Are Double Wall Nanotubes Luttinger Liquids?

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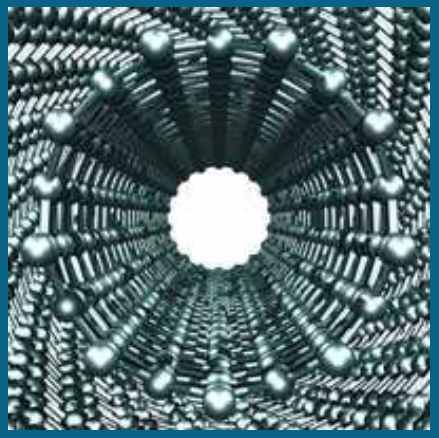

Outline

- Introduction
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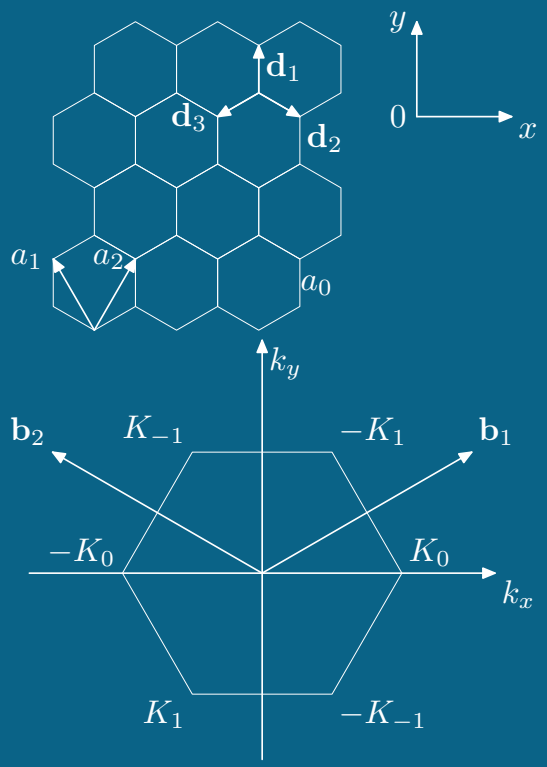
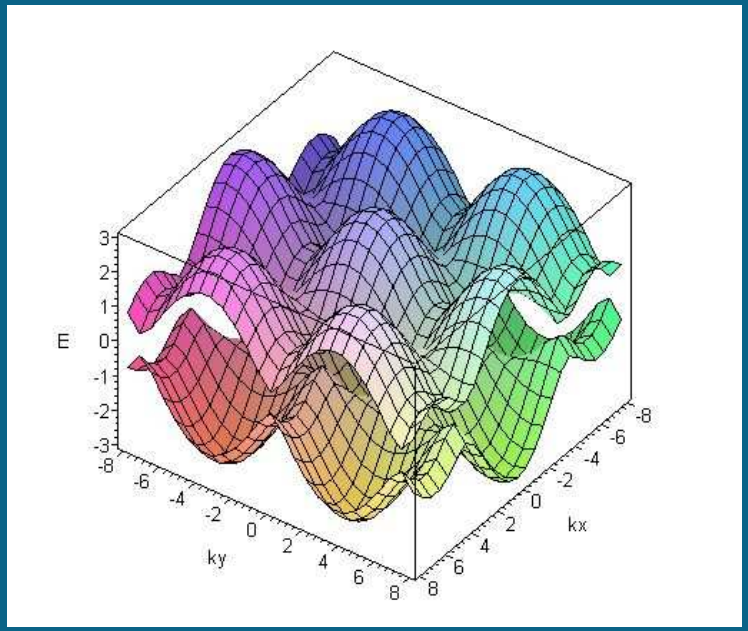
  - ⋆ Tomonaga-Luttinger model
  - ⋆ Single wall nanotubes as Luttinger liquids
- Electronic structures of double wall nanotubes
- Double wall nanotubes as Luttinger liquids
- Transport properties of double wall nanotubes
- Summary and outlook

Electronic structure of graphene sheets





$$H_0 = t_0 \sum_{\mathbf{k}} \gamma_{\mathbf{k}} c^{\dagger}_{A\mathbf{k}} c_{B\mathbf{k}} + \text{H.c.}, \qquad \gamma_{\mathbf{k}} = \sum_{j=1}^{3} e^{i\mathbf{k}\cdot\mathbf{d}_j}$$

Low energy theory

Expand around the Fermi point — $\mathbf{k} = \alpha\mathbf{K}_0 + \mathbf{q}, \quad \alpha = \pm,$

$$\gamma_{\mathbf{k}} = \sum_{j=1}^{3} e^{i\mathbf{k}\cdot\mathbf{d}_j} = \sum_{j=1}^{3} e^{i(\alpha\mathbf{K}_0+\mathbf{q})\cdot\mathbf{d}_j} \approx -\frac{3a_0}{2}(\alpha q_x - iq_y)$$

