



Numerical studies of quantum transport in carbon nanotubes

François Triozon and Stephan Roche

Commissariat à l'Energie Atomique, Grenoble, France

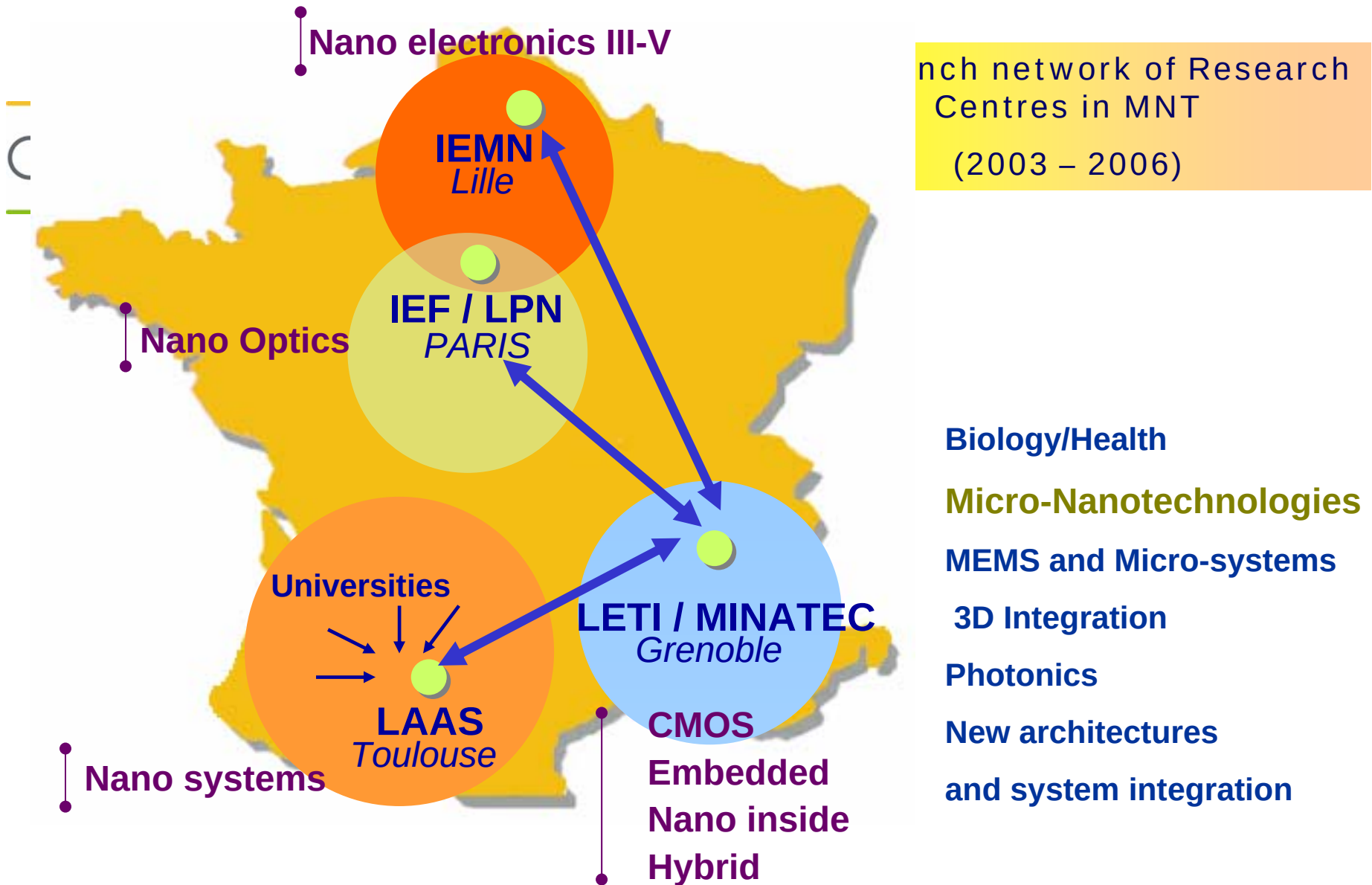


Outline



- Research environment : « RTB » projects
- Intrinsic transport in CNTs : role of disorder
- Magnetotransport : generalities and experiments
- Magnetotransport simulation: combined role of disorder + parallel magnetic field
- Perspectives

Recherche Technologique de Base



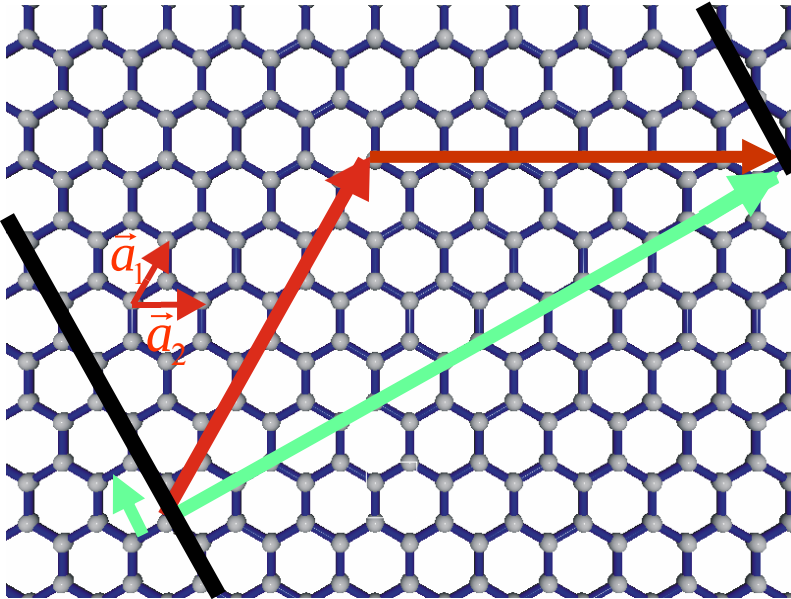
Our scientific interests

- 
- Quantum simulation of CNT field effect transistors : transmission through metal-nanotube interfaces, gate electrostatics
 - Intrinsic transport properties of CNTs : role of disorder and magnetic field
 - Generalization to other nano-systems : nanowires, molecules...

