Probing a Single Quantum Dot & Negative Differential Capacitance

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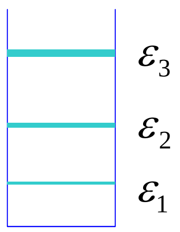
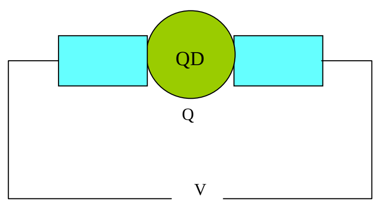
Two Issues



Probing a single quantum dot



$$Q(V) = ?$$

Outline

- **Introduction I**
  - **Why nano-structures? Similarity between QD & atoms; Optical and transport measurements**
- **Photon-Assistant Tunnelling Measurement**
- **Analysis and Results**
- **Introduction II**
  - **Negative differential capacitance**
  - **Ways of applying bias**
- **Model of a QD Connected to Two Leads**
- **Analysis and Results**
  - **Existence of negative differential capacitance**
- **Summary**

Why 'nano'?

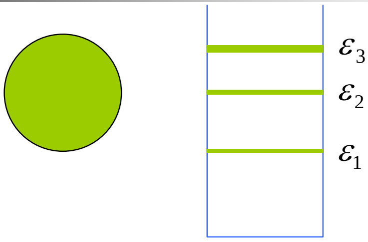
Trivial reasons

- Space
- Time
- Energy

Non-trivial reasons

- Quantum devices
- Quantum information processing
  - Quantum computing
  - Quantum cryptography
  - Quantum error correction
  - Quantum teleportation

How to Probe a Quantum Dot?



- Similar to atoms
- Difficult to make two identical QD
- Strong e-e interaction

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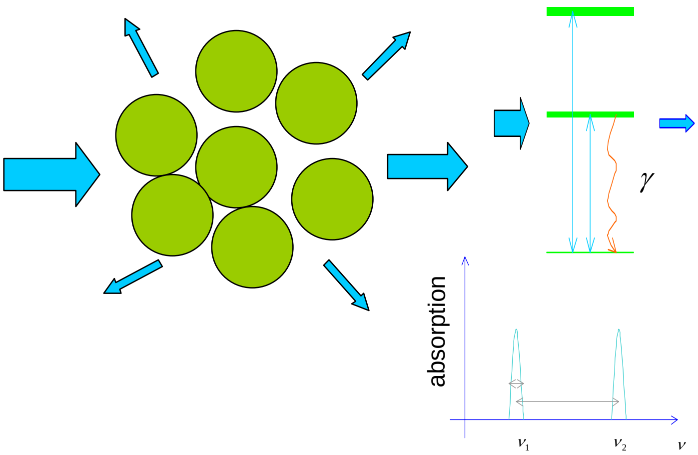
Optical Measurement



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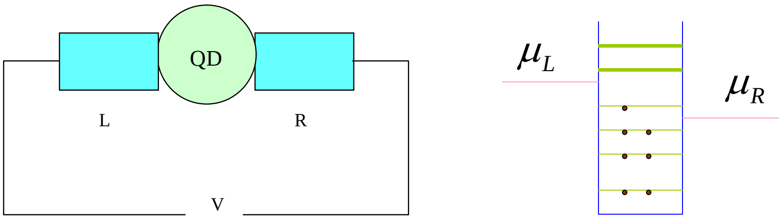
Transport Measurement

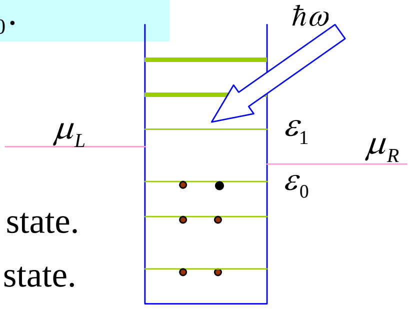


N-electron

Photon-assistant Tunneling

Assumption : Strong e - e interaction,

$$e^2/C >> \varepsilon_1 - \varepsilon_0.$$



ε_0 : N - electron ground state.

ε_1 : N - electron excited state.