The Hamiltonian becomes

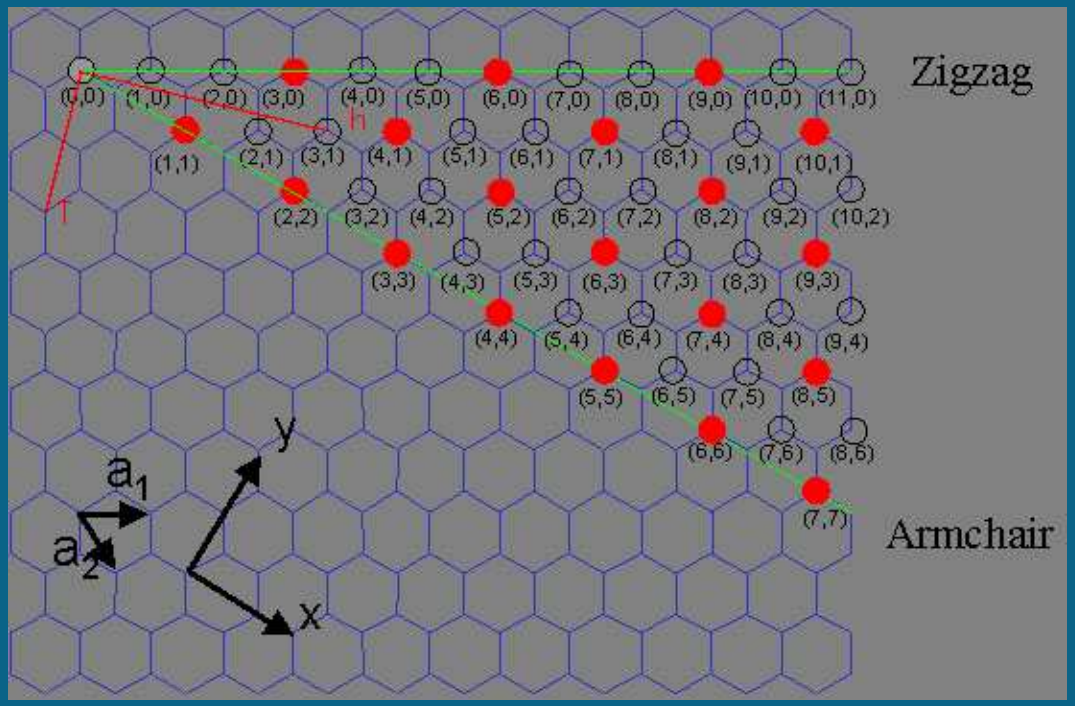
$$H_0 = v\psi^{\dagger}_{\alpha\mathbf{k}} \left(\alpha q_x \sigma_x + q_y \sigma_y\right) \psi_{\alpha\mathbf{k}}$$

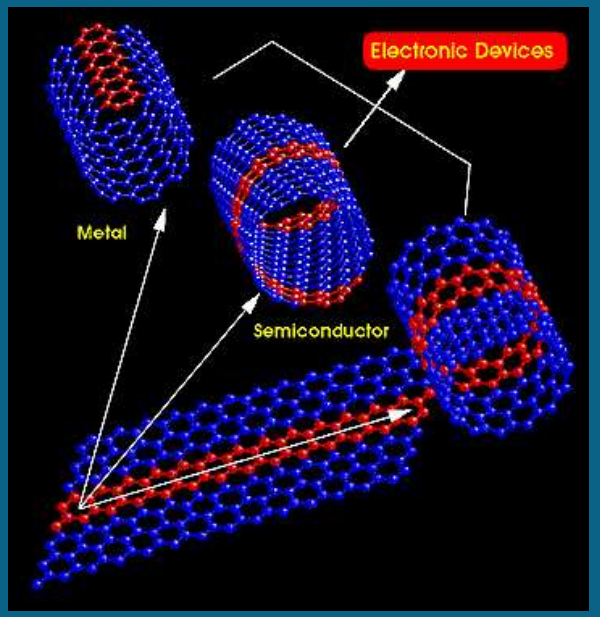
with

$$v = 3t_0 a_0/2, \qquad \psi^{\dagger}_{\alpha\mathbf{k}} = (c^{\dagger}_{A\mathbf{k}},\ c^{\dagger}_{B\mathbf{k}}), \qquad q_y = \mu G_1$$

σ_x, σ_y Pauli Matrices

Structure of nanotubes (n, m) I





$\mathbf{h} = n\mathbf{a}_1 + m\mathbf{a}_2$ Chiral (wrapping) vector for a tube (n,m)

Structure of nanotubes (n, m) II

- Unit lattice vectors ($a_0 = 1.42\,\text{Å}$)

$$\mathbf{a}_1 = \sqrt{3}a_0(1/2, \sqrt{3}/2), \quad \mathbf{a}_2 = \sqrt{3}a_0(-1/2, \sqrt{3}/2)$$

- Chiral (wrapping) vector

$$\mathbf{h} = n\mathbf{a}_1 + m\mathbf{a}_2, \quad |\mathbf{h}| = \sqrt{3}a_0\mathcal{L}, \qquad \mathcal{L} = \sqrt{n^2 + m^2 + nm}$$

- The unit vector $\mathbf{T}$ along the tube axis

$$\mathbf{T} = \big((2n + m)\mathbf{a}_1 - (2m + n)\mathbf{a}_2\big)/d_R, \qquad T = 3a_0\mathcal{L}/d_R$$

d_R is the greatest common divisor of $(2n + m)$ and $(2m + n)$.

$$T = \sqrt{3}a_0, \quad \text{armchair } (m, n), \qquad T = 3a_0 \quad \text{zigzag } (0, m)$$

Structure of nanotubes (n, m) III

- Number of hexagons in the nanotube unit cell

$$N = \frac{2(n^2 + m^2 + nm)}{d_R} = \frac{2\mathcal{L}^2}{d_R}$$

$N = 2m$ armchair (m, m), $N = 2m$ zigzag $(0, m)$

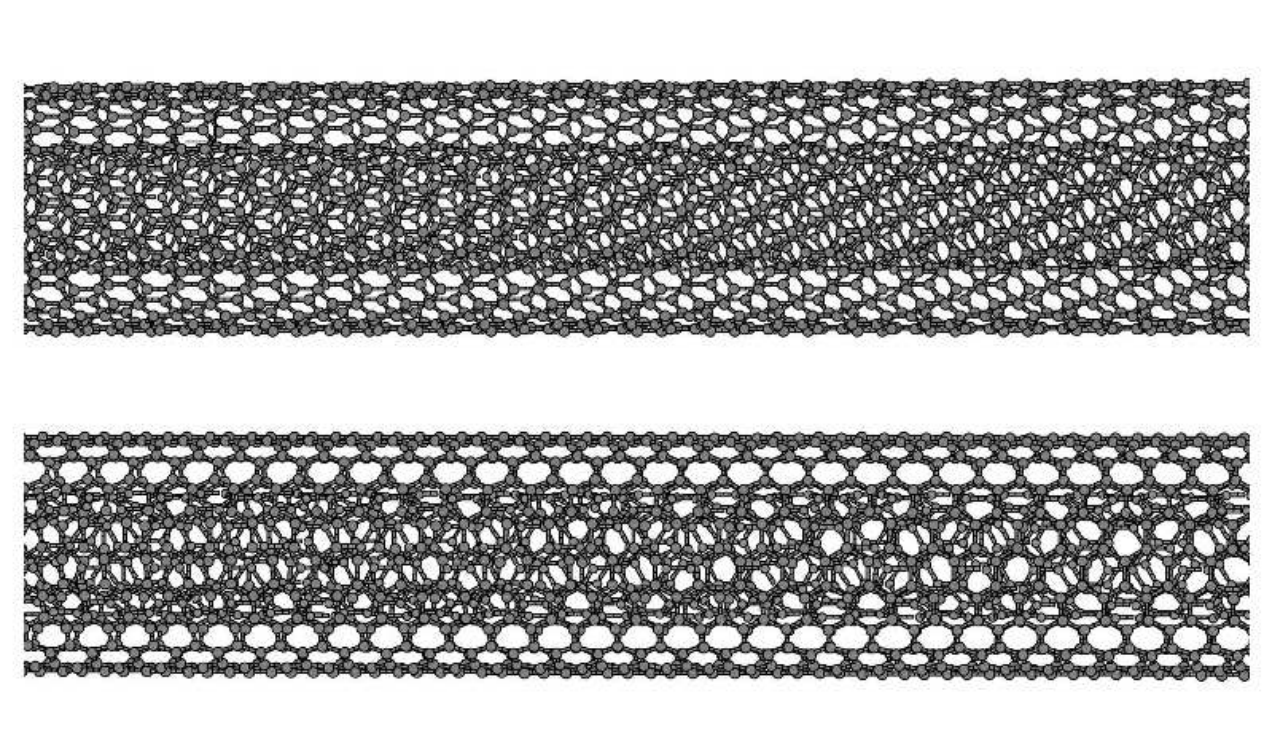
- Discrete unit wave vector along $\mathbf{h}$:

