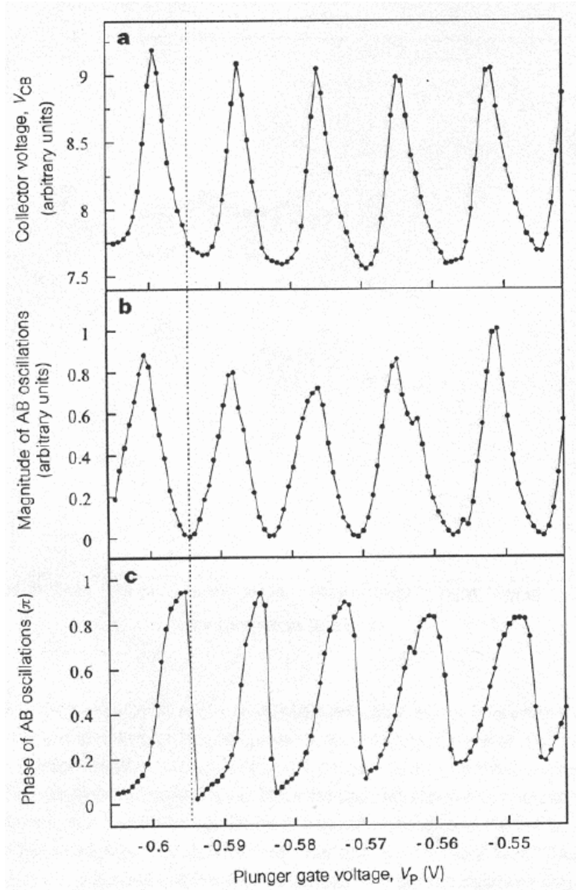
- AB oscillation of the conductance → Phase coherence of transmission through a QD
- Onsager's relation $G(-B) = G(B) \longrightarrow$ Phase rigidity ($\varphi_{qd} = 0$ or π) Yeyati & Büttiker, PRB (1995)

Phase Measurement of a Quantum Dot

- AB Interferometer with Open Geometry (Schuster *et al.*, Nature (1997))



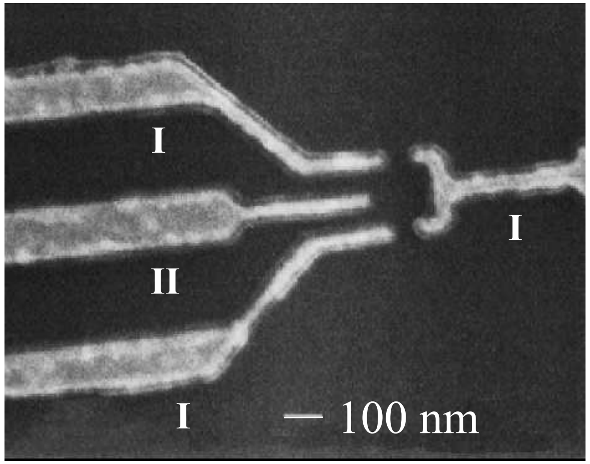


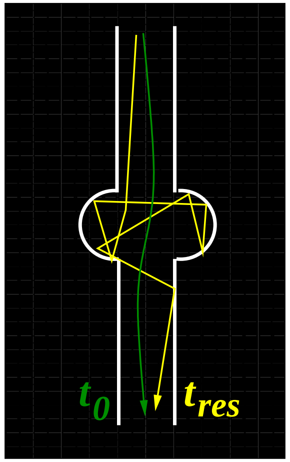


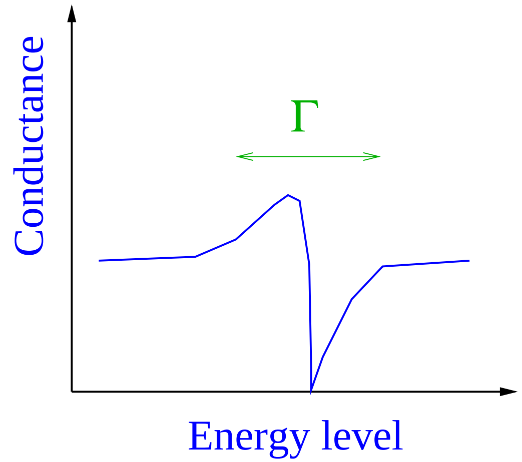
- Open geometry allows two-path interference and continuous phase evolution of t_{qd}
- Abrupt phase drop (π) accompanied by transmission zero / In-phase resonances
- Generic transmission zero in a single channel transport with a time-reversal symmetry (H. W. Lee (1999))

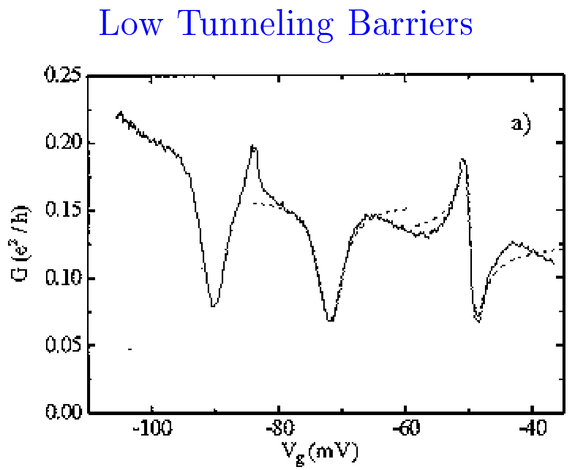
Detecting the Phase Coherence II

- Fano Resonances (Göres *et al.*, PRB (2000))









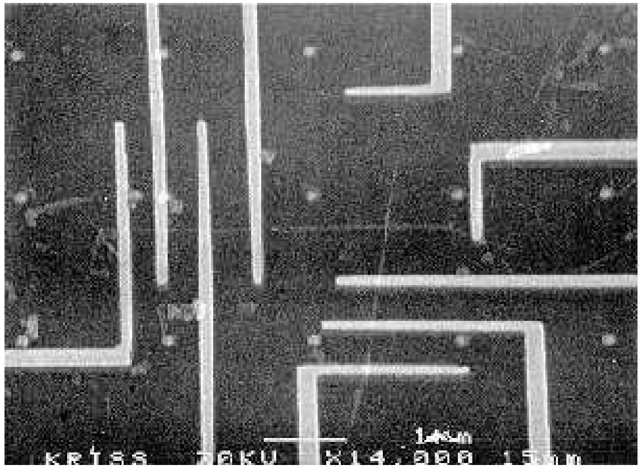
- Fano-resonance as a result of the interference between t_{res} and t_0

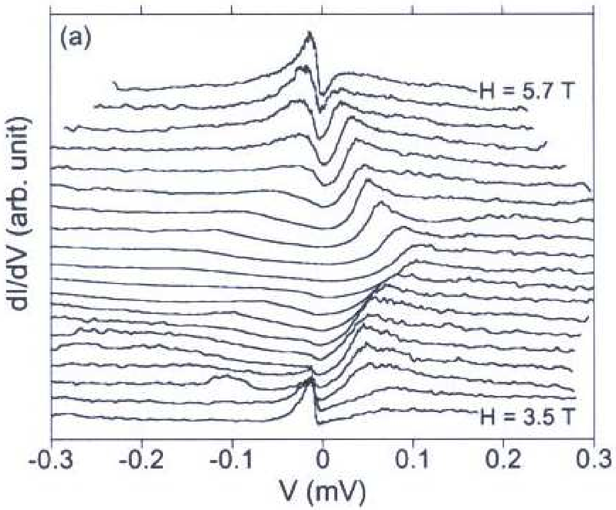
$$G \simeq \frac{2e^2}{h}|t_0 + t_{res}|^2 = G_0 \frac{|e_0 + Q|^2}{e_0^2 + 1}$$

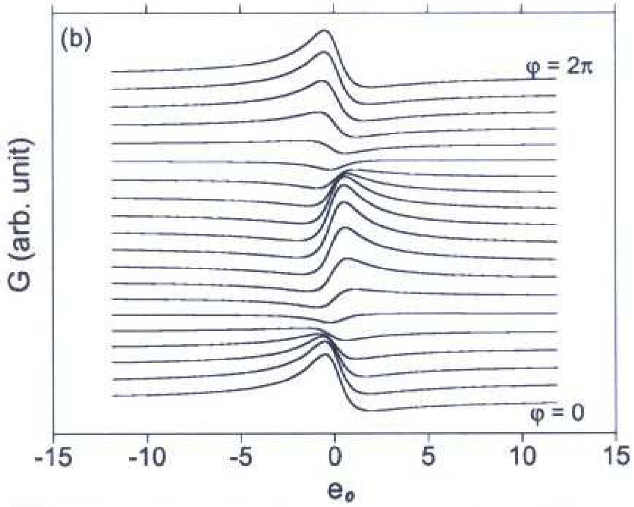
- Asymmetric line shape : Evidence of the phase coherence

Fano Resonance in Carbon Nanotubes

(J. Kim, Kicheon Kang, and coworkers, PRL (2003))