Methods :

- tight-binding hamiltonians → large systems
- coupling with ab-initio calculations (DFT) to improve tight-binding parameters

Carbon nanotubes



Helicity

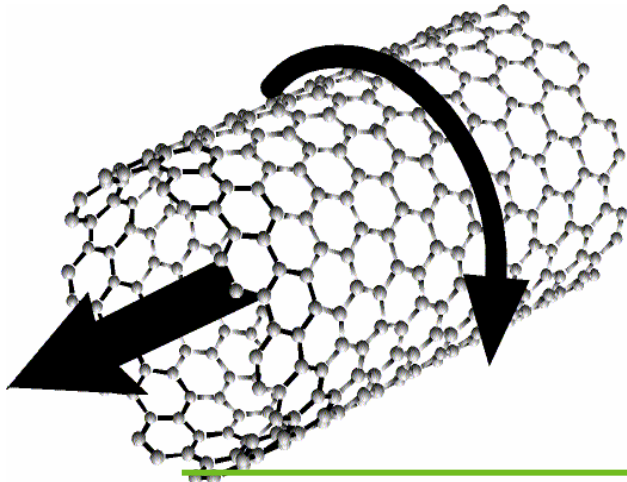
$$\vec{C}_{(l,m)} = l\vec{a}_1 + m\vec{a}_2$$

Diameter

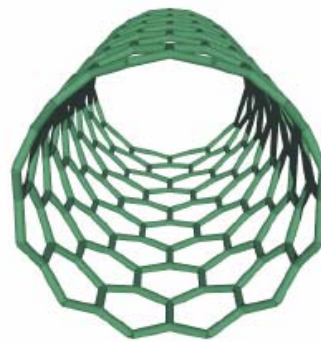
$$d_{(l,m)} = \frac{|\vec{C}_{(l,m)}|}{\pi}$$

Unit-cell length

$$|\vec{T}_{(l,m)}|$$



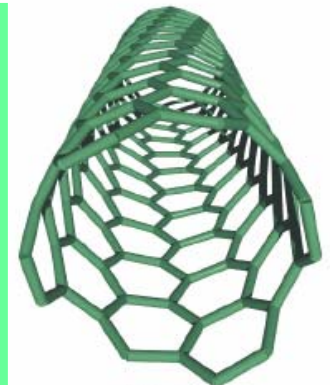
DSM/DRFMC



(12,0)



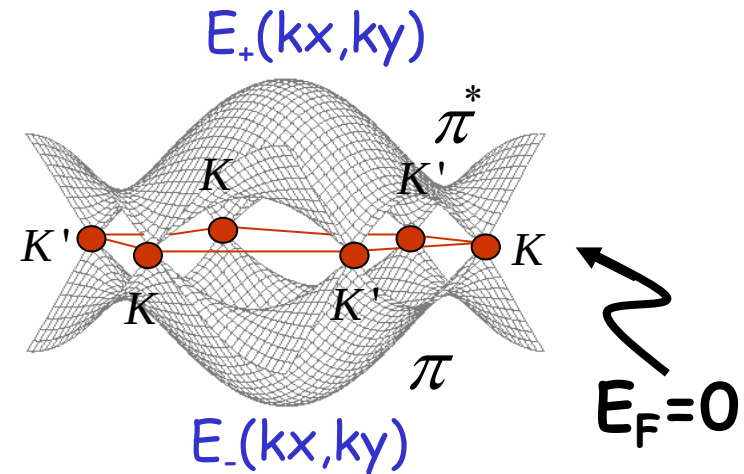
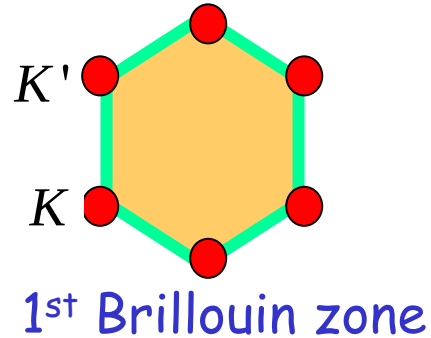
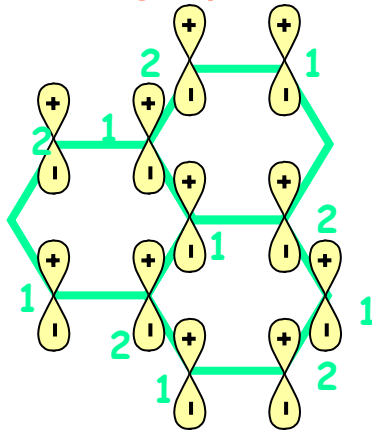
(6,6)



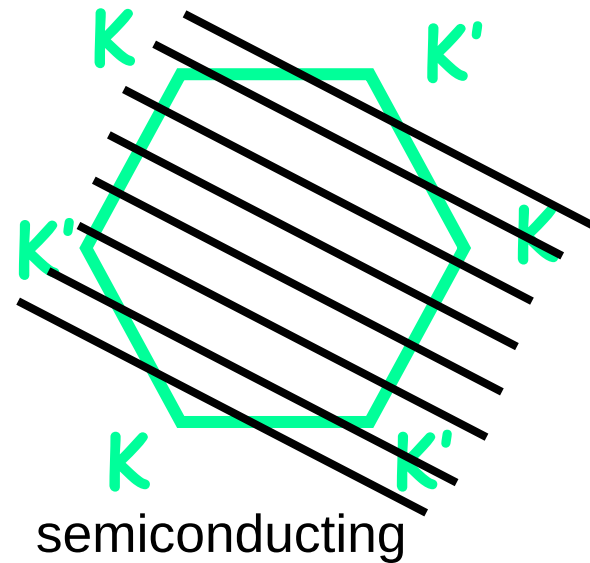
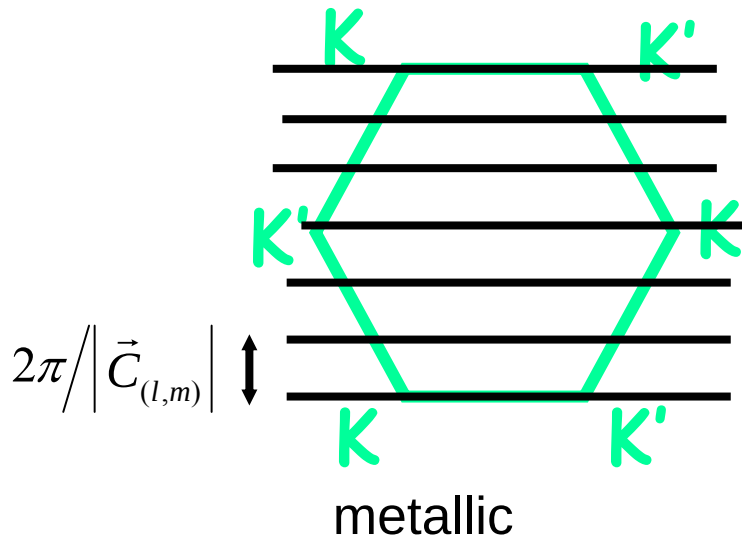
(6,4)

Electronic properties

from graphene :