$$\mathbf{G}_1 = \frac{(2n + m)\mathbf{b}_1 + (2m + n)\mathbf{b}_2}{N d_R}, \qquad |\mathbf{G}_1| = \frac{2\pi}{h}$$

- Unit reciprocal lattice vector along $\mathbf{T}$

$$\mathbf{G}_2 = \frac{(m\mathbf{b}_1 - n\mathbf{b}_2)}{N}, \qquad |\mathbf{G}_2| = \frac{2\pi}{T}$$

Structure of double wall nanotubes (DWNT)



$(9,0)@(18,0)$ (up) and $(9,0)@(10,10)$ (down)

*S. Roche et al., Phys. Lett. A **285**, 94 (2001)

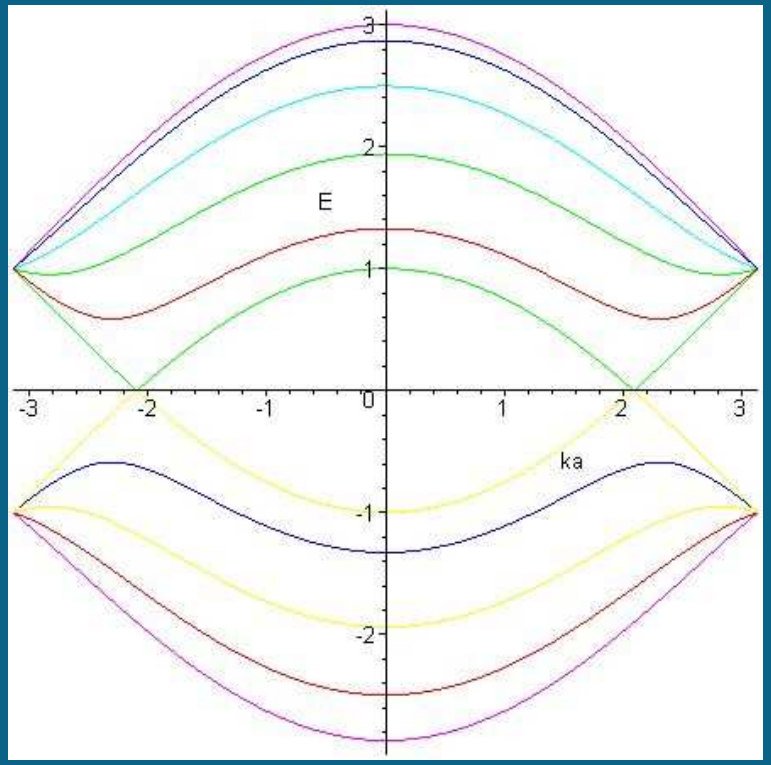
Electronic structure of single wall nanotube I

$$E_\mu(k) = E_{2D}(\mathbf{k})$$

$$\mathbf{k} = k\mathbf{G}_2/G_2 + \mu\mathbf{G}_1$$

$$\begin{cases} -\pi/T < k < \pi/T, \\ \mu = 1, 2, \ldots, N \end{cases}$$

N Number of subbands



$(5, 5)$ nanotube

Electronic structure of single wall nanotube II

Condition for metallic tubes

$$\mathbf{G}_1 \cdot \mathbf{K}_0 / G_1 = \text{Integer}$$

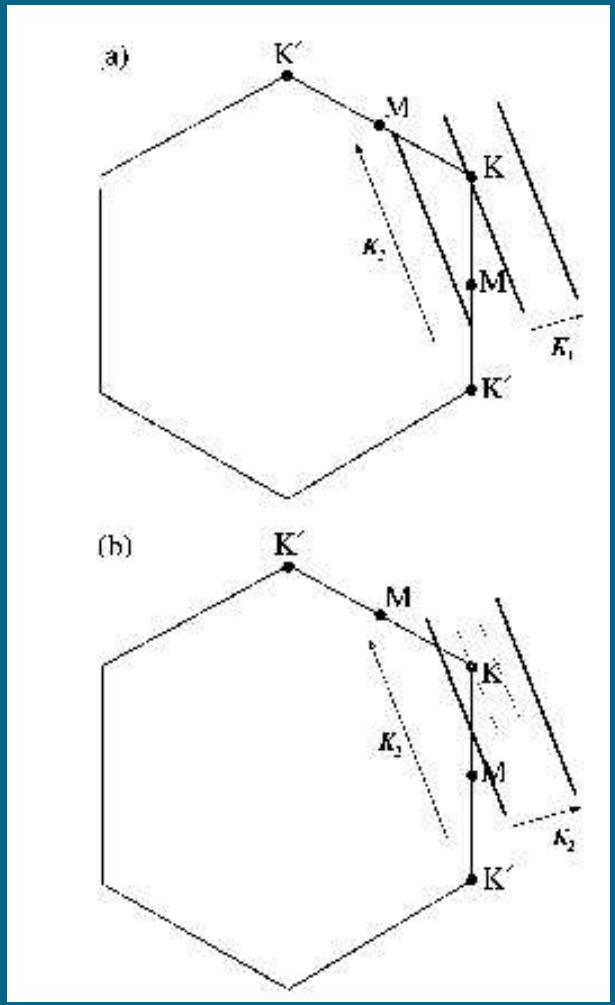
$$\frac{\mathbf{G}_1 \cdot \mathbf{K}_0}{G_1} = \frac{n-m}{3},$$