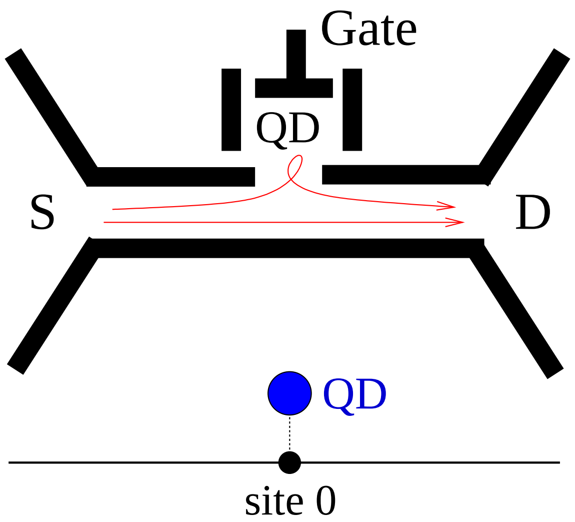


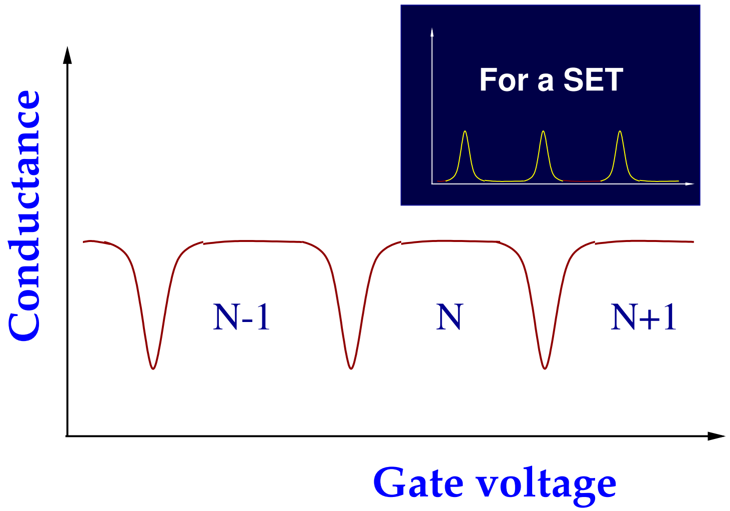
- Resonant "cavity" at the contact between the two CNTs
- "Aharonov-Bohm oscillation" of the Fano resonances
- Agrees well with the simple theory of Fano-resonance with AB phase:

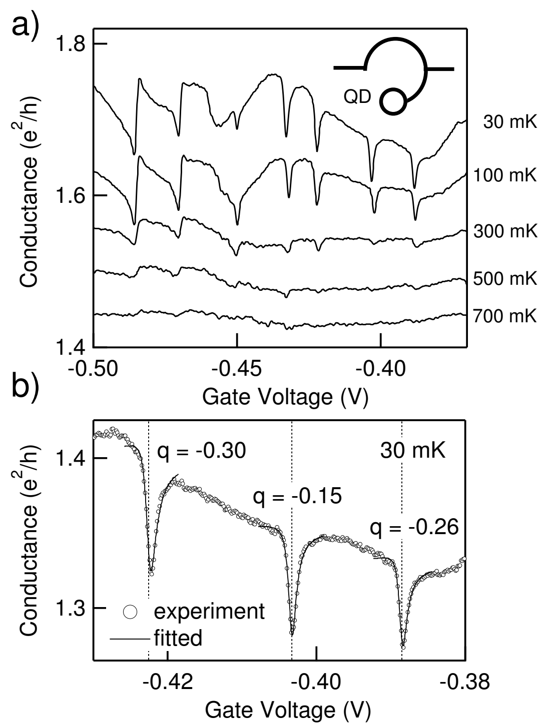
$$G \simeq G_0 \frac{|e_0 + Q|^2}{e_0^2 + 1}, \qquad Q = Q_R \cos\varphi + iQ_I \sin\varphi$$

“Anti-Coulomb-Blockade”?

Theory: Kicheon Kang *et al.*, PRB(2001), Experiment: K. Kobayashi *et al.*, cond-mat/0311497







- Theoretical prediction for 1D wire + dot:

$$G = (e^2/h)\cos^2 \pi n_d$$

- Anti-resonances around $n_d = N + 1/2 \longleftarrow$ “destructive interference”
- Quasi-periodic “anti-resonance”: verified experimentally

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- Coherent transport in quantum dots
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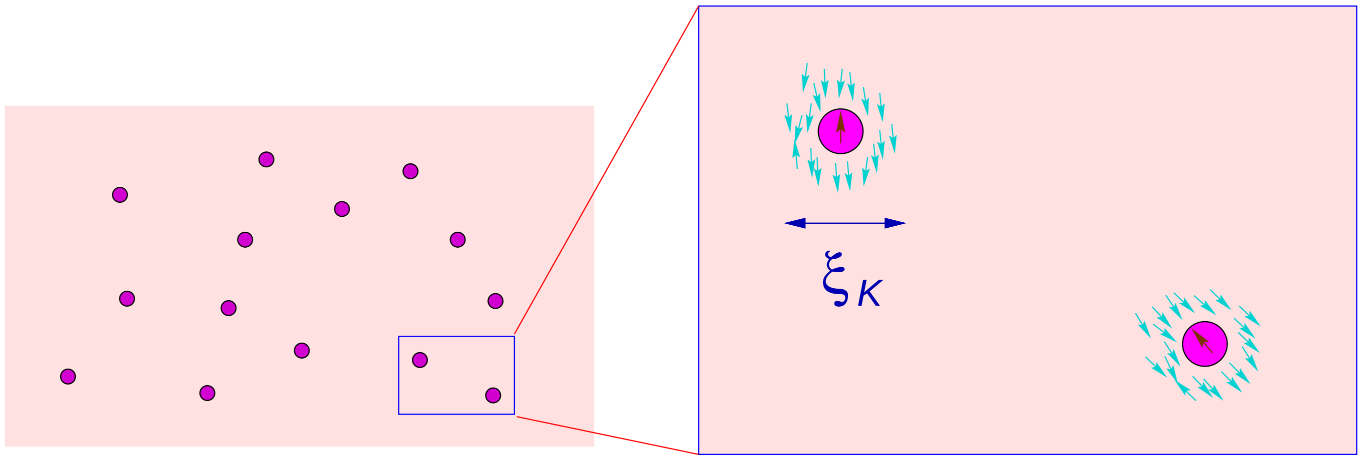
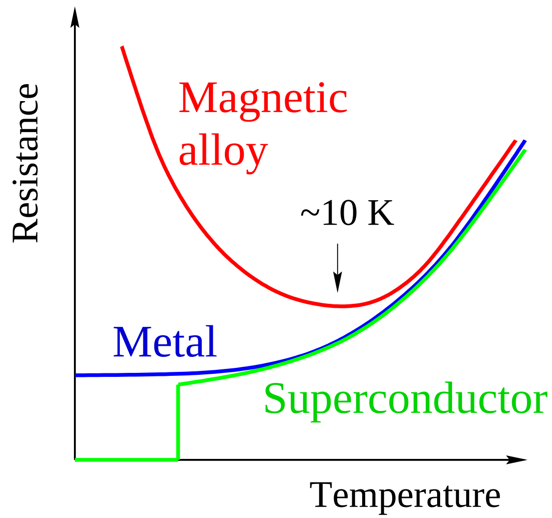
- Phase coherence of the Kondo effect
- “Mesoscopic” Kondo effect and spin-charge separation

Kondo Effect in Dilute Magnetic Alloys

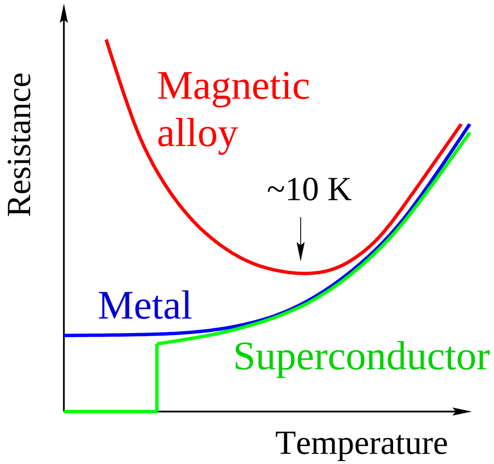


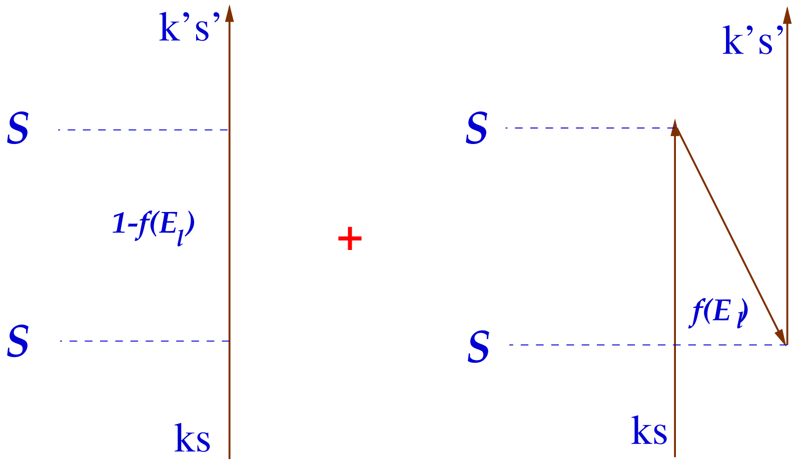


- Antiferromagnetic exchange interactions between the impurity spin and the conduction electron spin
- (Entangled) spin singlet ground state is formed between the impurity and the conduction electrons
- Screening cloud around the impurity (with its size ξ_K and the binding energy T_K) screens the local magnetic moment
- Enhanced resistivity at low temperatures due to the large spin-flip scattering cross section