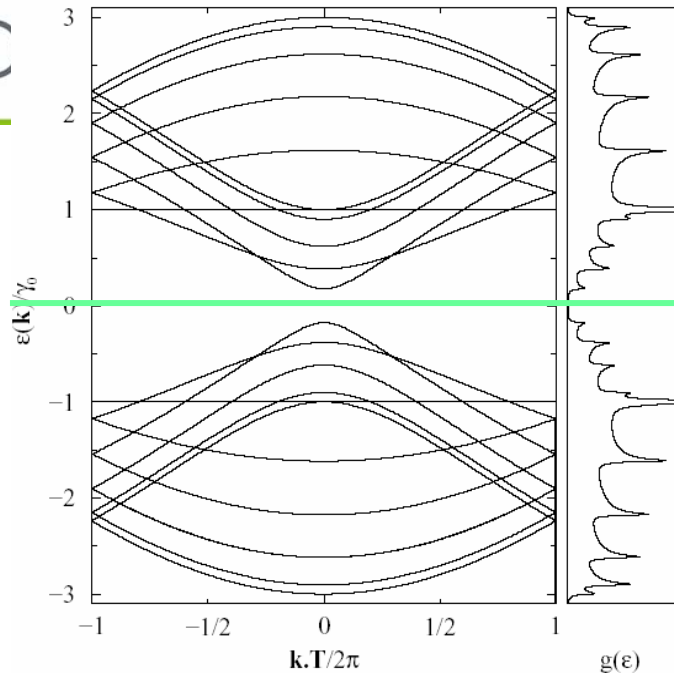


to nanotubes :

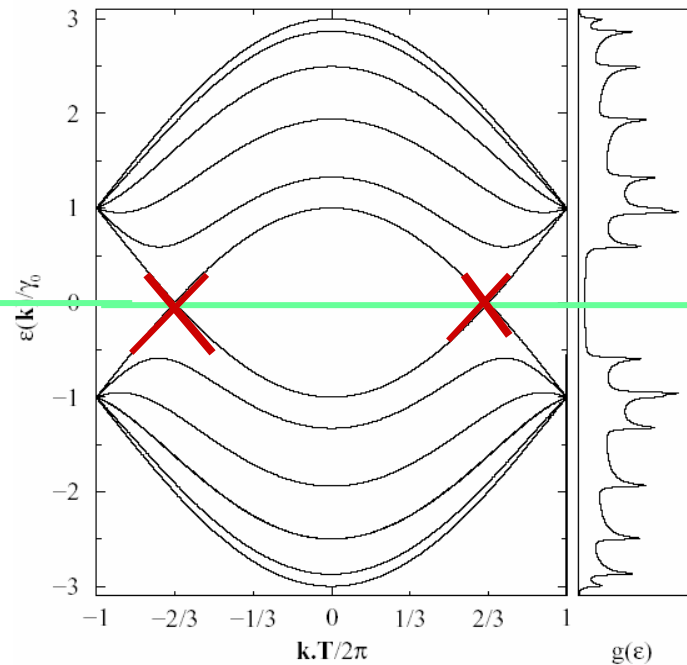


Band structure and density of states

semiconducting (10,0)



metallic (5,5)

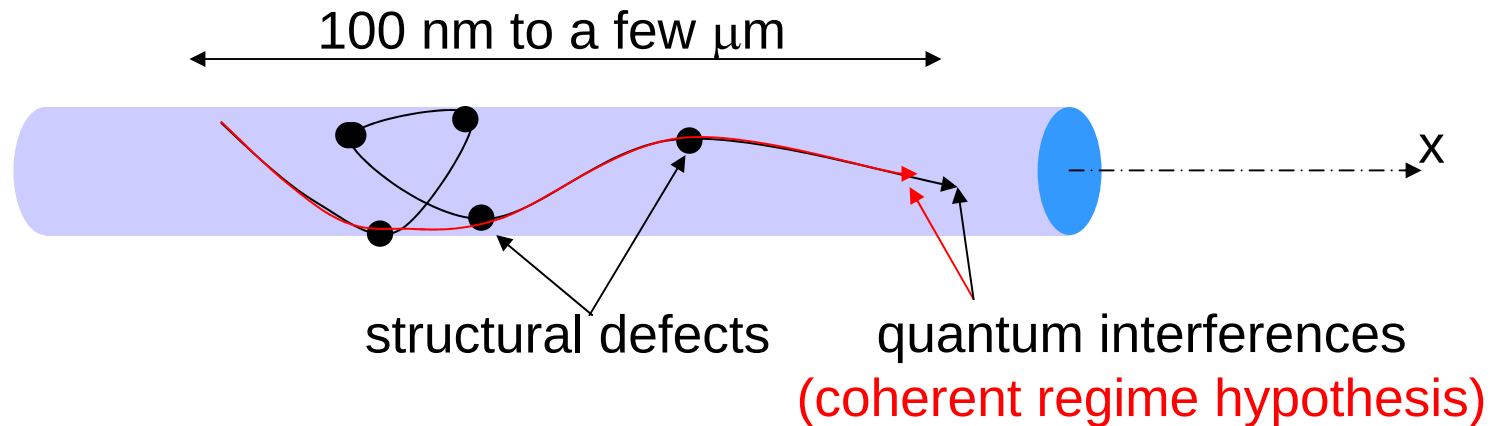


$$E_+(k=0) - E_-(k=0) \equiv \frac{2\pi a \gamma_0}{\sqrt{3} \|\vec{C}_h\|}$$

$$d_{\text{t}} = 1.4 \text{ nm} \Rightarrow \Delta_g = 0.6 \text{ eV}$$

$$E_{\pm}(\delta k) \equiv \pm \frac{\sqrt{3}a}{2} \gamma_0 \|\delta \vec{k}\|$$

Scattering and non-ballistic transport



Key quantity : quadratic spreading of wave-packets

$$\Delta X^2(E, t) = \frac{1}{\rho(E)} \text{Tr}[\delta(E - \hat{H})(\hat{X}(t) - \hat{X}(0))^2]$$

$$\neq \text{regimes : } \begin{cases} \Delta X^2(E, t) = v^2(E) \times t^2 & \text{ballistic} \\ \Delta X^2(E, t) = D(E) \times t & \text{diffusive} \\ \dots\dots\dots \end{cases}$$

Numerical simulation : - tight-binding models
- single or multi-wall CNTs

(Coll. : D. Mayou, S. Latil)

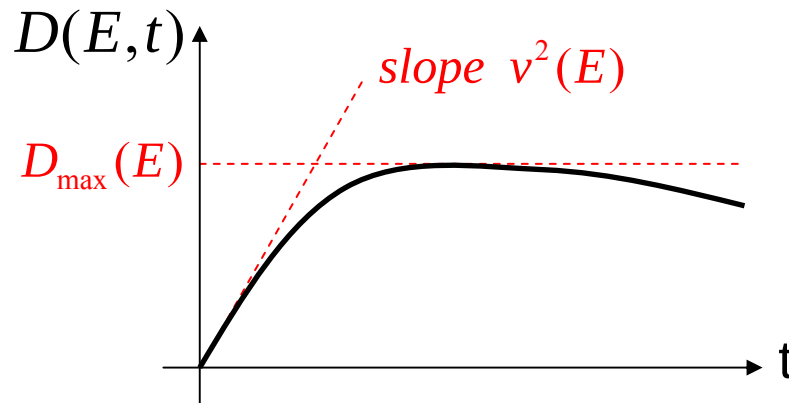
disordered CNTs : mean free path

Hamiltonian : Anderson disorder, no e-e interaction

$$\hat{H} = \gamma_0 \sum_{\langle i,j \rangle} |j\rangle\langle i| + \sum_i \varepsilon_i |i\rangle\langle i|$$

$$\gamma_0 = 2.9 \text{ eV} \quad \text{hopping energy} \quad \varepsilon_i \in [-W/2, W/2]$$