$$\frac{\mathbf{G}_1 \cdot \mathbf{K}_1}{G_1} = \frac{2m+n}{3},$$

$$\frac{\mathbf{G}_1 \cdot \mathbf{K}_{-1}}{G_1} = -\frac{2n+m}{3}$$

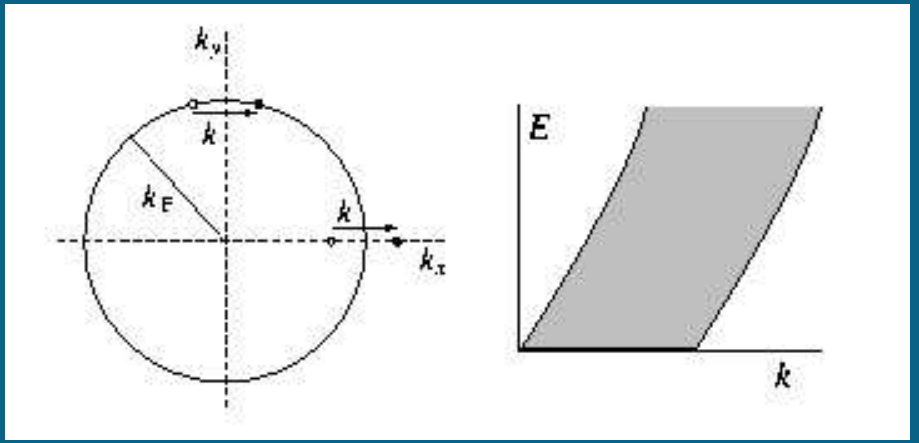
Armchair nanotubes (m, m) are always metallic!

Coulomb interaction, electronic structure?

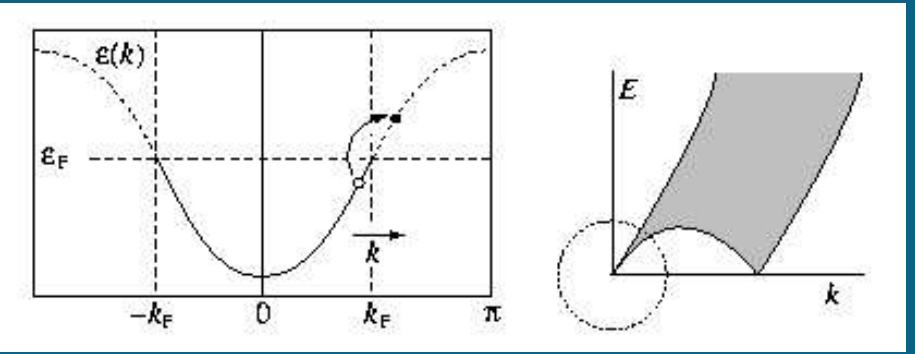


Why is 1D system special

- Fermi liquid
  - ⋆ Quasi particles
  - ⋆ Fermi surface



- Luttinger liquid
  - ⋆ No quasi particles
  - ⋆ No Fermi surface
  - ⋆ Tomonaga-Luttinger model



Tomonaga-Luttinger(TL) model

- It only describes low energy excitation near the Fermi energy E_F,

$$E(k) \approx E_F + \frac{\hbar^2 k_F}{m}(k - k_F).$$

- The dispersion relation becomes linearized

$$E(k) = \hbar v_F (k - k_F), \qquad v_F = \hbar \frac{k_F}{m}$$

- Right- and left-movers

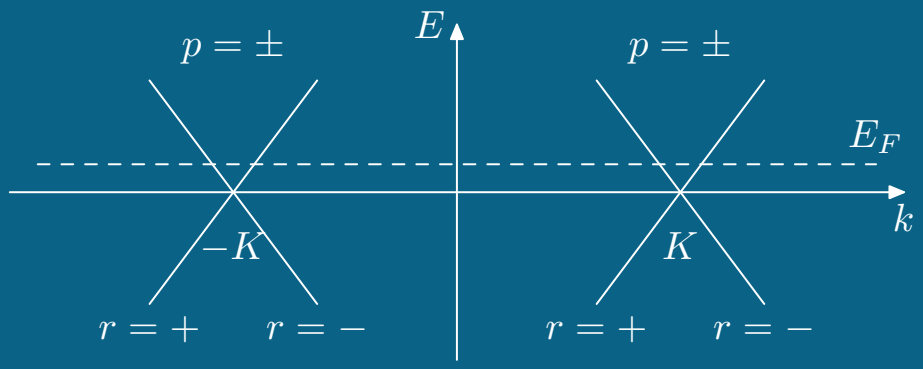
$$c_k = c_{+,k}\Theta(k) + c_{-,k}\Theta(-k)$$

- It is exactly solvable by bosonization technique!