Origin of the Resistance Anomaly

J. Kondo (1964)





- Spin-exchange Hamiltonian

$$H = -J\,\sigma \cdot S \quad (J < 0)$$

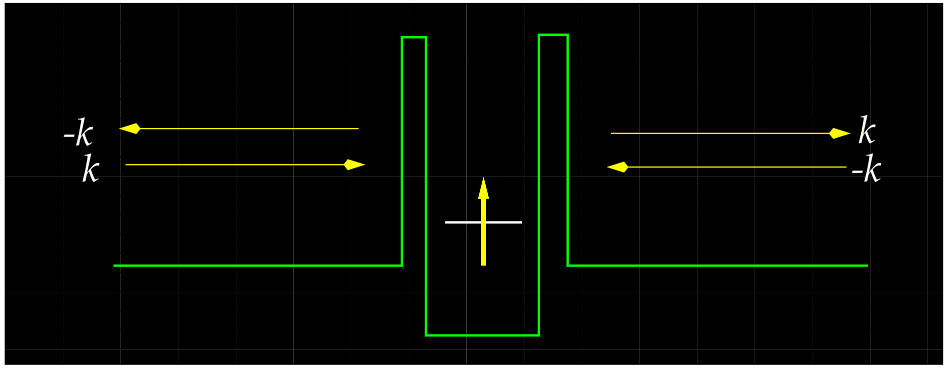
- 2nd-order spin-exchange scattering amplitude:

$$\sim J^2 \sum_l \frac{f(E_l)}{E_F - E_l} \sim -J^2 \rho(E_F) \log\left(k_B T / D\right)$$

- Logarithmic singularity $\longleftarrow$ time-reversal asymmetry of the two processes!

Kondo-Resonant Transmission

Langreth (1966), Glazman & Reich (1988), Ng & Lee (1988)



$$|out\rangle = S|in\rangle, \qquad S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

- Generalized Friedel sum rule: No inelastic scattering takes place at $T = 0$

$$|\mathsf{t}|^2 = \sin^2 \frac{\pi}{2}\mathsf{n}$$

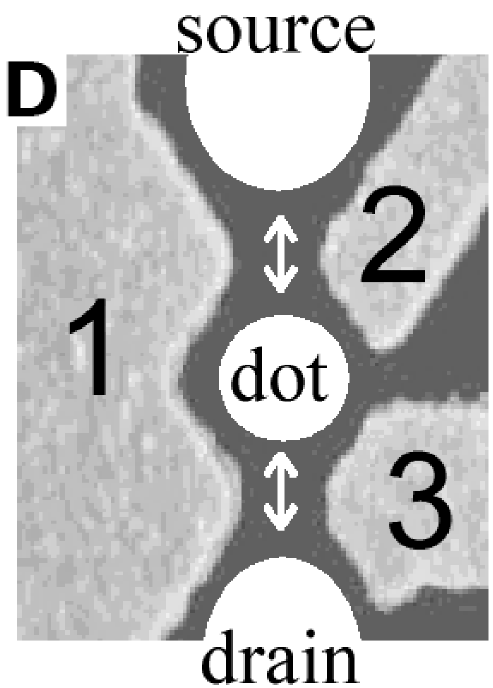
- Conductance formula

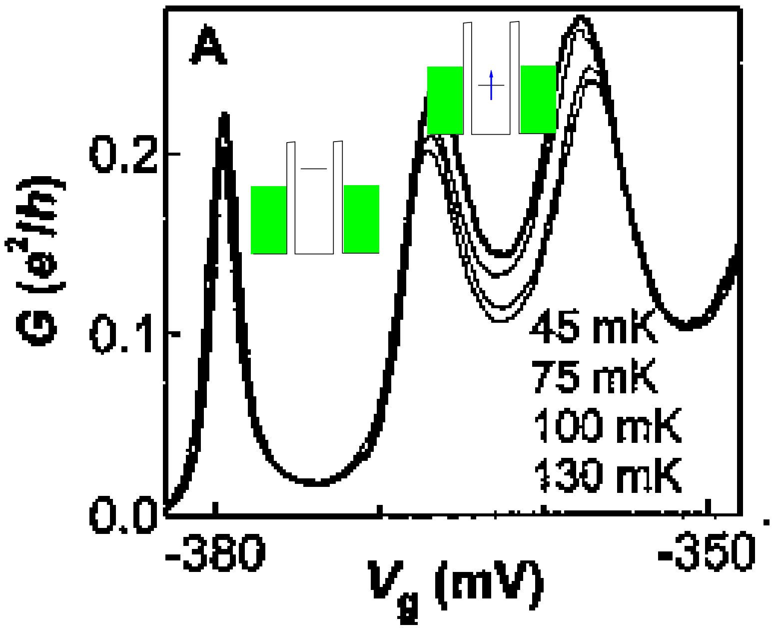
$$\mathsf{G} = \frac{2\mathsf{e}^2}{\mathsf{h}}|\mathsf{t}|^2 \longrightarrow 2\mathsf{e}^2/\mathsf{h} \text{ for } n = 1 \text{ (Unitary limit)}$$

- Coherent superposition of "spin-flip" cotunneling events

Kondo Effect in Quantum Dots

Goldhaber-Gordon *et al.* (Nature 1998), Cronenwett *et al.* (Science 1998), and more...

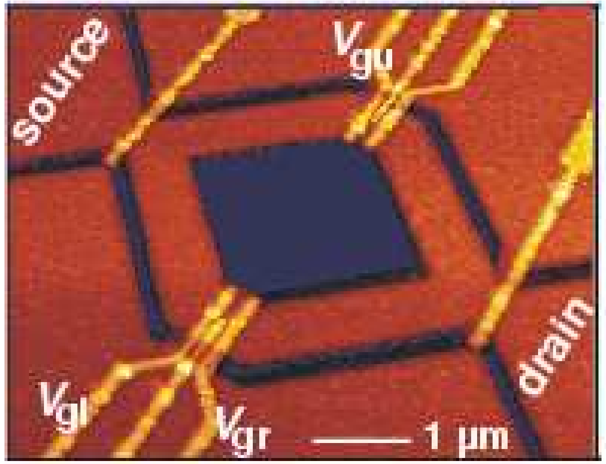


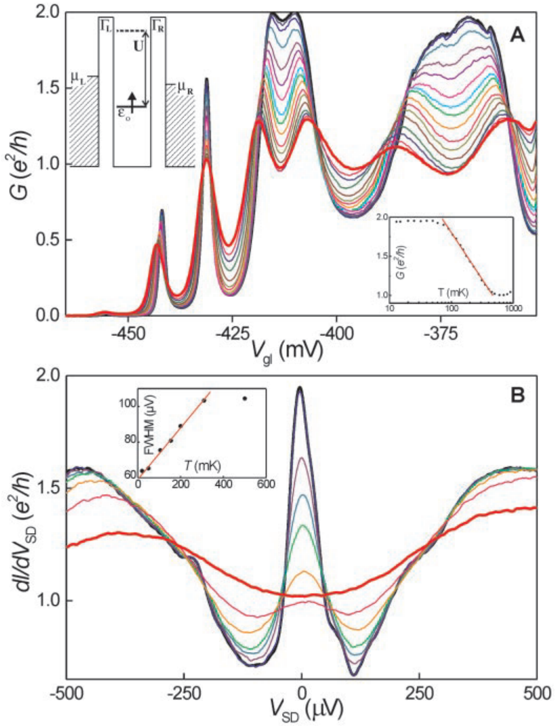


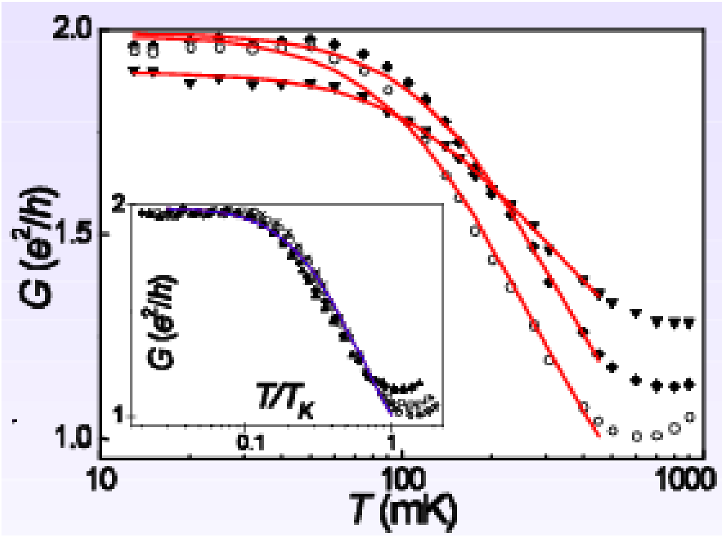
- Strong even-odd effect
- Conductance enhanced at low temperatures in the “Kondo” valley
- Observed also in carbon nanotubes, single molecules