We compute the diffusivity $D(E,t) = \frac{\Delta X^2(E,t)}{t}$ for each energy E



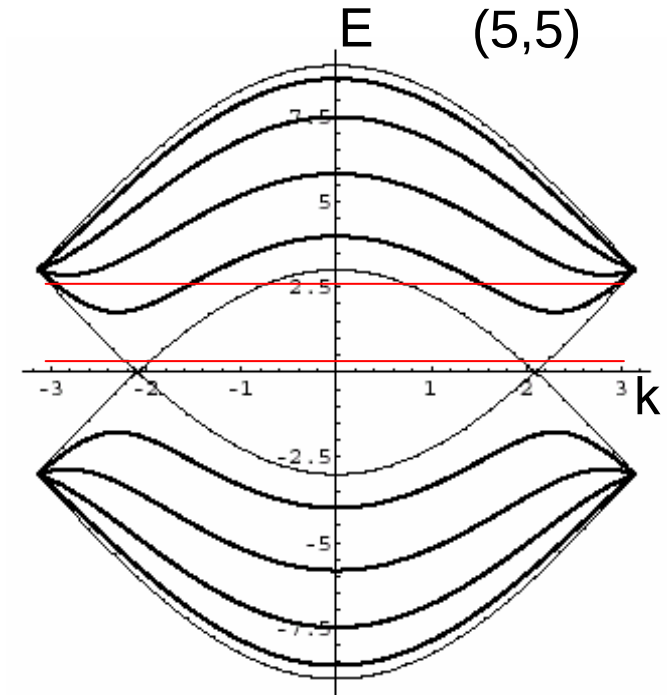
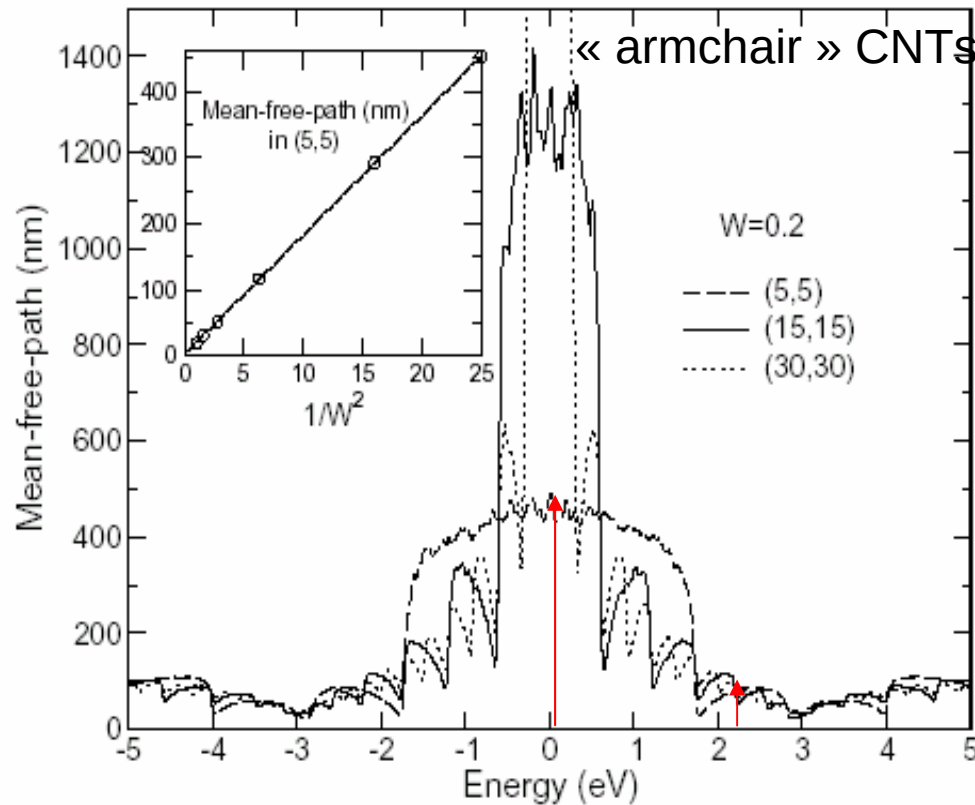
mean free path :

$$l_e(E) \equiv \frac{D_{\max}(E)}{v(E)}$$

$l_e(E)$ depends on helicity, disorder W, energy E

Energy dependence

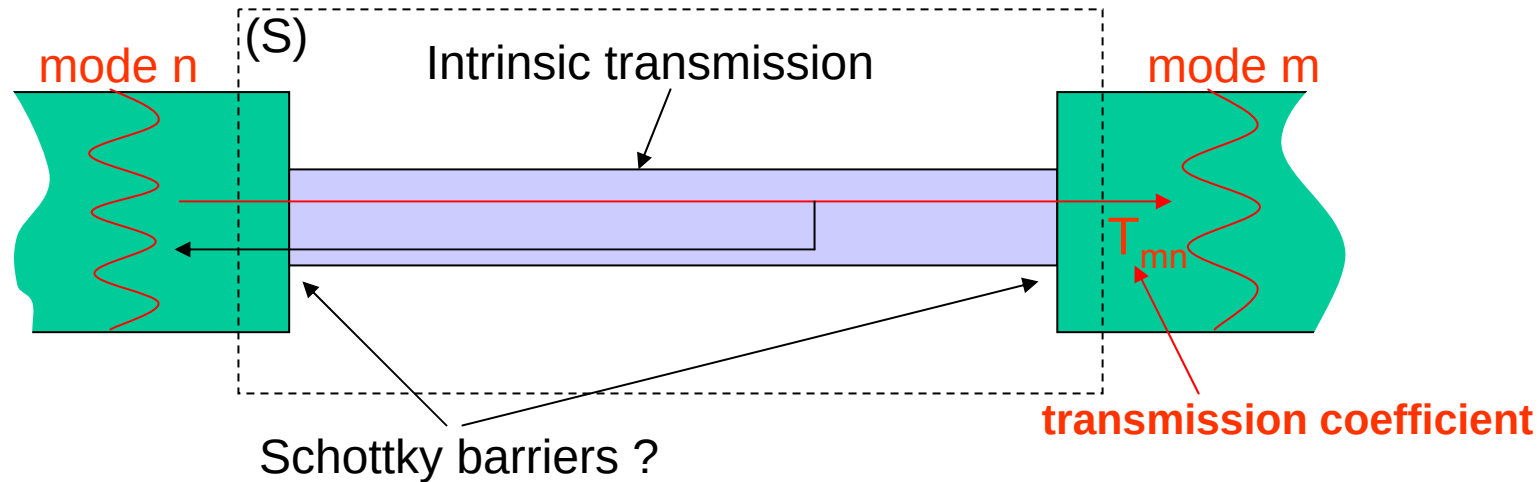
F.T., S. Roche, A. Rubio, D. Mayou, Phys. Rev. B (2004)



$$\text{At } E = 0 : \ell_e = \frac{18}{\sqrt{3}} \left(\frac{\gamma_0}{W} \right)^2 \left\| \vec{C}_h \right\|$$