Single wall nanotubes as Luttinger liquids I

The basis ($\alpha = \pm$)

$$\phi_{p\alpha}(x,y) = \frac{1}{\sqrt{2\pi R}}\, e^{-i\alpha K\cdot \mathbf{r}}$$



$$\phi_{p\alpha}(x,y)\phi_{-p\alpha'}(x,y) = 0$$

The Fermi field operator

$$\Psi_\sigma(x,y) = \sum_{p\alpha} \phi_{p\alpha}(x,y)\psi_{p\alpha\sigma}(x),$$

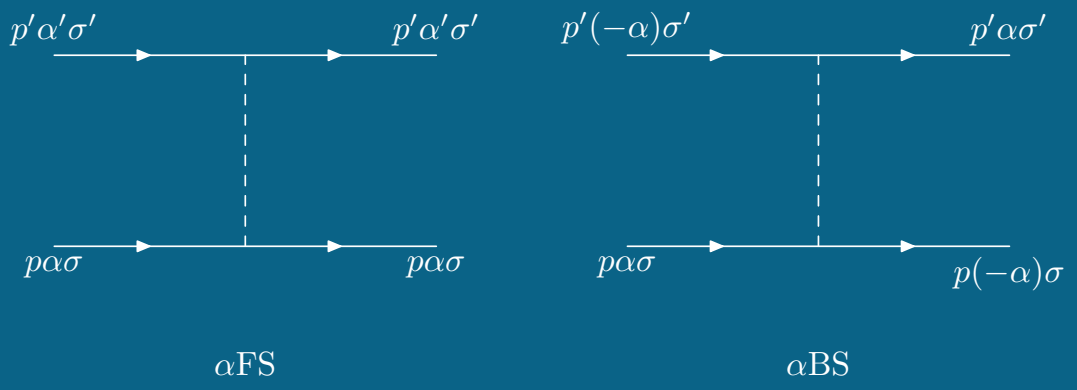
$$\psi_{r\alpha\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\psi_{p\alpha\sigma} \Leftrightarrow \psi_{p\alpha\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\psi_{r\alpha\sigma}.$$

$$H = -\hbar v \sum_{p\alpha\sigma} p \int dx\, \psi^\dagger_{p\alpha\sigma}\partial_x \psi_{-p\alpha\sigma} \quad \Rightarrow \quad H = -i\hbar v \sum_{r\alpha\sigma} r \int dx\, \psi^\dagger_{r\alpha\sigma}\partial_x \psi_{r\alpha\sigma}$$

Single wall nanotubes as Luttinger liquids II

The Coulomb interaction

$$H_I = \frac{1}{2}\sum_{\sigma\sigma'}\int d\mathbf{r}d\mathbf{r}'\Psi^\dagger_\sigma(\mathbf{r})\Psi^\dagger_{\sigma'}(\mathbf{r}')U(\mathbf{r}-\mathbf{r}')\Psi_{\sigma'}(\mathbf{r}')\Psi_\sigma(\mathbf{r}).$$



$$H^{(0)}_{\alpha\mathrm{FS}} = \frac{1}{2}\int dxdx'\ \rho(x)V_0(x-x')\rho(x')$$

Forward scattering

*R. Egger and A. O. Gogolin, Eur. Phys. J. B **3**, 281 (1998)

Single wall nanotubes as Luttinger liquids III

The density operators

$$\rho(x) = \sum_{r\alpha\sigma} \psi^{\dagger}_{r\alpha\sigma}(x)\psi_{r\alpha\sigma}(x) = \sum_{r\alpha\sigma} \rho_{r\alpha\sigma}(x)$$

The Fermi field operator

$$\psi_{r\alpha\sigma}(x) = \frac{\eta_{r\alpha\sigma}}{\sqrt{2\pi a}} \exp(i\alpha k_F x + irq_F x + i\varphi_{r\alpha\sigma}(x))$$

with $\eta_{r\alpha\sigma}$ — Klein factor, a — cutoff, and

$$\varphi_{r\alpha\sigma} = \frac{\sqrt{\pi}}{2}(r\phi_{c+} + r\alpha\phi_{c-} + r\sigma\phi_{s+} + r\alpha\sigma\phi_{s-} + \Theta_{c+} + \alpha\Theta_{c-} + \sigma\Theta_{s+} + \alpha\sigma\Theta_{s-})$$

Hamiltonian with interactions can be easily diagonalized by these Bosonic operators!

Single wall nanotubes as Luttinger liquids IV

New boson operators

$$\phi_{c+} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\varphi_{r\alpha\sigma}, \qquad \Theta_{c+} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \varphi_{r\alpha\sigma}, \qquad \text{total charge}$$

$$\phi_{c-} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\alpha\varphi_{r\alpha\sigma}, \qquad \Theta_{c-} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \alpha\varphi_{r\alpha\sigma}, \qquad \text{relative charge}$$

$$\phi_{s+} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\sigma\varphi_{r\alpha\sigma}, \qquad \Theta_{s+} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \sigma\varphi_{r\alpha\sigma}, \qquad \text{total spin}$$

$$\phi_{s-} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\alpha\sigma\varphi_{r\alpha\sigma}, \qquad \Theta_{s-} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \alpha\sigma\varphi_{r\alpha\sigma}, \qquad \text{relative spin}$$

The commutators of them are

$$[\phi_{j\delta}(x), \Theta_{j'\delta'}(x')] = i\frac{\pi}{2}\delta(x-x')\mathrm{sgn}(x-x')\delta_{j\delta,j'\delta'}.$$

Single wall nanotubes as Luttinger liquids V

Kinetic Hamiltonian

$$H_0 = \sum_{j\delta} \frac{\hbar v^0_{j\delta}}{2} \int dx \left(K^0_{j\delta} \left(\partial_x \Theta_{j\delta}(x)\right)^2 + \frac{1}{K^0_{j\delta}} \left(\partial_x \phi_{j\delta}(x)\right)^2 \right),$$

with $j = c, s$, $\delta = \pm$, $v^0_{j\delta} = v = v_F$ and $K^0_{j\delta} = 1$.

Interaction Hamiltonian (only affect $c+$ mode)

$$H^{(0)}_{\alpha \mathrm{FS}} = \frac{2}{\pi} \int dx dx' \, \rho(x) V(x - x') \rho(x')$$

$$= \int \frac{dk}{2\pi} \frac{2k^2 V(k)}{\pi} e^{-ikx} \phi_{c+}(k) \phi_{c+}(-k).$$

Forward scattering

with $\rho(x) = \frac{2}{\sqrt{\pi}} \partial_x \phi_{c+}$.