Kondo Effect in the Unitary limit

W. van der Wiel *et al.* (Science 2000)







- Unitarity / universal scaling

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- Coherent transport in quantum dots
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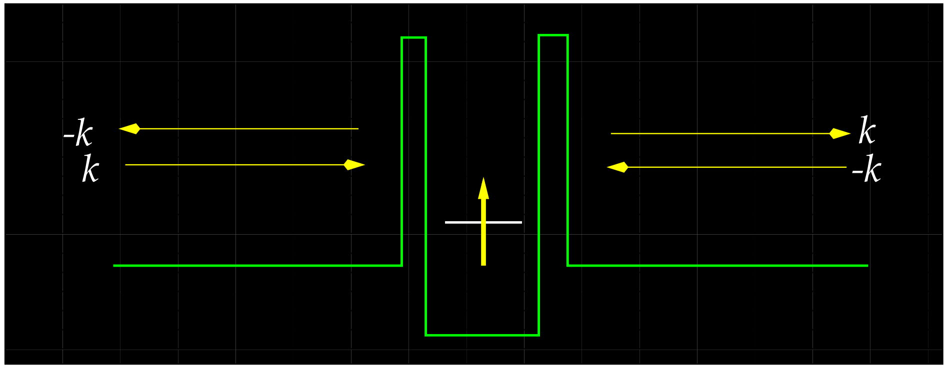
Kondo Resonance as a Scattering Problem

Langreth (1966), Glazman & Reich (1988), Ng & Lee (1988)



$$|out\rangle = S|in\rangle, \quad S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

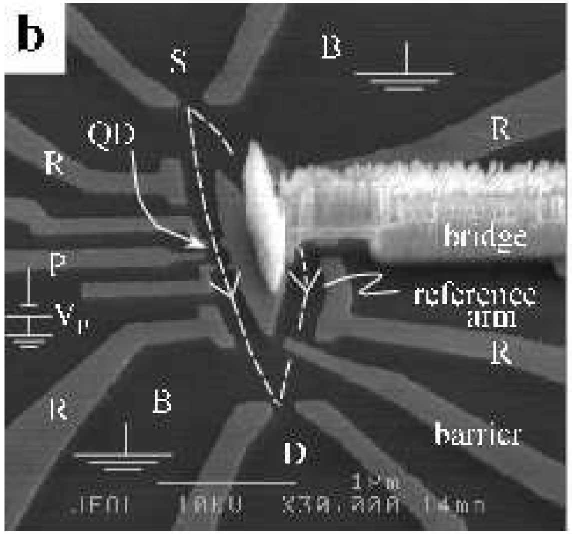
- Generalized Friedel sum rule: No inelastic scattering takes place at $T = 0$

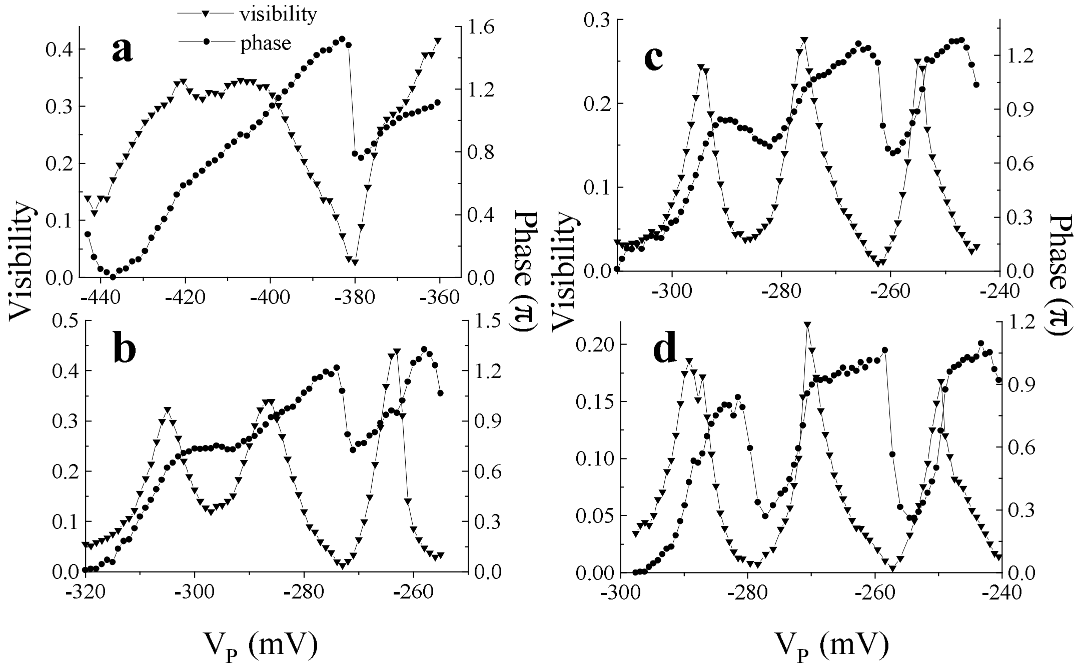
$$t = -i \sin\gamma \, e^{i\gamma}, \quad \gamma = \frac{\pi}{2} n$$

- $\boxed{\mathrm{G} = (2\mathrm{e}^2/\mathrm{h}) \sin^2\gamma}$: Conductance enhanced in the Kondo limit ($n = 1$)
- Phase shift of $\Delta\gamma \equiv \gamma(n = 1) - \gamma(n = 0) = \pi/2$

Phase Evolution in a Kondo System

Y. Ji *et al.*, Science (2000); PRL (2002)

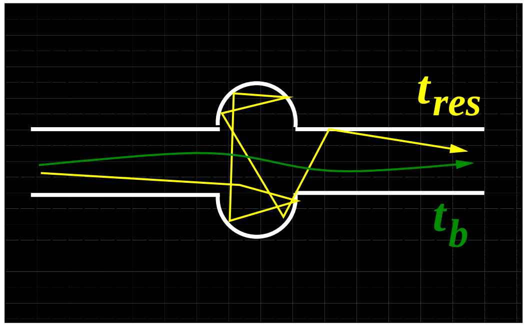




- Phase-coherent transmission: From the Coulomb blockade to the Kondo limit
- Anomalous phase shift at the Kondo plateaus with $\boxed{\Delta\gamma \sim \pi}$ instead of $\pi/2$
- Charge fluctuation is important in the unitary limit

Toward an Explanation of the Phase Evolution: Model

K. Kang *et al.*, cond-mat/0401169



$$
\begin{aligned}
H &= H_L + H_R + H_D + H_T \\
H_D &= \sum_\sigma \varepsilon_0 d_\sigma^\dagger d_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow \;, \\
H_T &= \sum_\sigma \left(w c_{L\sigma}^\dagger c_{R\sigma} + h.c. \right) + \sum_{\alpha \in L,R;\sigma} V_\alpha \left(d_\sigma^\dagger c_{\alpha\sigma} + c_{\alpha\sigma}^\dagger d_\sigma \right)
\end{aligned}
$$

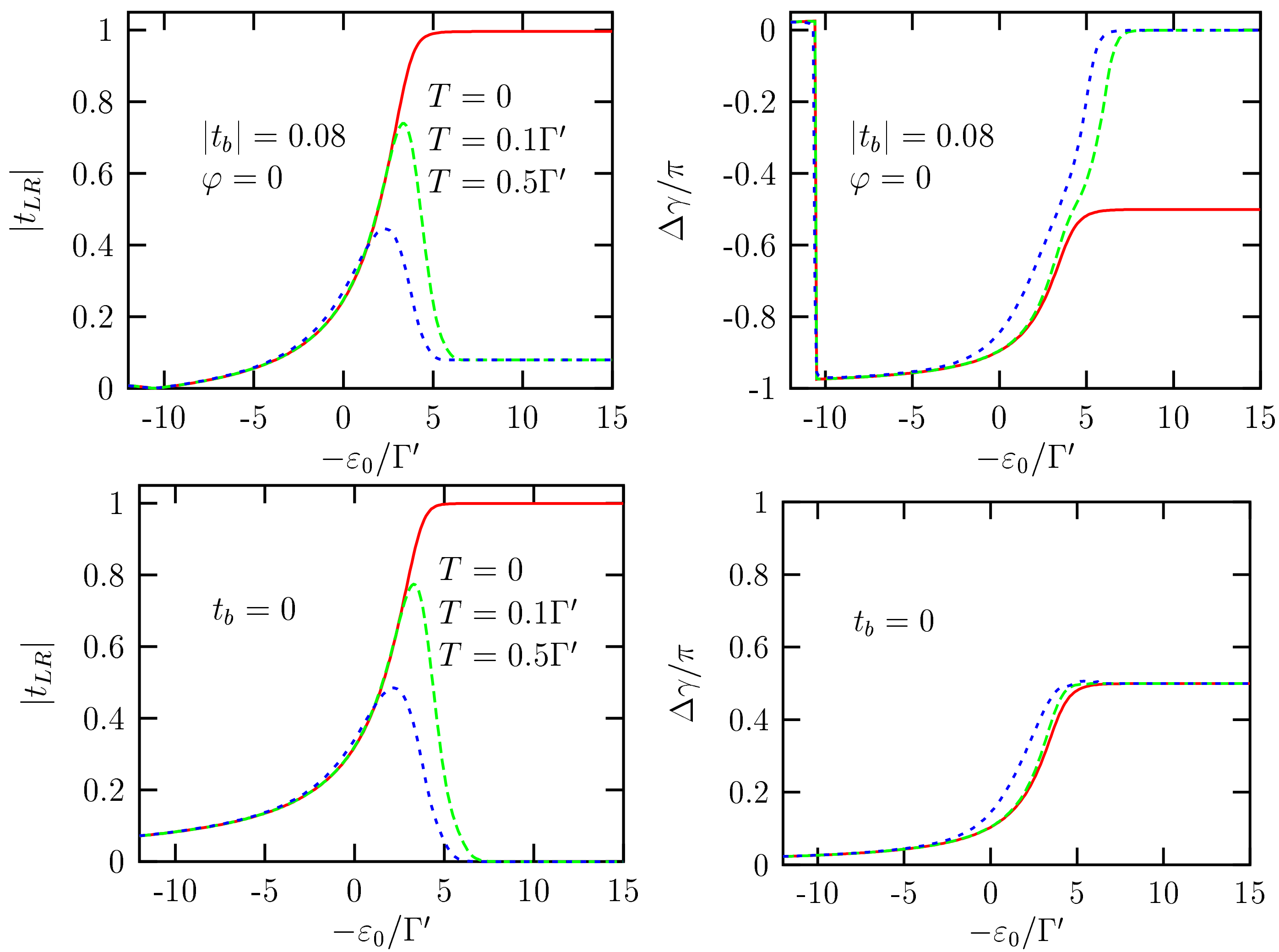
- Transmission amplitude at zero temperature (obtained from the generalized Friedel sum rule)

$$
\begin{aligned}
t_{LR} &= t_{LR}(0) = \frac{i e^{i\varphi} |t_b|}{e_0 - i} (e_0 + Q) \\
e_0 &= \cot(\pi n_d / 2), \quad Q = -\frac{|r_b|}{|t_b|} \cos\varphi + \frac{i}{|t_b|} \sin\varphi
\end{aligned}
$$