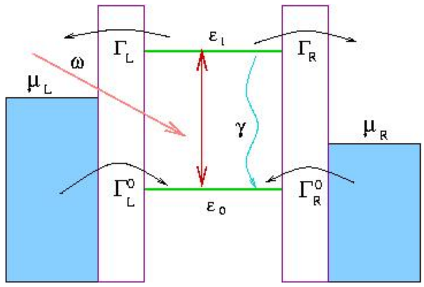
Model

$$H = H_0 + H_l + H_t$$

$$H_0 = \varepsilon_0 |0\rangle\langle 0| + \varepsilon_1 |1\rangle\langle 1|$$

$$H_t = \sum_{\alpha,i} (\Gamma_{\alpha,i} c^{\alpha,i} |E\rangle\langle i| + \text{c.c})$$

$$H_l = -\frac{\hbar}{2}\Omega e^{-i\omega t}|0\rangle\langle 1| + \text{H.c}$$



α : Label of leads; $|i>$: N - electron states

$|E\rangle : (N-1)$ – electron state; Ω is Rabi frequency

Approach – Equations of Motion

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}(H_{eff}\rho - \rho H_{eff}^{+}) + \gamma \sigma_- \rho \sigma_+$$

$$\sigma_+ = (\sigma_-)^+ = |i\rangle\langle j|$$

$$H_{eff} = H - \frac{i\hbar\gamma}{2}\sigma_+\sigma_-$$

$$\Gamma_0 = \Gamma_L^0 + \Gamma_R^0$$

$$\Gamma = \Gamma_L + \Gamma_R$$

$$\dot{\rho}_E = -\Gamma_0 \rho_E + \Gamma \rho_{11}$$

$$\dot{\rho}_{11} = -(\gamma + \Gamma)\rho_{11} + \text{Im}(\Omega \tilde{\rho}_{10})$$

$$\dot{\tilde{\rho}}_{10} = (i\delta - \frac{\gamma + \Gamma}{2})\tilde{\rho}_{10} + \frac{i}{2}\Omega(\rho_{00} - \rho_{11})$$

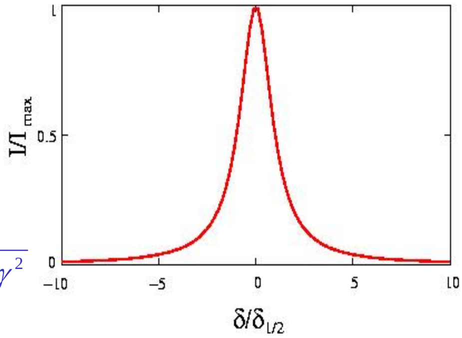
$$\dot{\rho}_{00} = \gamma\rho_{11} + \Gamma_0 \rho_E - \text{Im}(\Omega \tilde{\rho}_{10})$$

$$\rho_{11} + \rho_{00} + \rho_E = 1$$

$$\tilde{\rho}_{10} = \rho_{10} e^{i(\varepsilon_1 - \varepsilon_0)t/\hbar}$$

Monochromatic Light

$$I = I_{\max} \frac{\delta_{1/2}^2}{\delta_{1/2}^2 + \delta^2}$$



$$\delta_{1/2} = \frac{1}{2}\sqrt{\Gamma^2 + 2\gamma\Gamma + \Omega^2\Gamma/\Gamma_0 + 2\Omega^2 + \gamma^2}$$

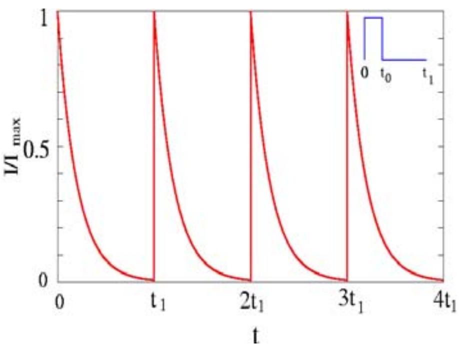
$$I_{\max} = \frac{(-e)\Omega^2[\Gamma_0(\Gamma_L - \Gamma_R) - \Gamma(\Gamma_L^0 - \Gamma_R^0)]}{\Gamma^2\Gamma_0 + 2\gamma\Gamma\Gamma_0 + \Omega^2\Gamma + 2\Omega^2\Gamma_0 + \gamma^2\Gamma_0}$$