Single wall nanotubes as Luttinger liquid VI

The total Hamiltonian

$$H_0 + H^{(0)}_{\alpha\mathrm{FS}} = \sum_{j\delta} \int \frac{dk}{2\pi} \frac{\hbar v_{j\delta}}{2} \left(K_{j\delta} \Pi_{j\delta}(k) \Pi_{j\delta}(-k) + \frac{k^2}{K_{j\delta}} \phi_{j\delta}(k) \phi_{j\delta}(-k) \right),$$

with

$$v_{c+}(k) = v\sqrt{1 + \frac{4V(k)}{\pi v}}, \quad K_{c+} = \sqrt{\frac{1}{1 + 4V(k)/\pi v}},$$

$$v_{j\delta} = v, \qquad K_{j\delta} = 1 \qquad \text{others}$$

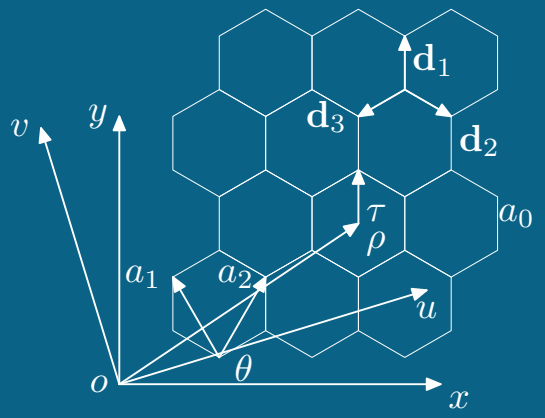
The Hamiltonian can be diagonalized by Bogliugov transformation. The energy spectrum of each mode is

$$E_{j\delta}(k) = \hbar v_{j\delta} k$$

Interlayer coupling in DWNTs I

Tunneling Hamiltonian

$$H_t = \frac{1}{\sqrt{N_a N_b}} \sum_{ab} \sum_{\mathbf{k}_a \mathbf{k}_b} t_{ai,bj} e^{i(\mathbf{k}_b \cdot \mathbf{r}_b - \mathbf{k}_a \cdot \mathbf{r}_a)} c^{\dagger}_{i\eta_a \mathbf{k}_a} c_{j\eta_b \mathbf{k}_b} + \text{H.c.},$$



a, b — indices for atoms, i, j — indices for two walls, N_a, N_b — numbers of unit cell.

The position of a lattice site $\mathbf{r}_i = \mathbf{R} + \boldsymbol{\rho} + \eta\boldsymbol{\tau}$, where R — lattice vector.

$$\frac{1}{A_{\text{cell}}} \sum_{\mathbf{G}} e^{-i\mathbf{G}\cdot\mathbf{R}} = \delta(R) \quad \Rightarrow \quad \sum_{\mathbf{R}} = \int d\mathbf{r}\delta(\mathbf{R}) = \frac{1}{A_{\text{cell}}} \sum_{\mathbf{G}} \int d\mathbf{r} e^{-i\mathbf{G}\cdot\mathbf{R}}$$

G — reciprocal lattice vector, A_{cell} — area of an unit cell

Interlayer coupling in DWNTs II

Expand around Fermi points $\mathbf{k} = \alpha\mathbf{K_0} + \mathbf{q}$, the Hamiltonian becomes

$$H_t = \sum_{\alpha_a\alpha_b}\sum_{\eta_a\eta_b}\sum_{\mathbf{q}_a} \psi^{\dagger}_{\alpha_a\mathbf{q}_a}\mathbf{q}_b T(\alpha_a\eta_a\mathbf{q}_a, \alpha_b\eta_b\mathbf{q}_b)\psi_{\alpha_b\mathbf{q}_b},$$

with $\psi^{\dagger}_{i\alpha\mathbf{q}} = (c^{\dagger}_{iA\mathbf{k}},\ c^{\dagger}_{iB\mathbf{k}})$, $\eta = \pm$ — indices for sublattices.

$$T(\alpha_a\eta_a\mathbf{q}_a, \alpha_b\eta_b\mathbf{q}_b) = \sum_{\mathbf{K}_a\mathbf{K}_b} e^{i\mathbf{K}_a\cdot(\boldsymbol{\rho}_a+\eta_a\boldsymbol{\tau}_a)-i\mathbf{K}_b\cdot(\boldsymbol{\rho}_b+\eta_b\boldsymbol{\tau}_b)} t_{\mathbf{q}_a+\mathbf{K}_a,\mathbf{q}_b+\mathbf{K}_b}$$

and

$$t_{\mathbf{k}_a,\mathbf{k}_b} = \frac{1}{A^2_{\text{cell}}\sqrt{N_aN_b}}\int d\mathbf{r}_a d\mathbf{r}_b e^{i(\mathbf{k}_b\cdot\mathbf{r}_b-\mathbf{k}_a\cdot\mathbf{r}_a)} t_{ai,bj}$$

*A. A. Maarouf et al., PRB **61**, 11156 (2000)

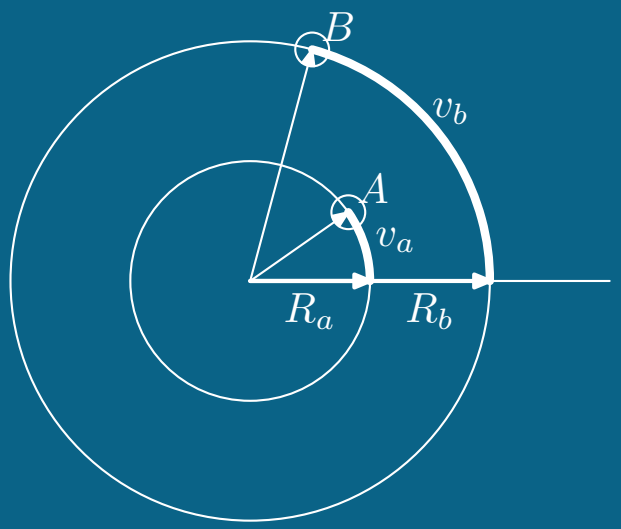
Interlayer coupling in DWNTs III

The tunneling matrix elements

$$t_{ij} = t_0 e^{-d_{ij}/a_t},$$

$$A = (R_a \cos v_a/R_a, R_a \sin v_a/R_a, u_a),$$

$$B = (R_b \cos v_b/R_b, R_b \sin v_b/R_b, u_b)$$



with $a_t \sim 0.5\,\text{Å}$ ($a_0 = 1.421\,\text{Å}$)

$$d_{ij} = \sqrt{(R_a - R_b)^2 + 4R_aR_b \sin^2\left(\frac{v_a}{2R_a} - \frac{v_b}{2R_b}\right) + (u_a - u_b)^2}$$

$$\approx \Delta + \frac{R_aR_b}{2\Delta}\left(\frac{v_a}{R_a} - \frac{v_b}{R_b}\right)^2 + \frac{(u_a - u_b)^2}{2\Delta},$$

with $\Delta = |R_a - R_b| \sim 3.4\,\text{Å}$

Interlayer coupling in DWNTs IV

Then the tunneling matrix element

$$t_{ai,bj} = t_0 \exp\left(-\frac{\Delta}{a_t} + \frac{R_a R_b}{2\Delta a_t}\left(\frac{v_a}{R_a} - \frac{v_b}{R_b}\right)^2 + \frac{(u_a - u_b)^2}{2\Delta a_t}\right).$$