- Phase difference between w and $V_L V_R$ $(\varphi) = 0$ or π in the presence of time-reversal symmetry
- For $t_b = 0$ (no background), $\Delta\gamma = \pi/2$ (Ordinary Kondo problem)
- At finite temperatures:

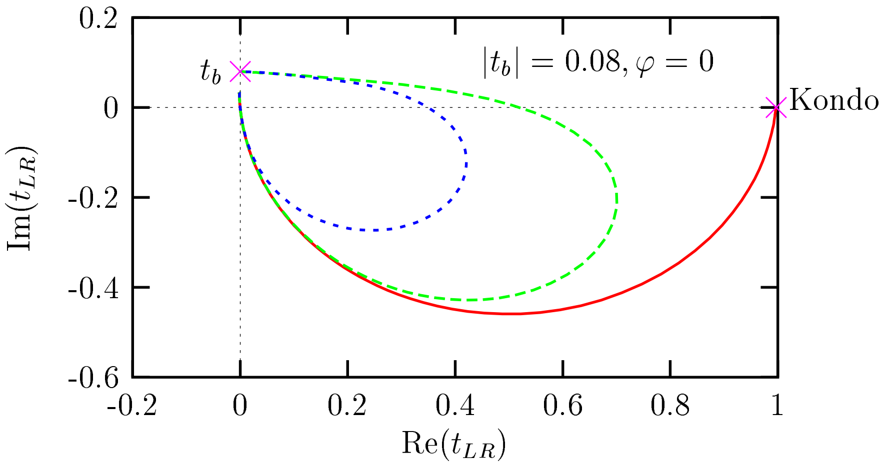
$$
t_{LR} = \int \left(-\frac{\partial f}{\partial \omega} \right) t_{LR}(\omega)\, d\omega
$$

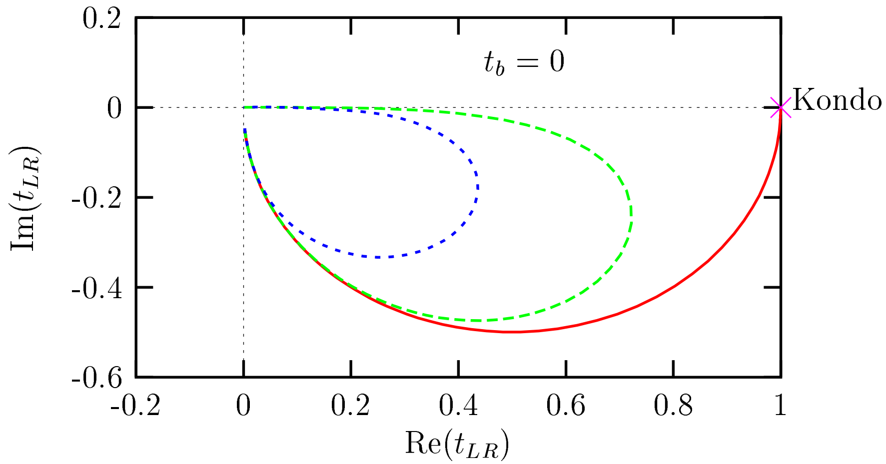
Toward an Explanation of the Phase Evolution: Result



- Slave boson mean-field approach with $U = \infty$
- For finite small t_b with $T \neq 0$, $\Delta\gamma \to \pi$ instead of $\pi/2$

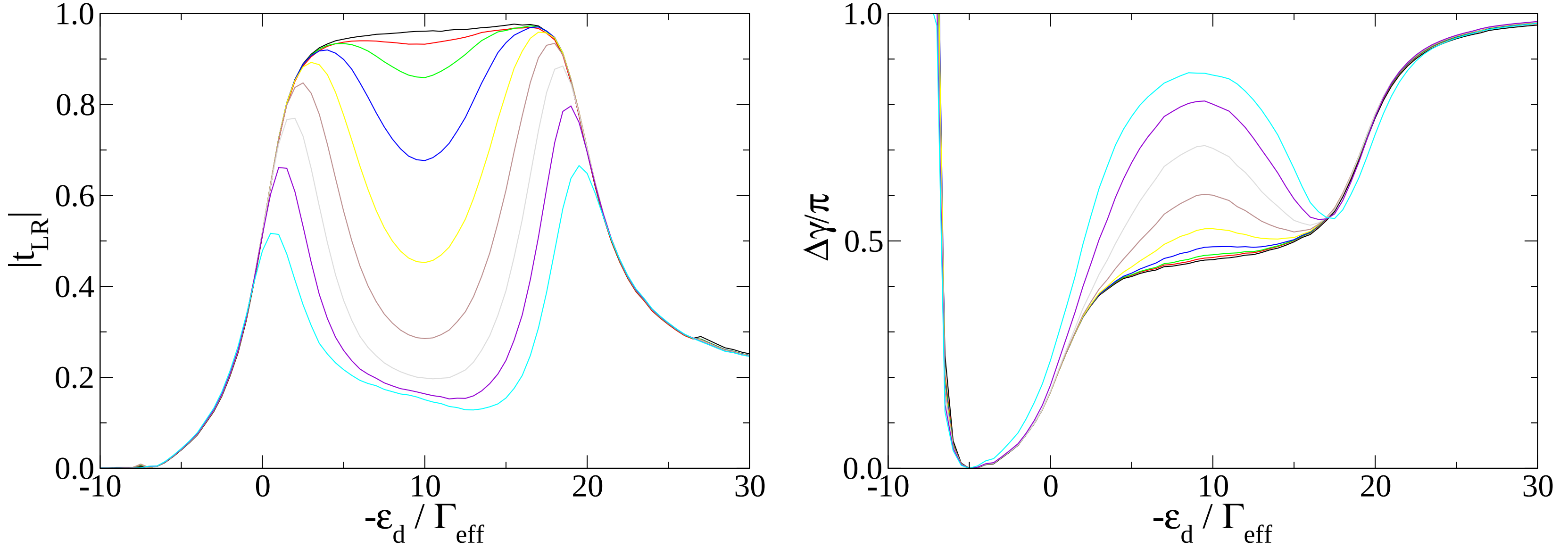
Toward an Explanation of the Phase Evolution: Trajectories





- Finite background t_b & time-reversal symmetry $\longrightarrow$ Anomalous phase evolution
- Universal feature independent of the approximation

NRG Result for finite U



$|t_b| = 0.2$, $\varphi = 0$, $T/\Gamma_{\mathbf{eff}} = 0, 10^{-4}, 10^{-3.5} \cdots 10^{-0.5}$

- Evolution from $\pi/2$-plateau to π-plateau as increasing temperatures

Crossover from $\Delta\gamma \sim \pi/2$ to $\Delta\gamma \sim \pi$ at the Kondo valley

- $T < T_1$: Unitary Kondo limit with $\Delta\gamma \sim \pi/2$
- $T > T_1$: High temperature Kondo region with $\Delta\gamma \sim \pi$
- From the Fermi-liquid expression of the Green's function we obtain

$$t_{LR} \simeq i|t_b| \frac{\pi T}{T_K + \pi T} + |r_b| \frac{T_K}{T_K + \pi T}$$

- Crossover takes place when the real and the imaginary part are comparable:

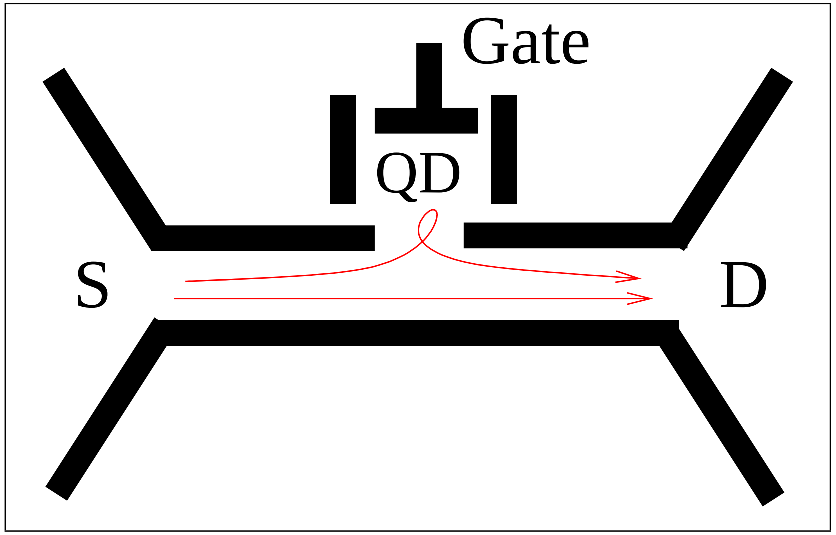
$$T_1 = \frac{|r_b|}{\pi |t_b|} \cdot T_K$$

