

# Quantum Transport Methods and Algorithms July 05-08, 2022



# **DFTBepy: A DFTB-based Approach for Electron-Phonon Coupling Calculations**

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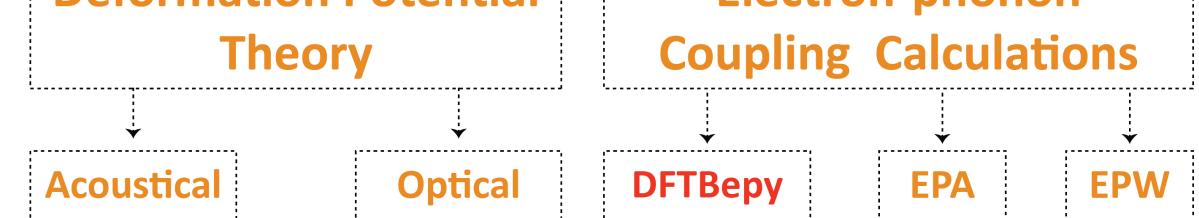
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#### Motivation

<b>Relaxation Tim</b>	e Estimation
$\checkmark$	•
<b>Deformation Potential</b>	Electron-phonon

Herein, we present a method for calculating the electron-phonon coupling matrix within the DFTB approach and demonstrate its implementation based on DFTB+ and phonopy. Exemplarily, we show results for  $\gamma$ -graphyne which was recently synthesized [Yiming Hu, Chenyu Wu, Qingyan Pan *et al* 2022 *Nat. Synth*]. Consistent with earlier predictions we obtain relaxation times on the order of 10<sup>-14</sup> s at room temperature.



## Electron-phonon Coupling

The deformed lattice is described in terms of the displacements  $\vec{u}_{ls}$ , as  $\vec{R} \rightarrow \vec{R} + \vec{u}_{ls}$ . The change in the Hamilton matrix elements and the overlap matrix elements can be defined by:

$$\delta H(\ell s, \ell' s') = \frac{\partial H(\ell s - \ell' s')}{\partial \vec{R}} \cdot (\vec{u}_{\ell s} - \vec{u}_{\ell' s'}) , \qquad (1)$$
  
$$\delta S(\ell s, \ell' s') = \frac{\partial S(\ell s - \ell' s')}{\partial \vec{R}} \cdot (\vec{u}_{\ell s} - \vec{u}_{\ell' s'}) , \qquad (2)$$

where the gradient is evaluated at the unperturbed positions. We first consider the two-center matrix elements:

$$H_{\mu,\nu}(\ell s, \ell' s') = \int dV \,\varphi_{\mu}^*(\vec{r} - \vec{R}_{\ell s}) \left[ T_e + V(\vec{r} - \vec{R}_{\ell s}) + V(\vec{r} - \vec{R}_{\ell' s'}) \right] \varphi_{\nu}(\vec{r} - \vec{R}_{\ell' s'}) \,. \tag{3}$$

## **Relaxation Time**

Here we use the self-energy relaxation time approximation (SERTA) [Samuel Poncé *et al* 2020 *Rep. Prog. Phys.* 83 036501] in which the scattering rate is defined as:

$$\tau_n^{-1}(\vec{k}) = \sum_m \int \frac{d^3q}{\Omega_{BZ}} \tau_{nm}^{-1}(\vec{k}, \vec{k} + \vec{q})$$

where  $\tau_{nm}^{-1}(\vec{k}, \vec{k} + \vec{q})$  is the partial decay rate which is given in terms of the electron-phonon coupling matrix g and the occupation factors of the involved states:

$$\tau_{nm}^{-1}(\vec{k},\vec{k}+\vec{q}) = \frac{2\pi}{\hbar} \sum_{\lambda} |g_{mn}^{\lambda}(\vec{k},\vec{q})|^2 \left[ \left( n_B(\omega_{\lambda}(\vec{q})) + 1 - f^0(\varepsilon_m(\vec{k}+\vec{q})) \right) \delta(\varepsilon_m(\vec{k}+\vec{q}) - \varepsilon_n(\vec{k}) + \hbar\omega_{\lambda}(\vec{q})) + \left( n_B(\omega_{\lambda}(\vec{q})) + f^0(\varepsilon_m(\vec{k}+\vec{q})) \right) \delta(\varepsilon_m(\vec{k}+\vec{q}) - \varepsilon_n(\vec{k}) - \hbar\omega_{\lambda}(\vec{q})) \right] .$$
(9)

Then the first order change of H and S are calculated by noting that the twocenter matrix elements depend on the distance vector and not the individual positions.

To find the electron-phonon coupling matrix, we consider the equation of motion of the electronic operator  $\hat{c}_v$  in the Heisenberg picture:

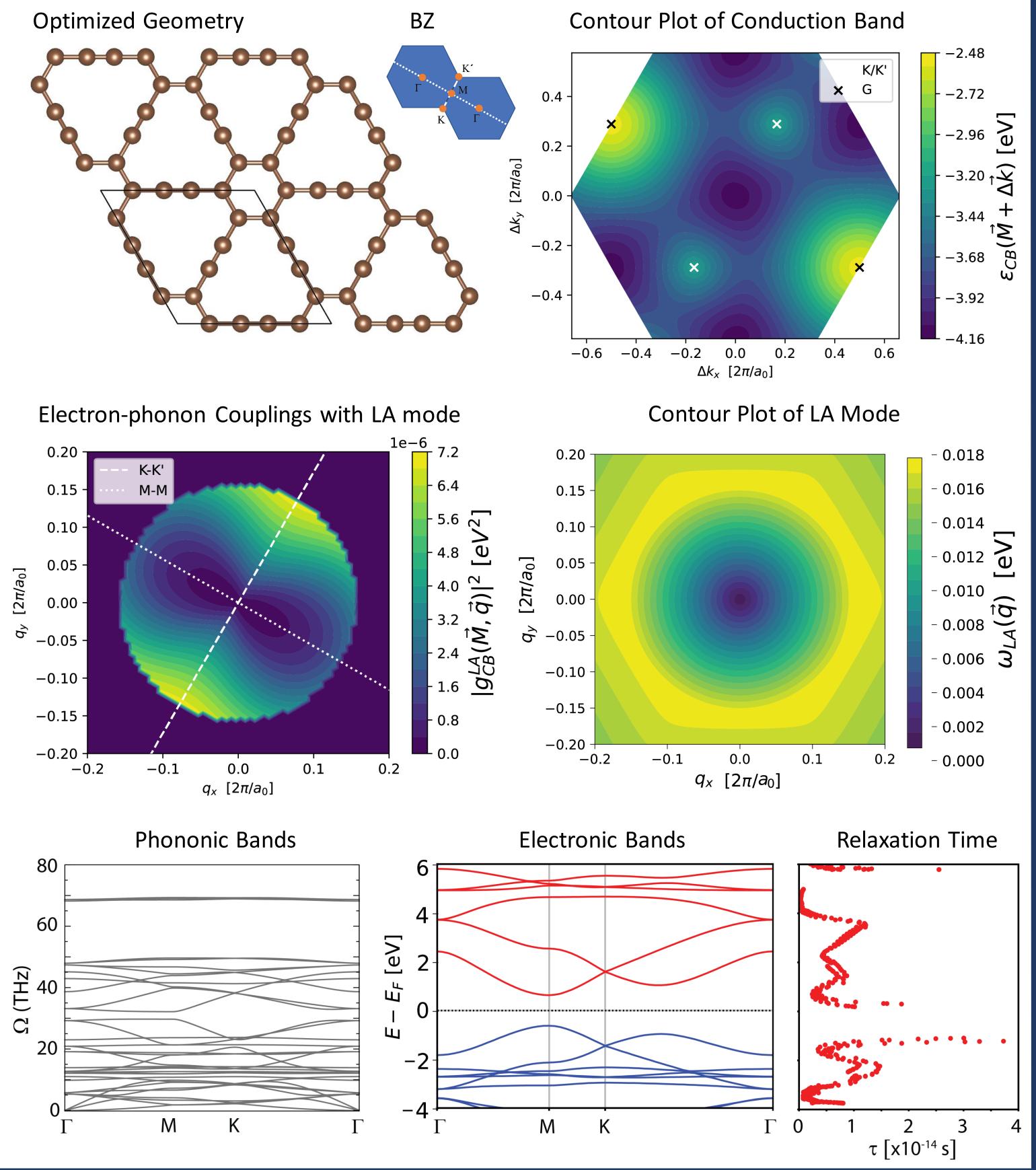
$$\hbar i \partial_t \hat{c}_{\nu} = [\hat{c}_{\nu}, \mathcal{H}] = [S^{-1}]_{\nu} H \hat{c} \longrightarrow \hbar i \partial_t \hat{c}_n(\vec{k'}) = \varepsilon_n(\vec{k'}) \hat{c}_n(\vec{k'})$$
For the unperturbed system
$$(4)$$

The displacements in terms of phonon creation and annihilation operators are described as usual:

$$\vec{u}_{\ell s} = \sum_{\lambda} \sum_{\vec{q}} e^{i\vec{q}\cdot\vec{R}_{\ell s}} \vec{\xi}_s^{\lambda}(\vec{q}) \sqrt{\frac{\hbar}{2N_c m_s \omega_{\lambda}(\vec{q})}} \left(\hat{b}_{\lambda}^{\dagger}(-\vec{q}) + \hat{b}_{\lambda}(\vec{q})\right)$$
(5)

where  $\xi_s^{\lambda}(\vec{q})$  denotes the polarization vector of atom *s* in phonon branch  $\lambda$  at wave-vector  $\vec{q}$ . Then the equation of motion of the perturbed system can be written as

#### Results



$$\hbar i \partial_t \hat{c}_n(\vec{k}') = \varepsilon_n(\vec{k}') \hat{c}_n(\vec{k}') + \sum_m \sum_{\vec{q};\lambda} g^{\lambda}_{nm}(\vec{k}' - \vec{q}, \vec{q}) \hat{c}_m(\vec{k}' - \vec{q}) \left( \hat{b}^{\dagger}_{\lambda}(-\vec{q}) + \hat{b}_{\lambda}(\vec{q}) \right)$$
(6)

where  $g_{nm}^{\lambda}$  is the electron-phonon coupling matrix:

$$g_{nm}^{\lambda}(\vec{k},\vec{q}) = \sqrt{\frac{\hbar}{2N_{c}\omega_{\lambda}(\vec{q})}} U^{*}(\vec{k}+\vec{q}) \left\{ \left[ \frac{\partial H}{\partial \vec{R}}(\vec{k}) - \frac{\partial S}{\partial \vec{R}}(\vec{k})\varepsilon_{m}(\vec{k}) \right] \cdot \frac{e^{i\vec{q}\cdot\vec{R}_{0}}\vec{\xi^{\lambda}}(\vec{q})}{\sqrt{m_{s}}} - \left[ \frac{\partial H}{\partial \vec{R}}(\vec{k}+\vec{q}) - \frac{\partial S}{\partial \vec{R}}(\vec{k}+\vec{q})\varepsilon_{m}(\vec{k}) \right] \cdot \frac{e^{i\vec{q}\cdot\vec{R}_{0}}\vec{\xi^{\lambda}}(\vec{q})}{\sqrt{m}} \right\} U(\vec{k})$$
(7)