C. White and T.N. Todorov, Nature (1998)

Nanotube + metallic contacts



Landauer-Büttiker formula :

$$G = G_0 \sum_{n,m} T_{mn} = \text{Tr} [\hat{\Gamma}_G \hat{G}^r \hat{\Gamma}_D \hat{G}^a]$$

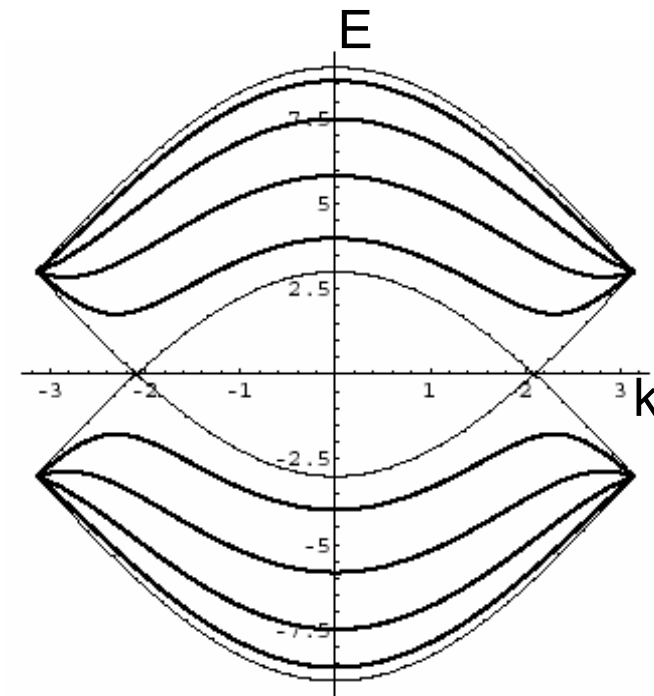
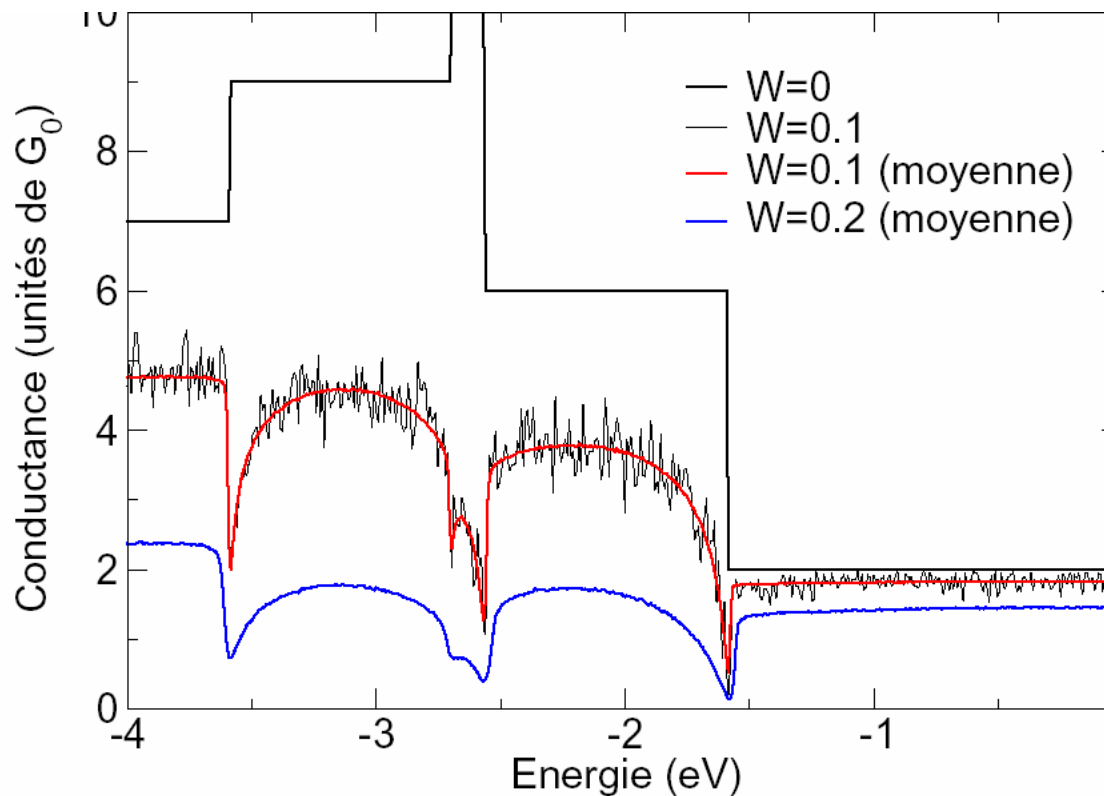
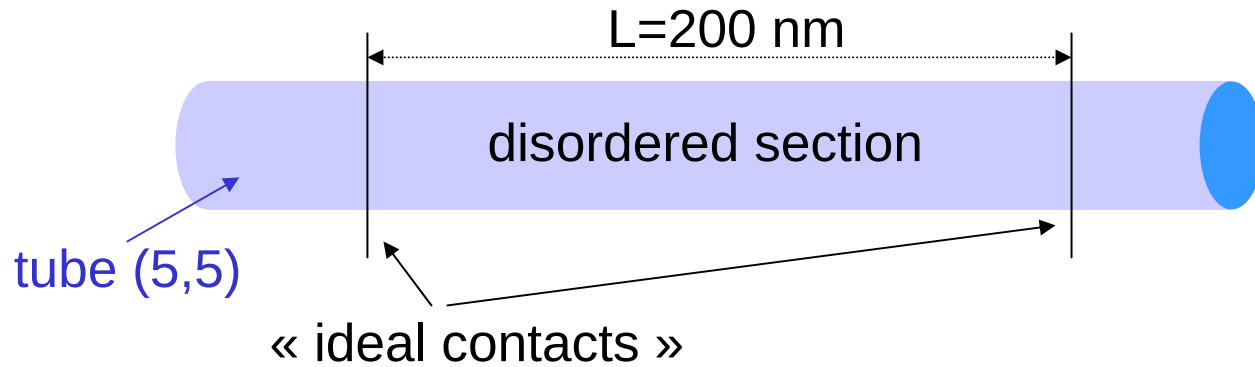
$$\hat{G}^{r,a}(E) = \frac{1}{E - \hat{H} - \hat{\Sigma}_G^{r,a}(E) - \hat{\Sigma}_D^{r,a}(E)}$$