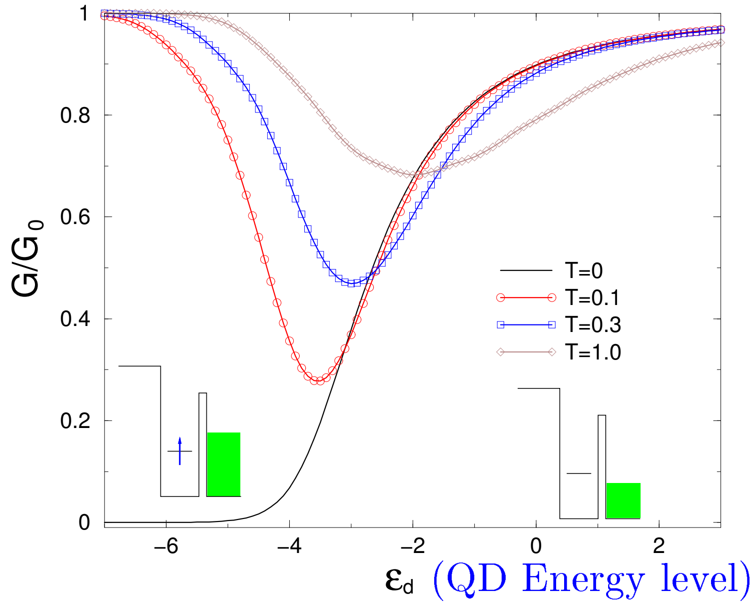
  - T_1 decreases as $|t_b|$ increases
  - For $t_b = 0$ only $\pi/2$-plateau exists

- $|t_b|$ and T_K might be extracted from the experimental data
  $\rightarrow$ Our prediction is experimentally testable

Anti-Kondo Resonance

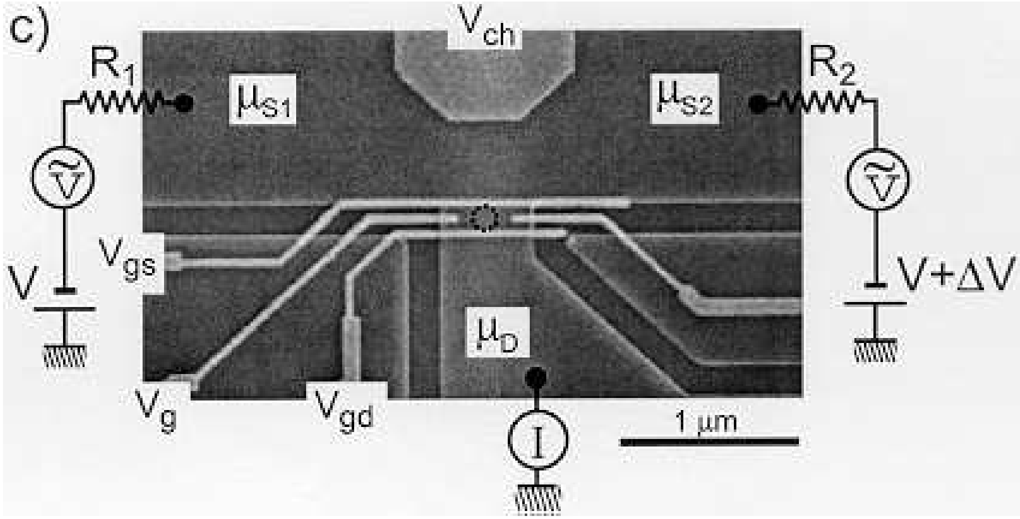
K. Kang *et al.*, PRB(2001)





- Theoretical prediction : $\boxed{G = (2e^2/h)\cos^2(\pi n_d/2)}$ (for $T = 0$)
- For an isolated one-dimensional ballistic wire: $G = G_0 \equiv 2e^2/h$
- For $S = 0$, $G \to G_0$: The wire and the dot is effectively decoupled
- For $S = 1/2$, $G \to 0$: Destructive interference between the ballistic channel and the spin (Kondo) channel
- Anti-resonance behavior at finite temperatures

Anti-Kondo Resonance : Experimental Challenges



Franceschi et al. (2002)

ETRI

- Delft University (Leo Kouwenhoven, PRL (2002))
- ETRI
- ISSP, U. of Tokyo with AB-type geometry

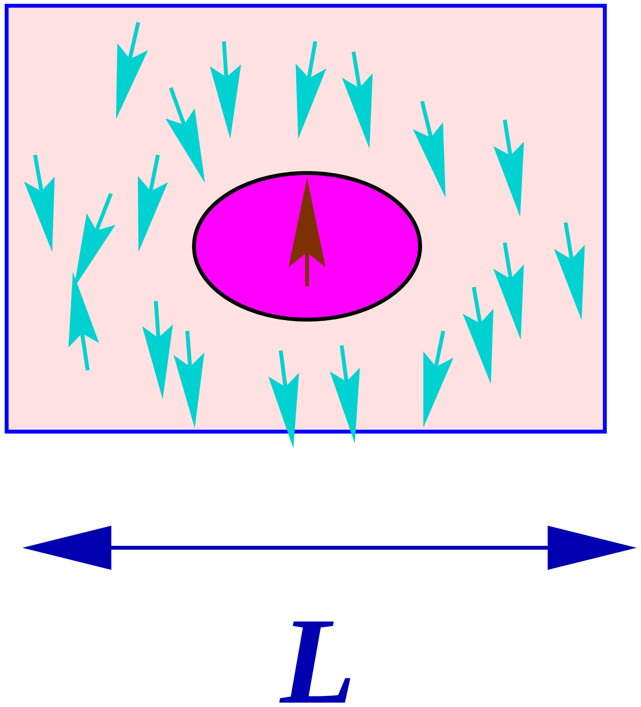
Outline

- Coherent transport in quantum dots
- Kondo effect in quantum dots
- Phase coherence of the Kondo effect
- **“Mesoscopic” Kondo effect and spin-charge separation**

Mesoscopic Kondo Effect

What Happens If $\xi_K \sim> L$ (or $T_K \sim< \delta$) ?

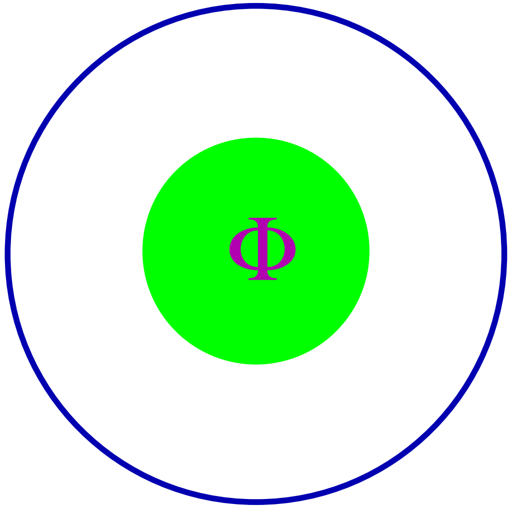
$$(\delta = hv_F/L, T_K = hv_F/\xi_K)$$

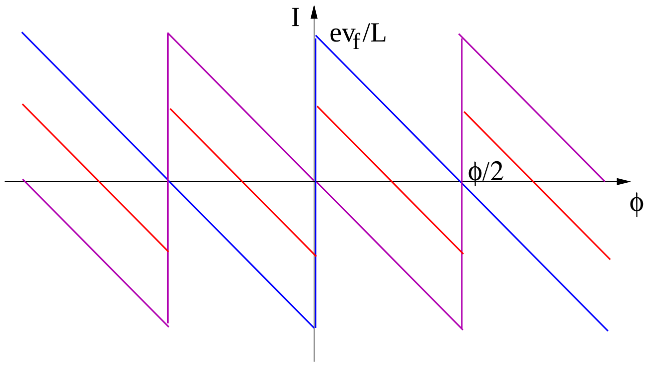


$\longrightarrow$ *Mesoscopic Kondo Effect, Detection of the Kondo cloud*

- Kondo Box: A single Kondo impurity embedded in a mesoscopic box (Thimm *et al.*, PRL (1999))
- Composite QD - AB ring: More systematic study via the gates, AB phase.