In k-space

$$t_{\mathbf{k}_x,\mathbf{k}_y} = t_G \exp\left(\frac{-\Delta a_t}{8R_aR_b}\left(k_{va}R_a + k_{vb}R_b\right)^2\right)\exp\left(+\frac{da_t}{2}|K_0|^2\right)$$
$$\cdot \exp\left(\frac{-\Delta a_t(k_{ua} + k_{ub})^2}{8}\right)\delta(k_{va}R_a - k_{vb}R_b)\delta(k_{ua} - k_{ub})$$

with

$$t_G = C\exp\left(-\frac{d}{a_t}\right)\exp\left(-\frac{da_t}{2}|K_0|^2\right) \sim 0.1\,\mathrm{eV} \qquad d = 3.4\,\text{Å}$$

Electronic structure of DWNTs

Two δ functions

$$\delta(k_{va}R_a - k_{vb}R_b)\delta(k_{ua} - k_{ub}) \quad \Rightarrow \quad k_{va}R_a - k_{vb}R_b = 0, \quad k_{ua} - k_{ub} = 0$$

and

$$k_{va}R_a = \ell_a, \quad k_{vb}R_b = \ell_b, \qquad \ell_a, \ell_b = \text{Integer} \qquad \Rightarrow \qquad \ell_a = \ell_b$$

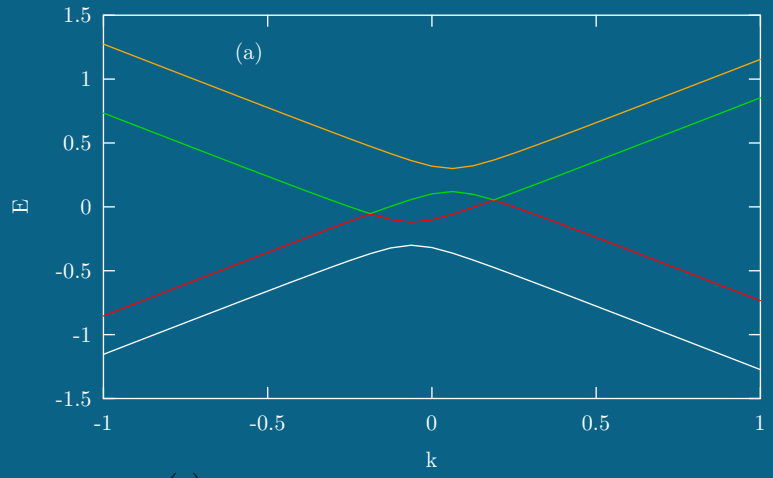
DWNTs with two armchair walls

$$\ell_a = \ell_b = 0 \qquad \Rightarrow \qquad t_{\mathbf{k}_i,\mathbf{k}_j} \neq 0$$

Other DWNTs

$$\ell_a \neq \ell_b \qquad \Rightarrow \qquad t_{\mathbf{k}_i,\mathbf{k}_j} = 0$$

Electronic structure of DWNTs with two armchair walls



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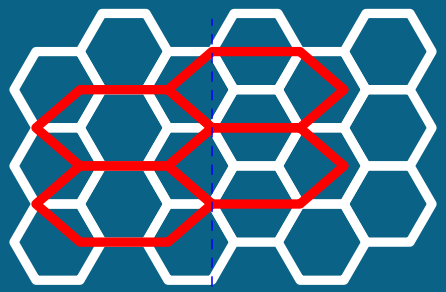
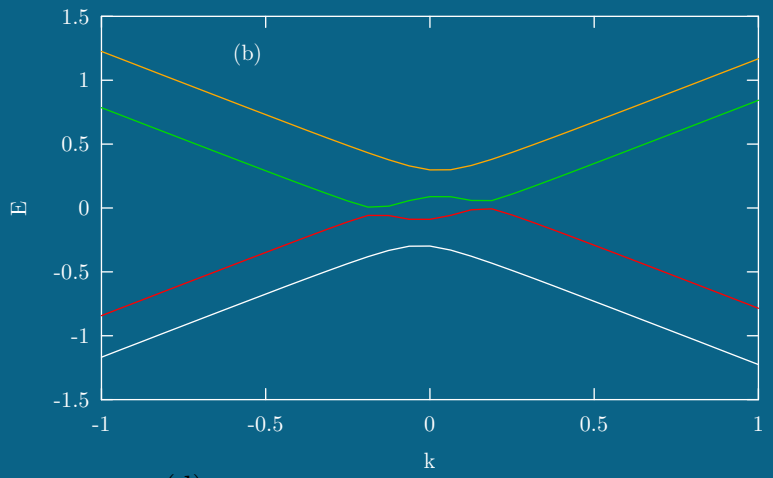
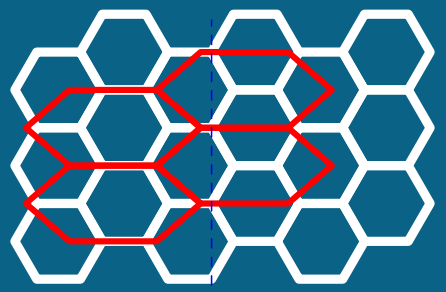


$\rho_a = \tau/2$ and $\rho_b = -\tau/2$



(d)