Green functions

hamiltonien of finite system (S) self-energies due to coupling of (S) with electrodes

$$\hat{\Gamma}_{G,D}(E) = i(\hat{\Sigma}_{G,D}^r - \hat{\Sigma}_{G,D}^a)$$

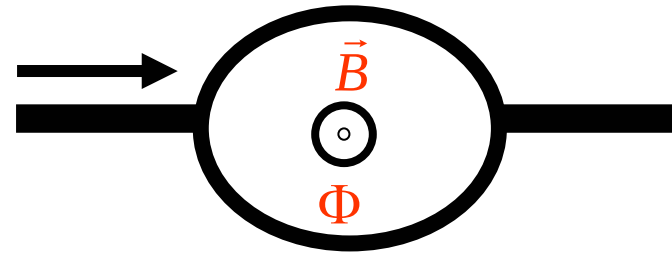
Conductance through a disordered section



Magnetotransport : Aharonov-Bohm phenomena



Doubly connected systems (rings),
cylinders

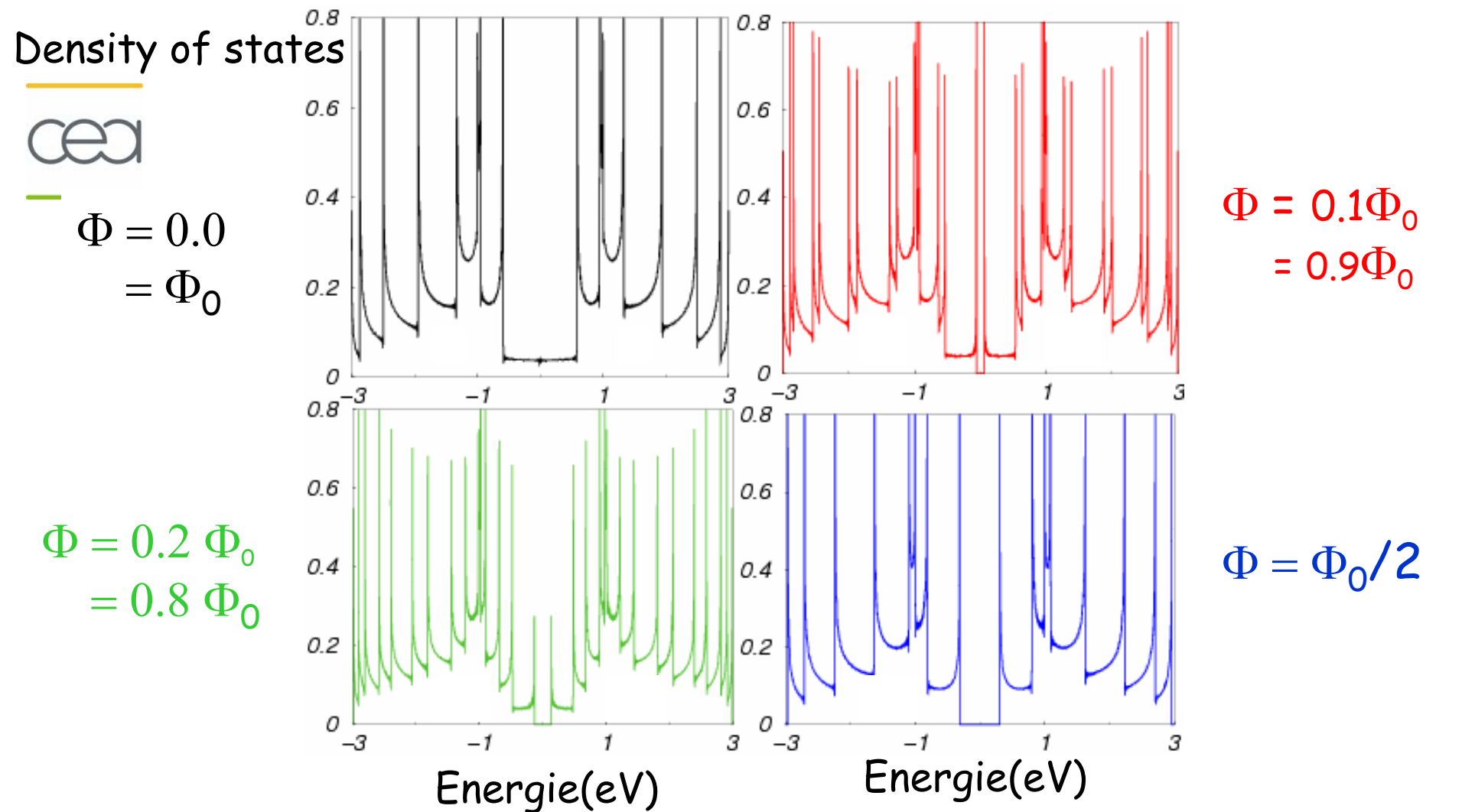


I- Electronic structure : periodic oscillations
of period $\Phi_0 (=h/e)$

II- Weak localization :
 $\Phi_0/2$ -periodic oscillations

III- Persistent currents : Φ_0 -periodic oscillations