Persistent Current in a Perfect 1D Ring

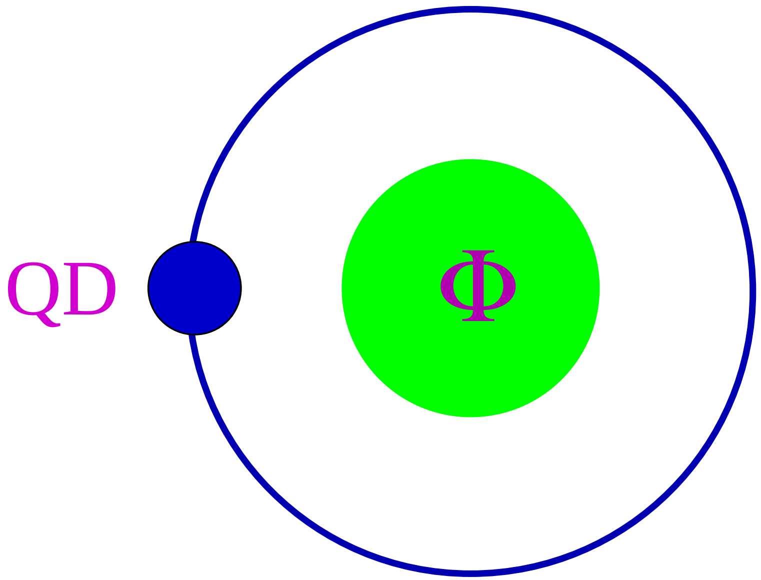


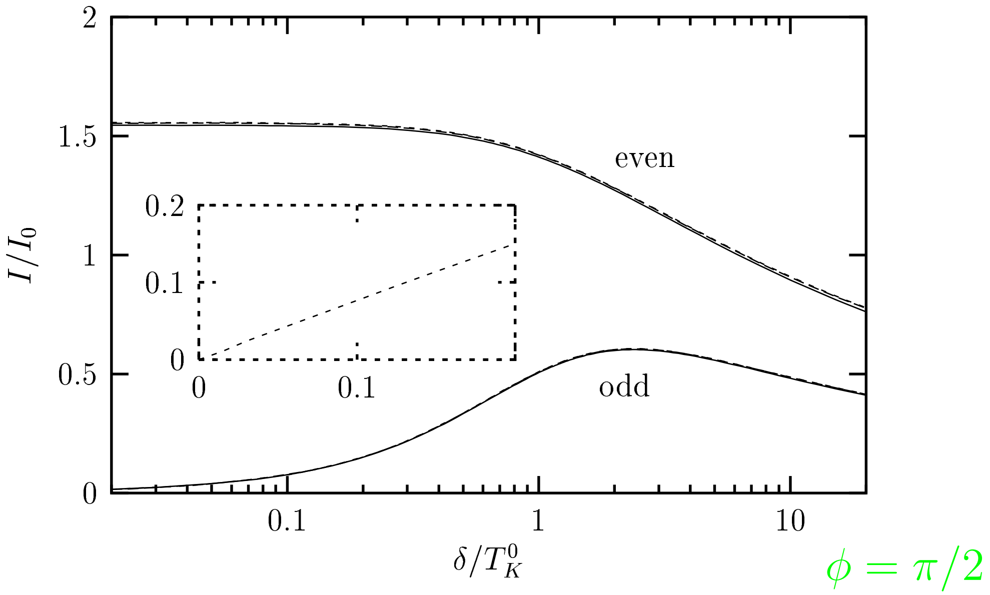


- Persistent current (PC): Indicator of quantum coherence in mesoscopic scale
- Elastic scattering (e.g. due to geometric imperfection) reduces the current, but does not destroy the phase-coherence ($L_\phi \neq l$)

Kondo Effect and the Persistent Current I

Kang & Shin, PRL (2000)





- Leading order $1/N_s$ expansion (N_s: Spin degeneracy)
- I_0 : current amplitude for an ideal ring
- Kondo assisted tunneling through the QD provides persistent current (I)
- Current suppressed for $\delta/T_K \gg 1$: Destruction of the screening cloud $\longrightarrow$ *Mesoscopic Kondo effect*
- Universal scaling with strong even-odd effect for the total number of electrons
- Suppression of the current for odd parity : because $I_\downarrow \sim -I_\uparrow$

Leggett's Theorem

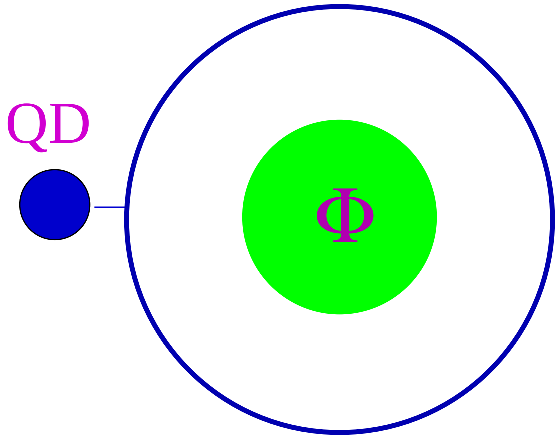
- For any 1D mesoscopic ring with N spinless electrons, the persistent current is

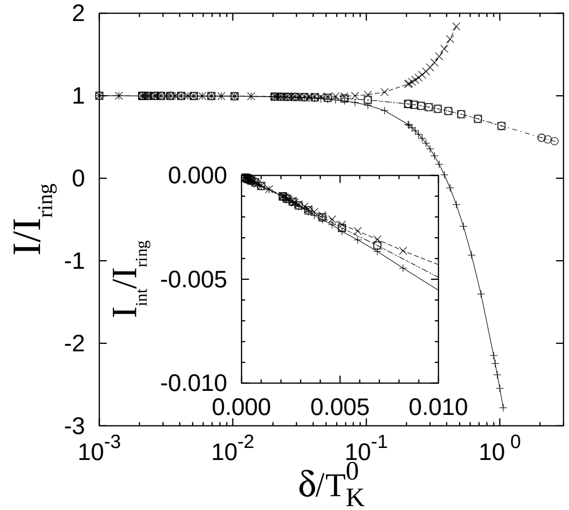
- Diamagnetic (Ground state at $\Phi = n\Phi_0$) for odd N, and
- Paramagnetic (Ground state at $\Phi = (n + 1/2)\Phi_0$) for even N,

regardless of interactions, disorder, etc.

Kondo Effect and the Persistent Current II

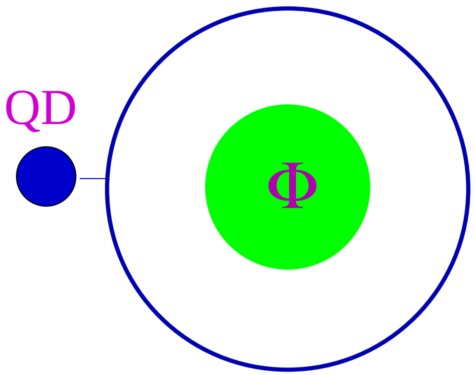
Cho, Kang and coworkers, PRB (2001)



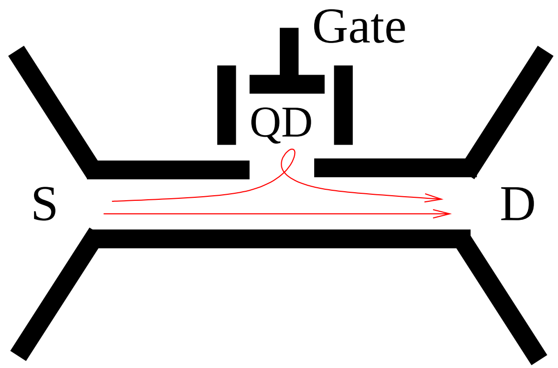


- $\mathsf{I}(\phi) = \mathsf{I}_{\mathsf{ring}}(\phi) + \mathsf{I}_{\mathsf{int}}(\phi)$ ($\mathsf{I}_{\mathsf{ring}}$: from the ideal ring, $\mathsf{I}_{\mathsf{int}}$: from the interaction)
- $\mathsf{I} \longrightarrow \mathsf{I}_{\mathsf{ring}}$ in the continuum limit ($\delta/T_K \longrightarrow 0$):
  Agrees with the exact Bethe ansatz result (H.P. Eckle *et al.*, PRL (2001))
- Linear reduction of the normalized current for small δ/T_K: $\mathsf{I}/\mathsf{I}_{\mathsf{ring}} = \mathsf{I}_{\mathsf{ring}} - \mathsf{c} \cdot \delta/\mathsf{T}_{\mathsf{K}}$
- For $\delta \gg T_K$, $\mathsf{I} \longrightarrow \mathsf{I}_{\mathsf{ring}}^{\mathsf{N}-1}$ (PC of an ideal ring with an electron missing):
  Effective decoupling of the ring and the dot

Remarks on Some Theoretical Debates



vs.



Behavior of the Persistent Current (PC) in the Limit of $L \to \infty$

- For any 1D noninteracting system, a correspondence exists between the PC and transport in an open system (Gogolin (1994)): If $\mathsf{T}(\mathsf{E}_\mathsf{F}) = 1$, then $\mathsf{I}/\mathsf{I}_\mathsf{ring} = 1$
- A Kondo-correlated QD does not seem to meet this condition

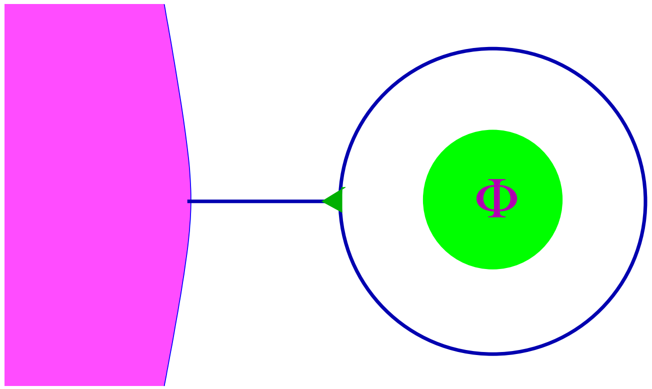
$$\mathsf{T}(\mathsf{E}_\mathsf{F}) = 0, \quad \text{but} \;\; \mathsf{I}/\mathsf{I}_\mathsf{ring} = 1$$