$\rho_a = \tau/2$ and $\rho_b = \tau/8 - \tau/2$

DWNTs as Luttinger liquid I

16 new Bosonic operators (i index for two walls)

$$\phi_{c+i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\varphi_{r\alpha\sigma i}, \qquad \Theta_{c+i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \varphi_{r\alpha\sigma i},$$

$$\phi_{c-i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\alpha\varphi_{r\alpha\sigma i}, \qquad \Theta_{c-i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \alpha\varphi_{r\alpha\sigma i},$$

$$\phi_{s+i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\sigma\varphi_{r\alpha\sigma i}, \qquad \Theta_{s+i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \sigma\varphi_{r\alpha\sigma i},$$

$$\phi_{s-i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} r\alpha\sigma\varphi_{r\alpha\sigma i}, \qquad \Theta_{s-i} = \frac{1}{4\sqrt{\pi}} \sum_{r\alpha\sigma} \alpha\sigma\varphi_{r\alpha\sigma i}$$

The field operator

$$\varphi_{r\alpha\sigma i} = \frac{\sqrt{\pi}}{2}(r\phi_{c+i} + r\alpha\phi_{c-i} + r\sigma\phi_{s+i} + r\alpha\sigma\phi_{s-i} + \Theta_{c+i} + \alpha\Theta_{c-i} + \sigma\Theta_{s+i} + \alpha\sigma\Theta_{s-i})$$

DWNTs as Luttinger liquids II

Coulomb interaction is

$$H_I = \frac{1}{2}\sum_{\sigma\sigma'}\int d\mathbf{r}d\mathbf{r}'\Psi^\dagger_\sigma(\mathbf{r})\Psi^\dagger_{\sigma'}(\mathbf{r}')U(\mathbf{r}-\mathbf{r}')\Psi_{\sigma'}(\mathbf{r}')\Psi_\sigma(\mathbf{r}).$$

Forward scattering in a wall

$$H^{(0)}_{\alpha\mathrm{FS}} = \sum_i \frac{1}{2}\int dxdx'\ \rho_i(x)V_i(x-x')\rho_i(x'),$$

Forward scattering between walls

$$H'^{(0)}_{\alpha\mathrm{FS}} = \frac{1}{2}\int dxdx'\ \rho_1(x)V_{12}(x-x')\rho_2(x'),$$

DWNTs as Luttinger liquids III

The Hamiltonian is

$$H_0 + H^{(0)}_{\alpha\mathrm{FS}} + H'^{(0)}_{\alpha\mathrm{FS}} = \sum_{j\delta i} \int \frac{dk}{2\pi} \frac{\hbar v_{j\delta i}}{2} \left(\Pi_{j\delta i}(k)\Pi_{j\delta i}(-k) + k^2 \phi_{j\delta i}(k)\phi_{j\delta i}(-k) \right)$$

$$+ \sum_{il} \int \frac{dk}{2\pi} \frac{4k^2 V_{il}(k)}{\pi} \phi_{c+i}(k)\phi_{c+l}(-k)$$

Coulomb interaction only affect $(c+\pm)$ modes.

$$\frac{\hbar v'_{c+\pm} K'_{c+\pm}}{2} = \frac{\hbar v}{2}, \quad \frac{\hbar v'_{c+\pm}}{2K'_{c+\pm}} = \frac{\hbar v}{2} \left(1 + \frac{2(V_1+V_2)}{\hbar\pi v} \pm \sqrt{\left(\frac{2(V_1-V_2)}{\hbar\pi v} \right)^2 + \left(\frac{4V_{12}^2}{\hbar\pi v} \right)^2} \right)$$

$$v_{j\delta i} = v, \qquad K_{j\delta i} = 1, \qquad \text{Other modes}$$

Double wall nanotubes are Luttinger liquids!

Tunneling density of states

Tunneling density of states (TDOS) is

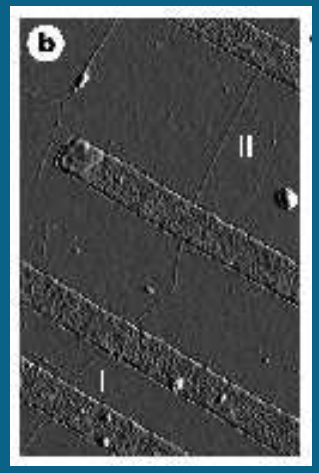
$$\rho(\epsilon, x) = \frac{1}{2\pi\hbar} dt\, e^{-i\epsilon t/\hbar} \langle \psi^\dagger(x,t)\psi(x,0)\rangle$$

$$\rho(\epsilon) \sim (kT)^\alpha \cosh\left(\frac{\epsilon}{2kT}\right) \left|\Gamma\left(\frac{1}{2}(1+\alpha) + i\frac{\epsilon}{2\pi kT}\right)\right|^2$$

$$\rho_{\rm end}(\epsilon) \sim \epsilon^{\alpha_{end}}, \qquad \rho_{\rm bulk}(\epsilon) \sim \epsilon^{\alpha_{bulk}} \qquad T \to 0$$

The current is given as

$$I \propto \int_0^V d\epsilon\, \rho_1(\epsilon)\rho_2(\epsilon) \quad \Rightarrow \quad \frac{dI}{dV} \propto \rho_1(\epsilon)\rho_2(\epsilon)$$



Tunneling density of state of DWNT

The exponents of a DWNT — only out wall is connected to leads

$$\alpha_{\mathrm{end,out}} = \frac{1}{4}\sin^2\xi\left(\frac{1}{K'_{c++}} - 1\right) + \frac{1}{4}\cos^2\xi\left(\frac{1}{K'_{c+-}} - 1\right),$$

$$\alpha_{\mathrm{bulk,out}} = \frac{1}{8}\sin^2\xi\left(K'_{c++} + \frac{1}{K'_{c++}} - 2\right) + \frac{1}{8}\cos^2\xi\left(K'_{c+-} + \frac{1}{K'_{c+-}} - 2\right),$$

For examples

$$\alpha_{\mathrm{end,out}} = 1.033, \qquad \alpha_{\mathrm{bulk,out}} = 0.429 \qquad (5,5)@(10,10)$$

$$\alpha_{\mathrm{end,out}} = 1.254, \qquad \alpha_{\mathrm{bulk,out}} = 0.523 \qquad (10,10)$$

Summary and outlook

- No interlayer coupling in DWNTs except those with two armchair walls
- Most DWNTs can be described as two Luttinger liquids coupled by Coulomb interaction
- The conductance has power law dependence on applied voltage. Exponents decrease in DWNTs.

To be continued

- DWNTs with interlayer tunneling, disorder, . . .