Pulse

$$I = I_{\max} e^{-(\gamma+\Gamma)t}$$

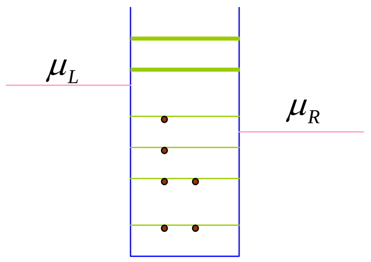
$$I_{\max} = \frac{(-e)\cos^2(\frac{\Omega t_0}{2})}{\Gamma_0 - \gamma - \Gamma}((\Gamma_0 - \gamma - \Gamma)$$

$$(\Gamma_L - \Gamma_R) - \Gamma(\Gamma_L^0 - \Gamma_L^0))$$



$$t_0 << \frac{1}{\gamma}, \frac{1}{\Gamma} << t_1$$

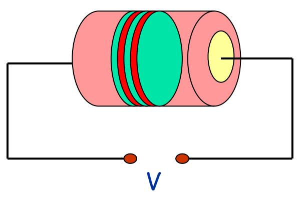
Transport Current



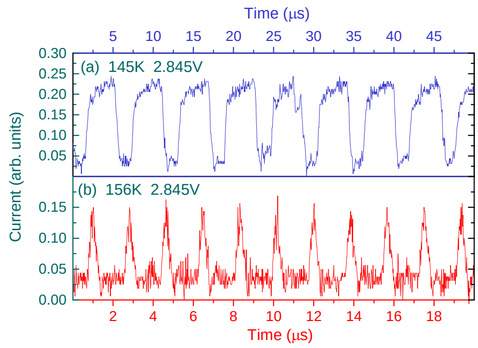
$$I = \frac{2e}{\hbar}\frac{\Gamma_L^0\Gamma_R^0}{\Gamma_L^0+\Gamma_R^0}$$

N-electron ground state

Why Negative Differential Capacitance?

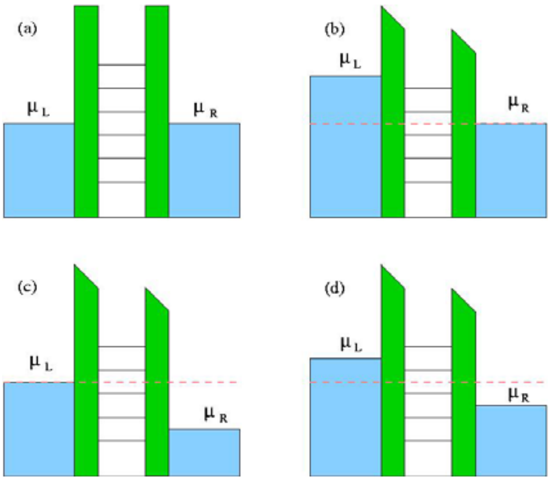


Like negative differential resistance, negative differential capacitance can also cause the Instability.



self-sustained current oscillation in superlattice caused by NDR

Is Negative Differential Capacitance Possible?



Three ways of applying a bias

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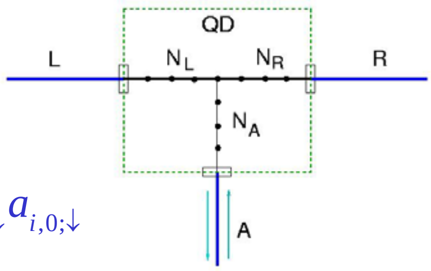
Model

$$H = H_0 + \sum_{i=-N_L}^{N_R} U a^+_{i,0;\uparrow} a_{i,0;\uparrow} a^+_{i,0;\downarrow} a_{i,0;\downarrow}$$

$$+ \sum_{m=1}^{N_A} U a^+_{0,m;\uparrow} a_{0,m;\uparrow} a^+_{0,m;\downarrow} a_{0,m,\downarrow}$$



Auxiliary lead

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Hartree-Fock Approximation

$$H_0^{eff} = \sum_{i,m,\sigma} \varepsilon_{i,m;\sigma}^{eff} a_{i,m;\sigma}^{+} a_{i,m;\sigma} + \sum_{\langle i,j\rangle\langle m,n\rangle,\sigma} t_{i,j,m,n} a_{i,m;\sigma}^{+} a_{j,n;\sigma}$$

$$+ \text{c.c},$$

$$\varepsilon_{i,m;\sigma}^{eff} = \varepsilon_{i,m;\sigma} + U\langle n_{i,m;\bar{\sigma}}\rangle \quad \sigma = -\bar{\sigma},$$

$$\langle n_{i,m;\sigma}\rangle = -\frac{1}{\pi}\int_{-\infty}^{\mu_A} \text{Im}\langle i,m;\sigma|G^{eff}(E)|i,m;\sigma\rangle dE$$