Nanotube in parallel field



S. R., G. Dresselhaus, M. Dresselhaus & R. Saito, Phys. Rev. B 62, 16092 (2000)

Nanotube in perpendicular field

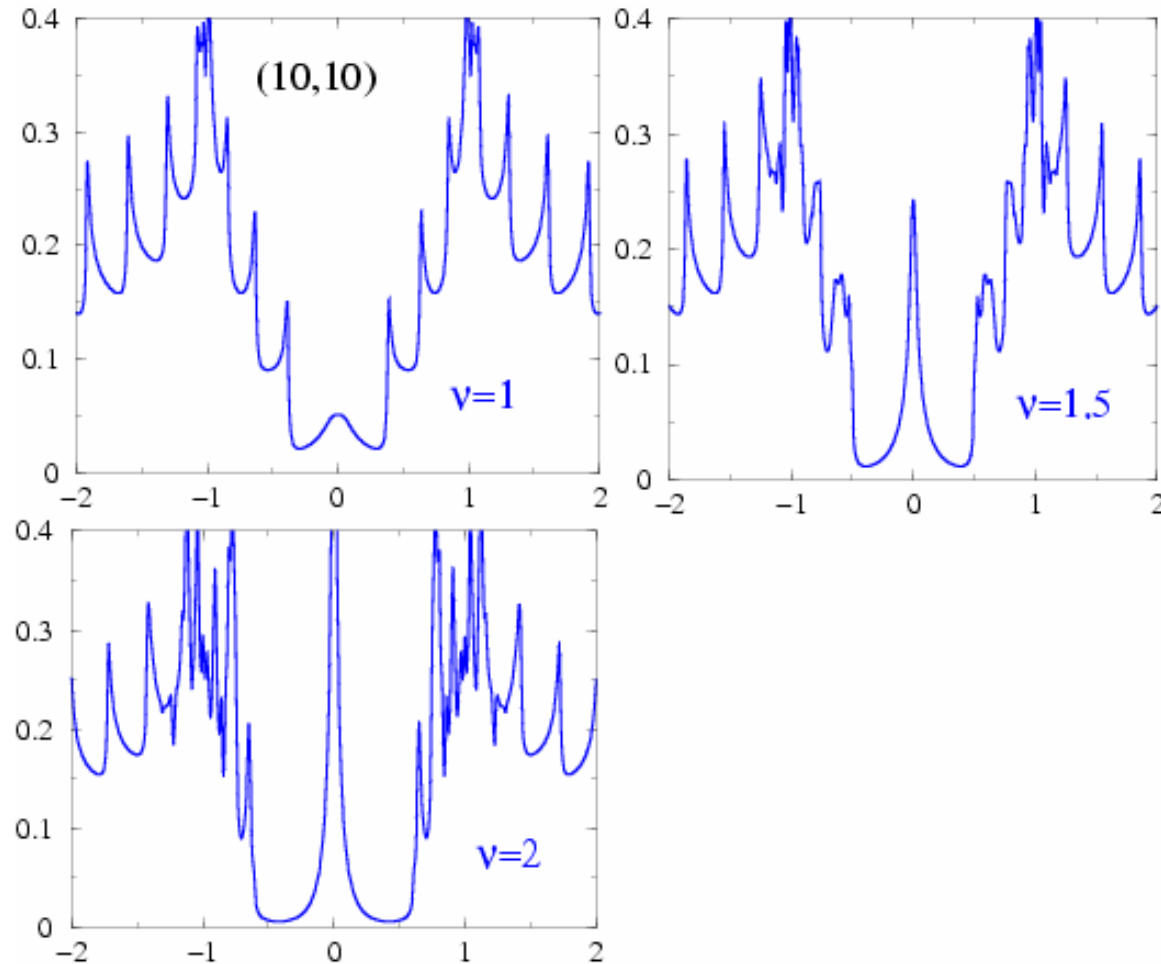
Formation of Landau levels



$$\nu = \frac{C_h}{2\pi l_m} \quad l_m = \sqrt{\frac{\hbar}{eB}}$$

$$\text{DoS}(\varepsilon = 0) \approx \frac{e^{2\nu^2}}{\sqrt{4\pi \nu^2}}$$

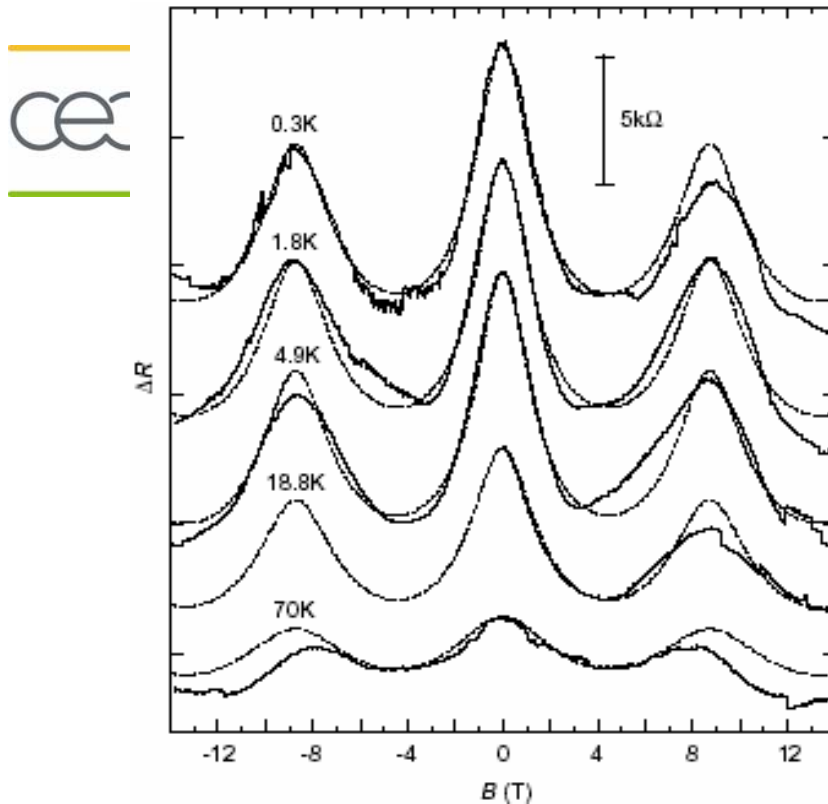
H. Ajiki & T. Ando,
J. Phys. Soc. Jpn 62, 1255 (1993)



S. R., G. Dresselhaus, M. Dresselhaus & R. Saito,
Phys. Rev. B 62, 16092 (2000)

Period of AB oscillation in parallel field ?