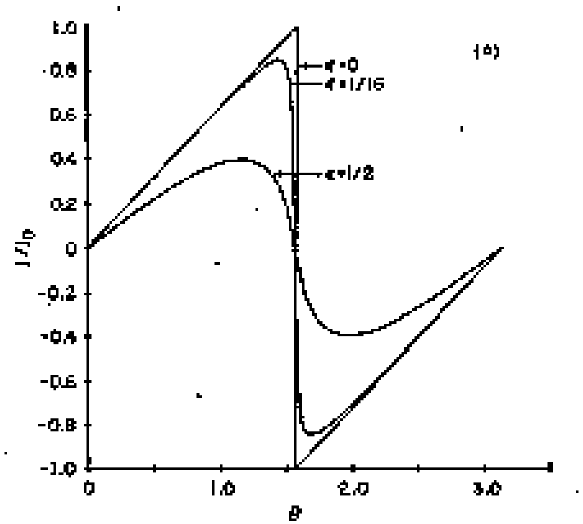
- Transport counts "electrons", but PC does not! (Cho et al., PRB (2001), Eckle et al., PRL (2001), etc.)
- Opposite result by Affleck et al., PRL (2001) based on RG

$$\mathsf{T}(\mathsf{E}_\mathsf{F}) = 0, \quad \mathsf{I}/\mathsf{I}_\mathsf{ring} = 0$$

Dephasing Induced by a Reservoir

M. Büttikker, PRB (1985)

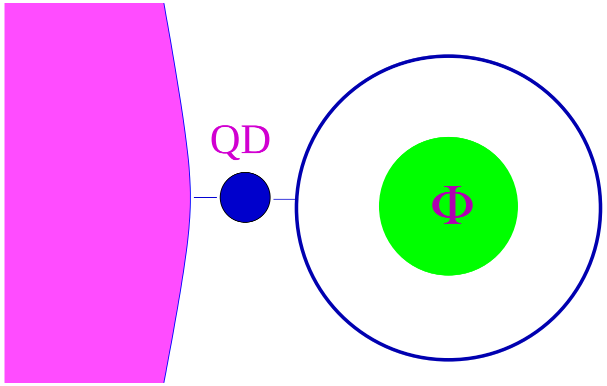


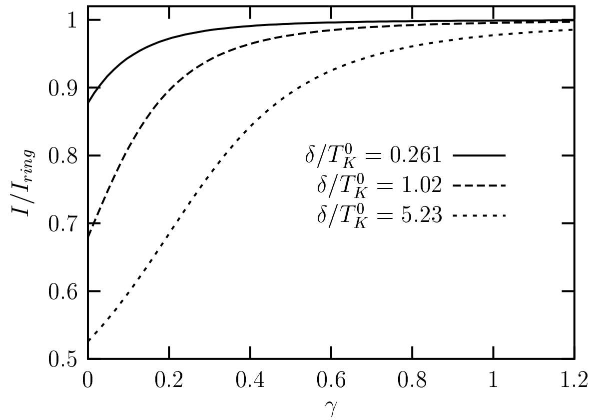


- Dissipative reservoir as an artificial dephasor: No phase coherence between the electrons absorbed by and those emitted by the reservoir → Reduces the AB oscillation

Mesoscopic Kondo Effect and Dephasing?

K. Kang *et al.*, PRB (2002)





$\phi = 0.1\pi$

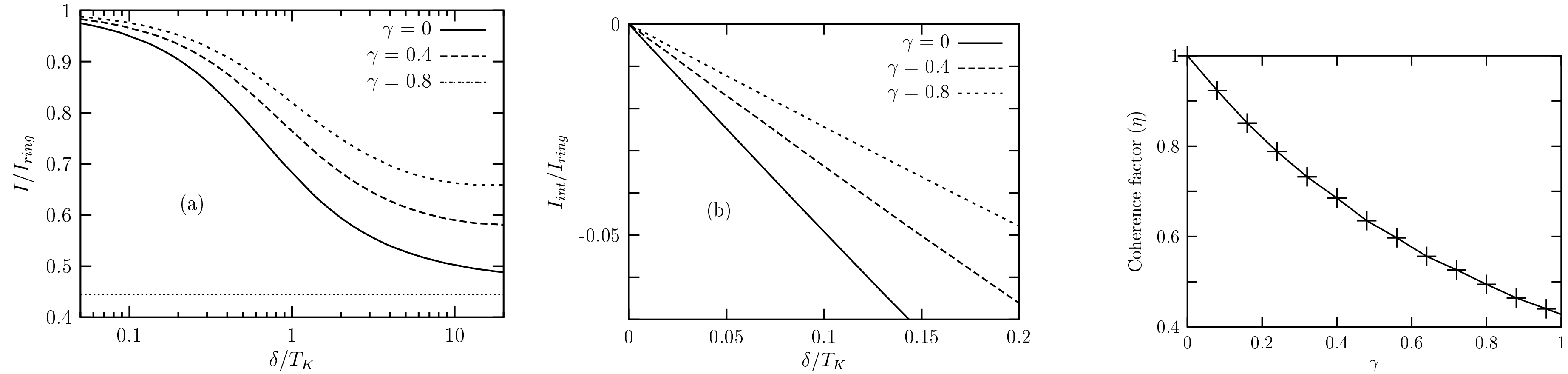
γ: relative coupling strength of dot-reservoir to dot-ring

- $\mathrm{I}(\phi) = \mathrm{I}_{\mathrm{ring}}(\phi) + \mathrm{I}_{\mathrm{int}}(\phi)$: Universal function of δ/T_K^0, γ
- The PC increases as γ increases: Counterintuitive!

Effects of the Dot-Reservoir Coupling

- To enhance the Kondo energy scale T_K (and thus reduces δ/T_K) $\rightarrow$ Enhances the PC
- To induce dephasing ?

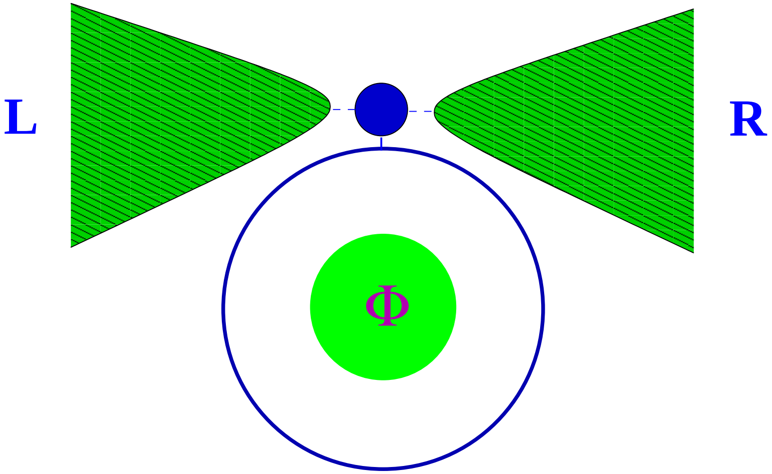
Spin Fluctuation Induced Dephasing

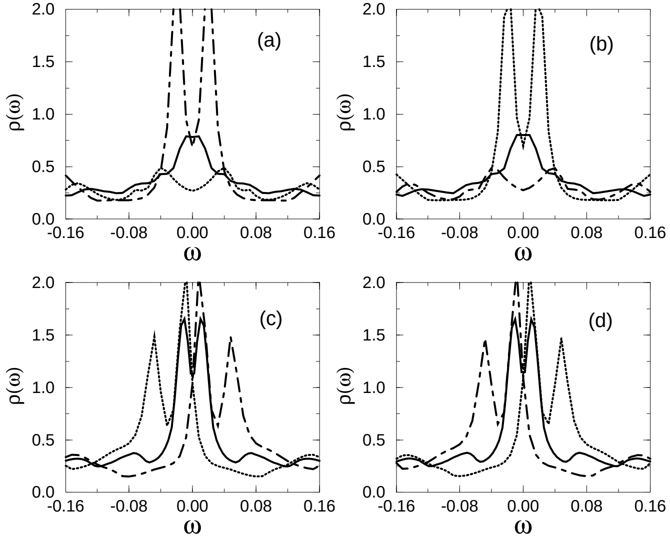


- Current as a function of δ/T_K: The effect of the rescaled Kondo energy extracted
- Even after subtracting the effect of renormalized T_K, the PC increases due to the dot-reservoir coupling $\longrightarrow$ Dephasing tends to enhance the AB oscillation !!
- Dot-reservoir coupling reduces I_{int} only: $I \rightarrow I_{ring}$ for $\gamma \gg 1$
- Can be interpreted as spin-charge separation: The reservoir degrades the coherence of spin degree of freedom (I_{int}) only, not affecting the charge one (I_{ring})
- Coherence factor defined by the relative strength of the slope in I_{int} vs. δ/T_K: $\eta \equiv c(\gamma)/c(0)$

Detecting the Mesoscopic Kondo Effect by Transport

Kang & Craco, PRB (2002)





(a) $N = 100$ (b) $N = 102$ (c) $N = 101$ (d) $N = 103$

$\varphi = 0$ (dot-dashed), $\varphi = \pi/2$ (solid) $\varphi = \pi$ (dotted)

- Mesoscopic Kondo effect can be detected by transport experiment for weak lead-dot coupling
- Phase-dependent Kondo resonance for small ring $(L < \xi_K)$ with modulo 4
- For larger rings the Kondo resonance is not phase-sensitive

Summary

- Coherent transport in quantum dots
- Kondo effect in quantum dots
- Phase coherence of the Kondo effect
- “Mesoscopic” Kondo effect and spin-charge separation

Vielen Dank!