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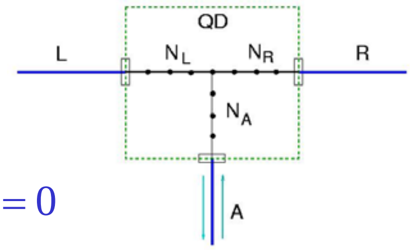
Calculation of Current

$$I_A = -\frac{2e}{h}\int_{\mu_A}^{\mu_L} T_{A\leftarrow L}(E)dE$$

$$+\frac{2e}{h}\int_{\mu_R}^{\mu_A} T_{R\leftarrow A}(E)dE = 0$$



$$I_{R\leftarrow L} = -\frac{2e}{h}\left(\int_{\mu_R}^{\mu_L} T_{R\leftarrow L}(E)dE + \int_{\mu_R}^{\mu_A} T_{R\leftarrow A}(E)dE\right)$$

$$Q = \sum_{\sigma,i=-N_L}^{N_R} e\langle n_{i,0;\sigma}\rangle + \sum_{\sigma;m=1}^{N_A} e\langle n_{0,m;\sigma}\rangle$$

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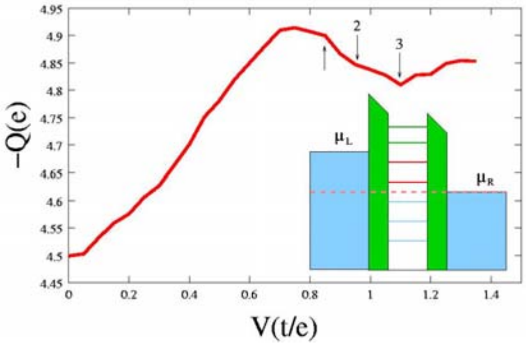
Numerical Evidence of NDC



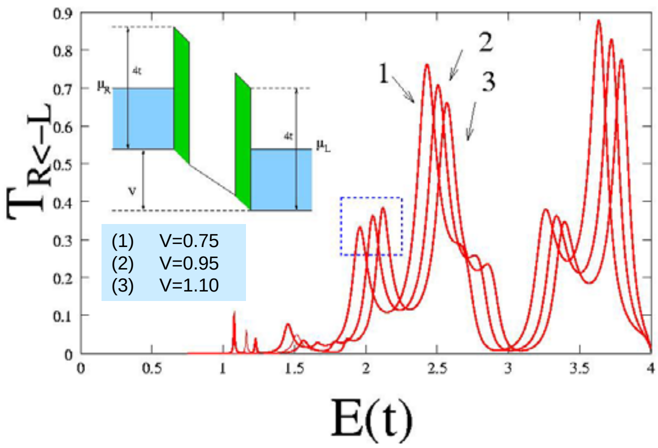
$N_L = N_R = 4, N_A = 3;$
$E_F = U = 2t;$
$U_L = V, U_R = U_A = 0;$
$b_L = b_R = b_A = 4t.$

Empty levels: $\varepsilon > \mu_L$

Conducting Channels: $\mu_R < \varepsilon < \mu_L$

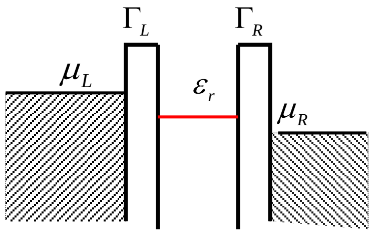
Fully occupied levels: $\varepsilon < \mu_R$

Transmission Coefficients



Why Does It Happen?

$$T = 0$$



$$I_{L\to D} = \frac{2e}{\hbar}\Gamma_L f_L(1 - f_r)$$

$$I_{D\to R} = \frac{2e}{\hbar}\Gamma_R f_r(1 - f_R)$$

$$I_{L\to D} = I_{D\to R}$$

$$Q = ef_r$$

$$\Rightarrow \quad Q_r = \frac{e\Gamma_L}{\Gamma_L + \Gamma_R} = \frac{e}{1 + \Gamma_R/\Gamma_L}$$

$dQ_r/dV < 0$ if Γ_R/Γ_L increases with bias V

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Summary

- Combining the microwave pumping and electron tunneling measurement, one can obtain the spontaneous emission rate of a single electron level of a quantum dot.
- Present numerical evidences of an NDC in a QD.
- NDC is cause by the bias dependence of the electron tunneling rates in a weakly coupled QD.

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