A. Bachtold et al, Nature 397, 673 (1999)

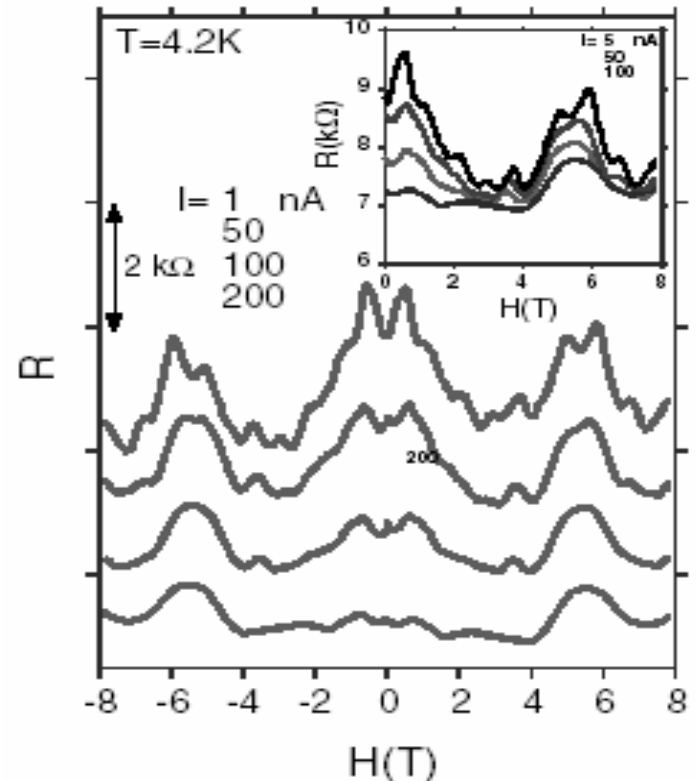


$$\frac{\Phi_0}{2}$$

$$l_e \leq \pi d_{tube}$$

Diffusive regime and weak localization

J. Lee et al, Sol. St Com 115, 467 (2000)



$$\Phi_0$$

$$l_e \geq L_{tube}$$

Ballistic regime
DOS effect

Numerical study of weak localization

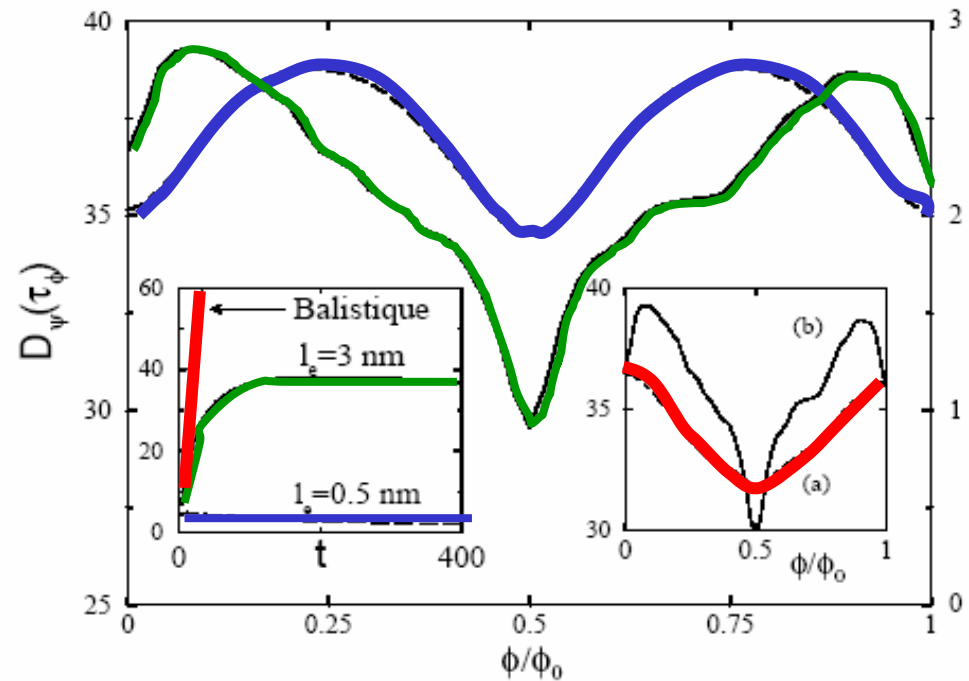
Approximation : no DOS effects !



$l_e < \text{tube circumference}$
Negative MR
Oscillations in $\Phi_0/2$

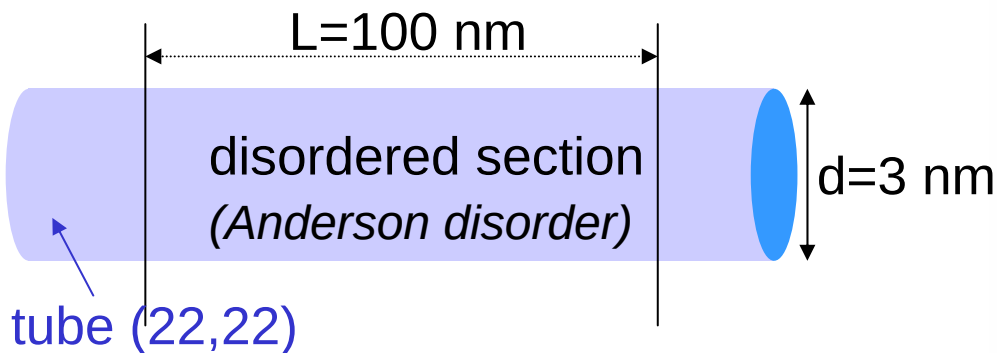
$L_{\text{tube}} > l_e > \text{tube circumference}$
Negative MR
Oscillations in Φ_0

$l_e > L_{\text{tube}}$
Positive MR
Oscillations in Φ_0

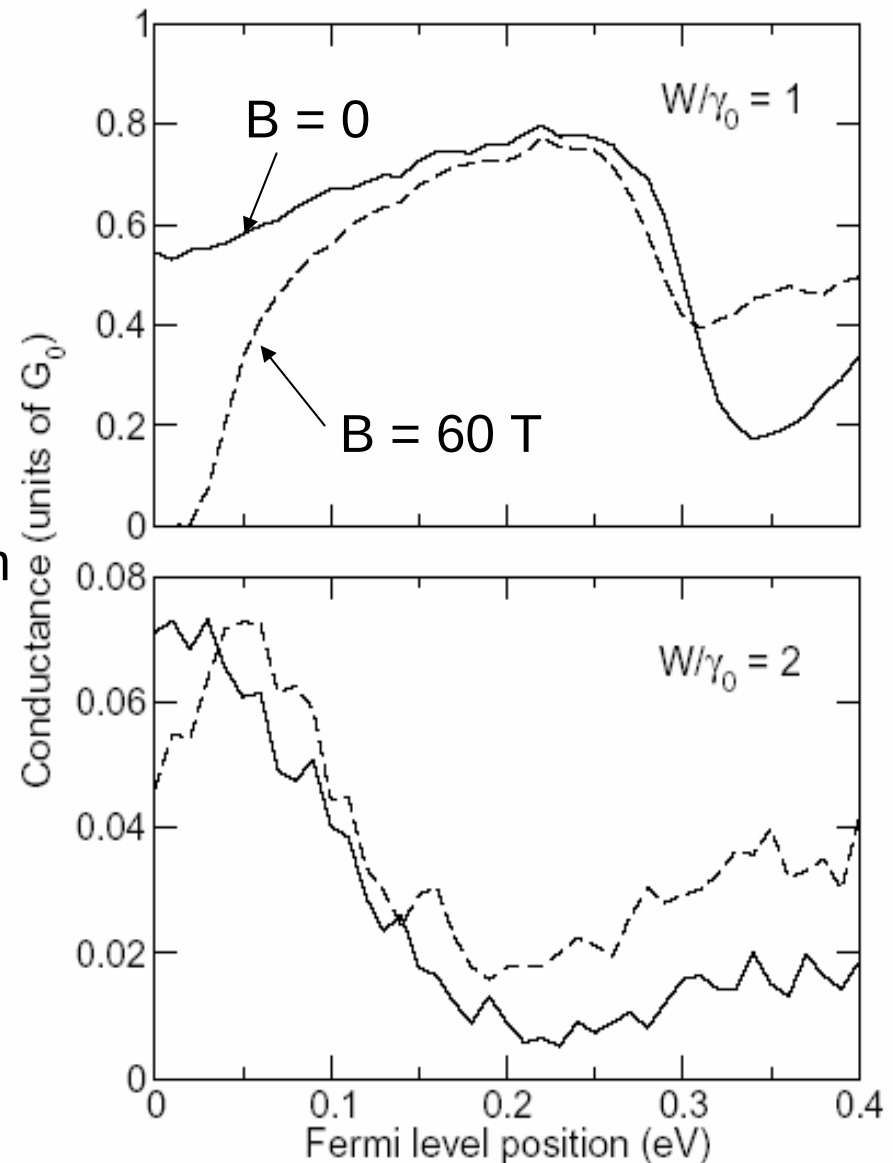


S. R, F. Triozon, A. Rubio, D. Mayou, Phys. Rev. B 64, 121401 (2001)

Simulation in parallel field



$$\Phi(B = 60T) = \Phi_0 / 10$$





h/e oscillation of the gap in parallel field ?

U.C. Coskun et al., Science 304, 1132 (2004)

Regensburg group

Experiments in parallel and perpendicular field,
T=20 mK, tube diameter=20 nm, strong gate coupling

Conclusion and perspectives



- Magnetotransport : DOS effects and localization effects → difficult interpretation
- Large diameter nanotubes give more information (several flux quanta) → future simulations for $d = 10$ nm or more
- The nature of disorder (structural or chemical) strongly influences the energy dependence of the mean-free-path → go beyond the Anderson model